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Subject :- Discrete Structure

Dept :- computer science

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-: Q1 :-

Part (a)

Solution:-

a is first term

d is the common difference of the arithmetic sequence.

$a_n = a + (n-1)d$

$a_3 = a + (3-1)d$

$a_8 = a + (8-1)d$

Given that

$a_3 = 7$ and $a_8 = 17$

therefore

$7 = a + 2d \longrightarrow (1)$

and

$17 = a + 7d \longrightarrow (2)$

Subtracting (1) from (2)

$17 = \cancel{a} + 7d$

$-7 = -\cancel{a} \pm 2d$

$10 = 5d$

$10 = 5d$

$d = \frac{\cancel{10}^{2}}{\cancel{5}}$

$\boxed{d = 2}$

putting value of d in eq$_n$ ①

$7 = a + 2(2)$

$7 = a + 4$

$a = 7 - 4$

$\boxed{a = 3}$

Thus $a_n = a + (n-1)d$

$a_n = 3 + (n-1)2$ $\therefore a = 3 / d = 2$

$a_{36} = 3 + (36-1)2$

$= 3 + (35)2$

$= 3 + 70$

$a_{36} = 73$

The 36th term of arithmetic Sequence is (73)

-: Q2 :-

Solution :-

$f(x) = 2x + 3$ and $g(x) = -x^2 + 5$

$$fog(x) = f(g(x))$$

$$= f(-x^2+5)$$

$$= 2(-x^2+5)+3$$

$$= -2x^2+10+3$$

$$fog(x) = -2x^2+13$$

Now

$$gof(x) = g(f(x))$$

$$= g(2x+3)$$

$$= -(2x+3)^2+5$$

$$= -((2x)^2+(3)^2+2(2x)(3))+5$$

$$= -(4x^2+9+12x)+5$$

$$= -4x^2-12x-9+5$$

$$gof(x) = -4x^2-12x-4$$

-: Q3 :-

$$1^2 + 2^2 + 3^2 + L + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Solution:-

$$1^2 + 2^2 + 3^2 + L + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Basic Step:-

For $n = 1$

$$L.H.S = 1^2 = 1$$

$$R.H.S = \frac{1(1+1)(2(1)+1)}{6}$$

$$= \frac{1(2)(2+1)}{6}$$

$$= \frac{1(2)(3)}{6}$$

$$= \frac{6}{6}$$

$$R.H.S = 1$$

Hence the equation is true for

$n = 1$

Inductive Step:-

Suppose given equation is true for $n = k$, that is

$$1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6} \rightarrow (1)$$

Again suppose that given equation is true for $n = k+1$, that is

$$1^2 + 2^2 + 3^2 + \cdots + (k+1)^2 = 1^2 + 2^2 + 3^2 + \cdots + k^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

Take L.H.S & $(k+1)$ common

$$= (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right]$$

$$= (k+1)\left[\frac{k(2k+1) + 6(k+1)}{6}\right]$$

$$= (k+1)\left[\frac{2k^2 + k + 6k + 6}{6}\right]$$

$$= \frac{(k+1)(2k^2 + 7k + 6)}{6}$$

$$= \frac{(k+1)(2k^2 + 4k + 3k + 6)}{6}$$

$$= \frac{(k+1)(2k(k+2) + 3(k+2))}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

Hence by mathematical induction the given equation is true for all integers $n \geq 1$.

-: Q 4 :-

Discuss different types & relations with example in details.

Solution:-

⇒ Binary Relation:-
let A and B be sets. The binary relation R from A to B is a subset of $A \times B$

Example:-
Let $A = \{1, 2\}$, $B = \{1, 2, 3\}$
then $A \times B = \{(1,1), (1,2), (1,3), (2,1) (2,2) (2,3)\}$
let
$R_1 = \{(1,1), (1,3), (2,2)\}$

$R_2 = \{(1,2), (2,1), (2,2), (2,3)\}$

$R_3 = \{(1,1)\}$

⇒ Domain of a Relation :-

The domain of a relation R from A to B is the set of all first elements of the ordered pairs which belong to R denoted by Dom(R).

Smbolically $Dom(R) = \{ a \in A \mid (a,b) \in R \}$

⇒ Range of a Relation :-

The range of a relation R from A to B is the set of all second elements of the ordered pairs which belong to R denoted Ran(R).

Smbolically $Ran(R) = \{ b \in B \mid (a,b) \in R \}$

⇒ Relation on a set :-

A relation on the set A is a relation from A to A. In other words, a relation on set A is a subset of $A \times A$.

Let $A = \{1,2,3,4\}$ Define a relation R on A.

$A \times A = \{1,2,3,4\} \times \{1,2,3,4\}$

$A \times A = \{ (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3)$
$(2,4), (3,1), (3,2), (3,3), (3,4), (4,1)$
$(4,2), (4,3), (4,4) \}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

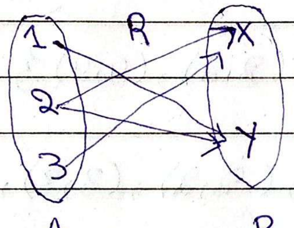
=) Arrow diagram of a Relation:-

let $A = \{1,2,3\}$

$B = \{x, y\}$ then

$R = \{(1,y), (2,x), (2,y), (3,x)\}$ be a relation from A to B

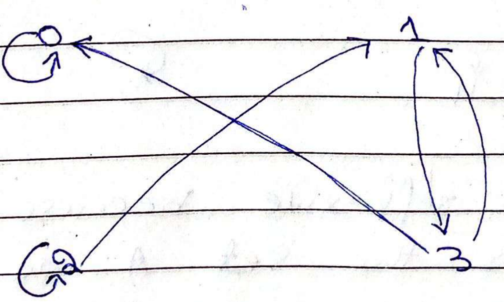
The arrow diagram of R is



=) Directed graph of a Relation:-

let

$A = \{0, 1, 2, 3\}$ and $R = \{(0,0), (1,3), (2,1), (2,2), (3,0), (3,1)\}$ be a binary relation on A



Reflexive relation :-

let R be a relation on a set A. R is reflexive if and only if, for all a ∈ A, (a,a) ∈ R That is, each element of A is related to itself.

Example :-

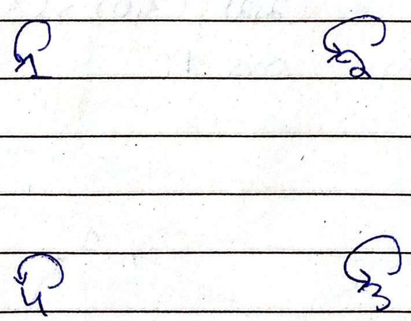
let A = {1, 2, 3, 4} and define relations R1, R2, R3, R4, on A as follows.

R1 = {(1,1), (3,3), (2,2), (4,4)}

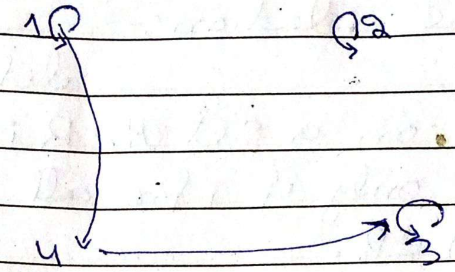
R2 = {(1,1), (1,4), (2,2), (3,3), (4,3)}

R3 = {(1,1), (1,2), (2,1) (2,2), (3,3), (4,4)}

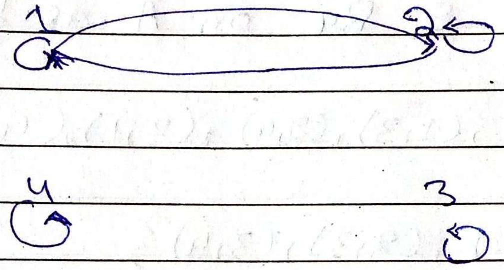
R4 = {(1,3), (2,2), (2,4), (3,1), (4,4)}



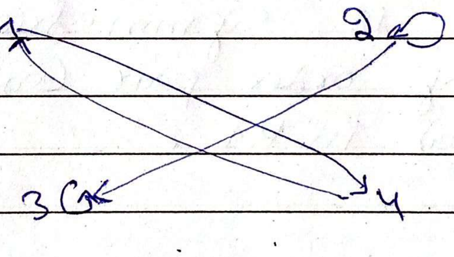
R1 is reflexive because at every point of the set A we have a loop in the graph.



R2 is not reflexive, as there is no loop at 4.



R3 is reflexive



R4 is not reflexive, as there are no loops at 1 and 3

⇒ Symmetric relation:-
Let R be a relation on a set A. R is symmetric if, and only if, for all $a,b \in A$, if $(a,b) \in R$ then $(b,a) \in R$.
R is not symmetric iff there are elements a and b in A such that $(a,b) \in R$ but $(b,a) \in R$.
Let $A = \{1,2,3,4\}$ and define relations R_1, R_2, R_3 & R_4 on A as follows.

$R_1 = \{(1,1),(1,3),(2,4),(3,1),(4,2)\}$

$R_2 = \{(2,2),(2,3),(3,4)\}$

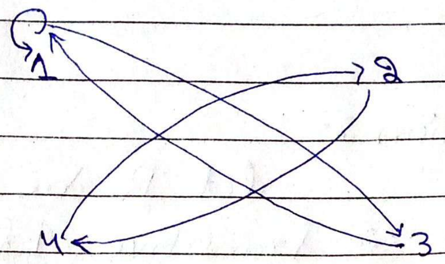
$R_3 = \{(1,1),(2,2),(3,3),(4,3),(4,4)\}$

Then R_1 is symmetric because for every order pair (a,b) in R_1 also have (b,a) in R_1.

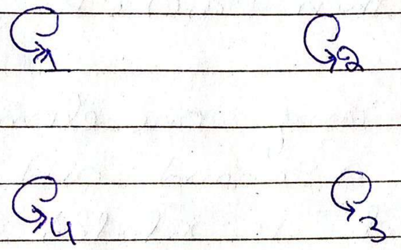
Example:-
We have $(1,3)$ in R_1 then we have $(3,1)$ in R_1. Similarly all other ordered pairs can be checked.

R_2 is not symmetric, because $(2,3) \in R_3$ but $(3,2) \notin R_3$.

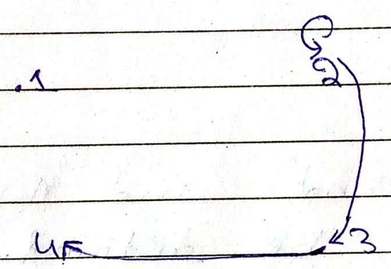
R_3 is not symmetric because $(4,3) \in R_4$ but $(3,4) \notin R_4$.



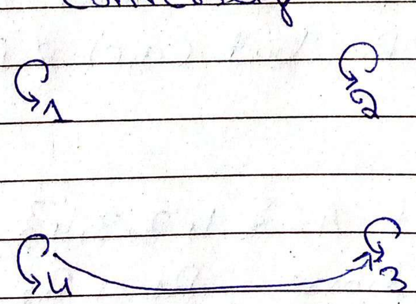
R_1 is symmetric



R_2 is symmetric



R_3 is not symmetric since there are arrows from 2 to 3 and from 3 to 4 but not conversely.



R_4 is not symmetric since there is an arrow from 4 to 3 but no arrow from 3 to 4.

Transitive relation :-

let R be a relation on a set A. R is transitive if and only if for all a, b, c ∈ A, if (a,b) ∈ R and (b,c) ∈ R then (a,c) ∈ R.

In words: if any one element is related to a second and that second element is related to a third then the first is related to the third

Note:

The "first" "second" and "third" element need not to be distinct.

R is not transitive iff there elements a, b, c in A such that if (a,b) ∈ R and (b,c) ∈ R but (a,c) ∉ R.

Example :-

let A = {1, 2, 3, 4} then define relations R1, R2, R3 on A

Sol:

$R_1 = \{(1,2), (2,3), (1,3)\}$

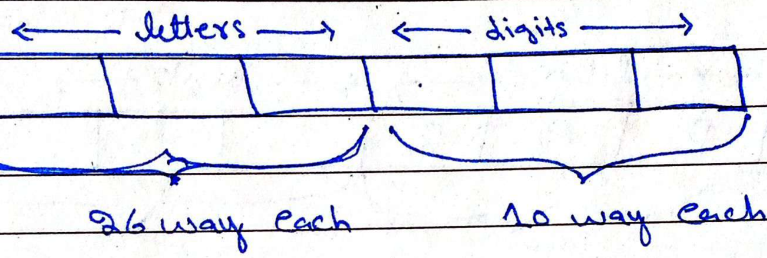
$R_2 = \{(1,2), (1,4), (2,3), (3,4)\}$

$R_3 = \{(2,1), (2,4), (2,3), (3,4)\}$

-: Q5 :-

Solution:-

(1) Each of the three letters can be written in 26 different ways, and each of the three digits can be written in 10 different ways.



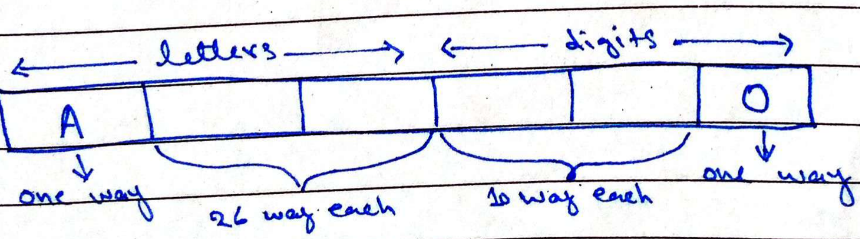
Hence by product rule, there is a total of

$$26 \times 26 \times 26 \times 10 \times 10 \times 10$$

$$= 17,576,000$$

different licence plates possible

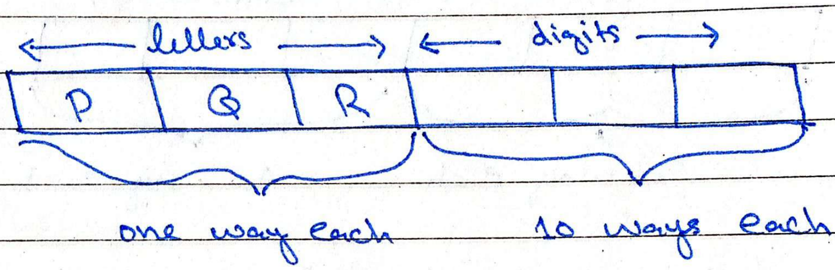
(2) The first and last place can be filled in one way only, while each of second and third place can be filled in 26 ways and each of fourth and fifth place can be filled in 10 ways.



Number of license plates that begin with A and end in 0 are

$1 \times 26 \times 26 \times 10 \times 10 = 67600$

(3) Number of licence plates that begin with PQR are



$1 \times 1 \times 1 \times 10 \times 10 \times 10 = 1000$

← end →