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Course Electromagnetic field (EMF)

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Date 22/06/2020

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Question (1)

State the difference and similarity between gradient and divergence Providing relevant example.

Gradient	Divergence.
-> Gradient is vector quantity.	-> Divergence is scaler quantity.
-> Gradient is applied on scaler quantity	-> Divergence is applied on vector quantity
-> Gradient of function (F) can be calculated by $Grad(F) = \nabla F = \frac{\partial F}{\partial x} i + \frac{\partial F}{\partial y} j + \frac{\partial F}{\partial z} k$	-> Divergence of function (F) can be calculated by $Div(\vec{F}) = \vec{\nabla} \cdot \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$
-> Gradient is a differential operator that operates on a scalar field	-> Divergence is a differential operator that operates on a vector field.

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→ The result of a gradient is a vector field.

→ The gradient is the vector that points towards the direction of greatest slope of the scalar field at each point.

Example The gradient of the distance from a given point is a vector field of unit length vectors pointing away from the given point.

→ The result of a divergence is a scalar field.

→ The divergence of a vector field is a scalar field that measure the net flow of the vector field at each given point in the space of said vector field.

Example: The divergence of a flow with no source or sink is 0. If there is a net source the divergence is possitive and if there is a net sink the divergence is negative.

Question (2)

find gradient of function F at point (1, 1, 2) for $F = x^3 + y^3 z$

Solution: We know that gradient of function is Calculated by

$$\text{Grad}(F) = \vec{\nabla} F = \frac{\partial F}{\partial x}\hat{i} + \frac{\partial F}{\partial y}\hat{j} + \frac{\partial F}{\partial z}\hat{k} \quad -(1)$$

put $F = x^3 + y^3 z$ in equation

$$\text{Grad}(F) = \frac{\partial}{\partial x}(x^3 + y^3 z)\hat{i} + \frac{\partial}{\partial y}(x^3 + y^3 z)\hat{j} + \left(\frac{\partial}{\partial z}(x^3 + y^3 z)\right)\hat{k}$$

Now apply partial defferential

So

$$\text{Grad}(F) = \vec{\nabla} F = (3x^2 + 0)\hat{i} + (0 + 3y^2 z)\hat{j} + (0 + y^3)\hat{k}$$

$$\text{Grad} F = \vec{\nabla} F = 3x^2\hat{i} + 3y^2 z\hat{j} + y^3\hat{k}$$

(4)

Now at point (1, 1, 2)

put $x = 1$, $y = 1$, and $z = 2$

So

$$\text{Grad}(F) = \vec{\nabla}F = 3x^2\,\hat{i} + 3y^2z\,\hat{j} + y^3\,\hat{k}$$

$$= 3(1)^2\,\hat{i} + 3(1)^2(2)\,\hat{j} + (1)^3\,\hat{k}$$

$$\text{Grad}(F) = \vec{\nabla}F = 3\hat{i} + 6\hat{j} + \hat{k}$$

Answer: Gradient of function (F) at point (1, 1, 2) is

$$\text{Grad}(F) = \vec{\nabla}F = 3\hat{i} + 6\hat{j} + \hat{k}$$

Question (3)

Compute div $(\vec{F})$ and curl for
$\vec{F} = x^2y\,\vec{i} - (z^3 - 3x)\,\vec{j} + 4y^2\,\vec{k}$

Solution we know that divergence of function can be calculated by

$$\text{Div}(\vec{F}) = \vec{\nabla}\cdot\vec{F} = \frac{\partial F}{\partial x}\cdot\vec{i} + \frac{\partial F}{\partial y}\cdot\vec{j} + \frac{\partial F}{\partial z}\cdot k$$

$$\text{Div}(\vec{F}) = \vec{\nabla}\cdot\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} \quad -(1)$$

put value of F_1, F_2 and F_3 in equation

$$\text{Div}(\vec{F}) = \vec{\nabla}\cdot\vec{F} = \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(-z^3+3x) + \frac{\partial}{\partial z}(4y^2)$$

Apply partial differential

so

$$= 2xy + 0 + 0$$

$$\text{Div}(\vec{F}) = \vec{\nabla}\cdot\vec{F} = 2xy$$

Now we know that curl of function $\vec{F}$ can be calculated as

$$\text{Curl}(\vec{F}) = \vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1 & F_2 & F_3 \end{vmatrix} \quad -(1)$$

Now put value in equation

$$\text{Curl}(\vec{F}) = \vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & -(z^3-3x) & 4y^2 \end{vmatrix}$$

$$= i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -(z^3-3x) & 4y^2 \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2 y & 4y^2 \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2 y & -(z^3-3x) \end{vmatrix}$$

$$= i\left(\partial/\partial y(4y^2) - \partial/\partial z(-z^3+3x)\right) - j\left(\partial/\partial x(4y^2) - \partial/\partial z(x^2 y)\right) + k$$

$$+k\left(\partial/\partial x(-z^3+3x) - \partial/\partial y(x^2 y)\right)$$

$$= i\left(8y - (-3z^2+0)\right) - j(0-0) + k\left((0+3) - x^2\right)$$

$$= i(8y+3z^2) - 0j + (3-x^2)k$$

So

$$\text{Curl}(\vec{F}) = \vec{\nabla} \times \vec{F} = (8y+3z^2)i - 0j + (3-x^2)k$$

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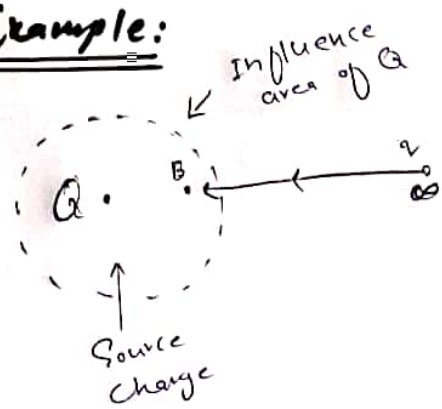
Question (4)

(7)

State the relationship between electric potential and potential difference with example.

★ **Electric potential:** An electric potential is the amount of work needed to move a unit of charge from a reference point to a specific point inside the field without producing an acceleration.

Example:



let this point is infinity and here a charge is placed suppose − q

Suppose A source charge is placed so it has some influence space around it where it can effect other charges around it.

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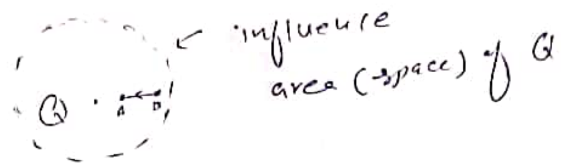
So if we have to bring one point charge which is placed outside influence area suppose placed at infinity, to inside space of source charge some work has to be done and that work is restored as a potential of that charge that potential is know as electric potential.

* Electric potential = work done / charge (q)

Potential Difference:

The difference in potential between two points that represents the work involved or the energy released in the transfer of a unit quantity of electricity from one point to another.



The figure show that the test charge is already placed inside influence area of source charge so it already got some amount of electric potential if test charge move between two point with in influence of source charge there will be differences in the potential across those two point so this is called potential difference.

* Potential difference = Work done / q

Where work is done to bring a charge q between two points within influence space of Q.

Question (5)

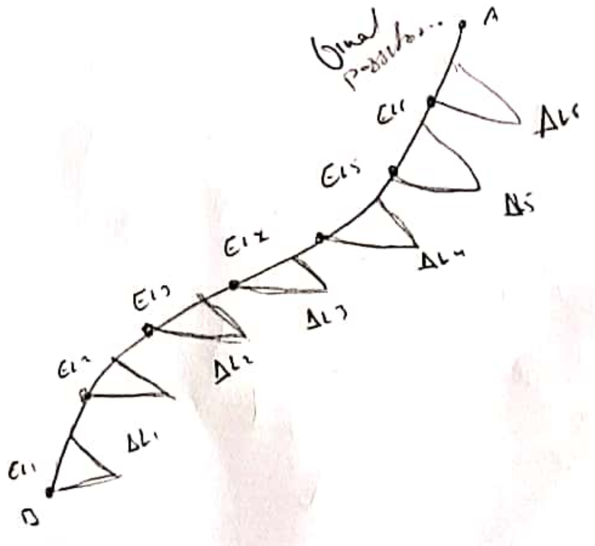
(10)

find the expression for moving a Point charge Q from one position to another by using line integral.

Answer: The integral expression for work done is for moving a point charge Q from one position to another is line integral. an example of which is vector analysis kelation always takes the form of the integral along Some prescibeel path of the Dot product.

A line integral is like many other integrals which appearing in advanced analysis the surface appearing in Gauss's law it tell us choose a path break it up into a large number of very small segments multiply the component of the field along each other the segment by the length of the segment and add the result of all segment.

The integral is obtained directly when the number of segment become infinite



The path is divided into six segment. $\Delta L_1, \Delta L_2 \ldots \ldots \Delta L_6$ And the component of E along each segment denoted by $E_{L1}, E_{L2} \ldots E_{L6}$. The work involved a charge Q from B to A is then approximately.

A graphical interpretation of a line integral is a uniform field. The line integral of E between point B and A is independent of the path selected.