NAME # SHAHKAR SALEEM

ID # 7943

SECTION # "B"

SUBJECT # MOS II

SEMESTER # 4

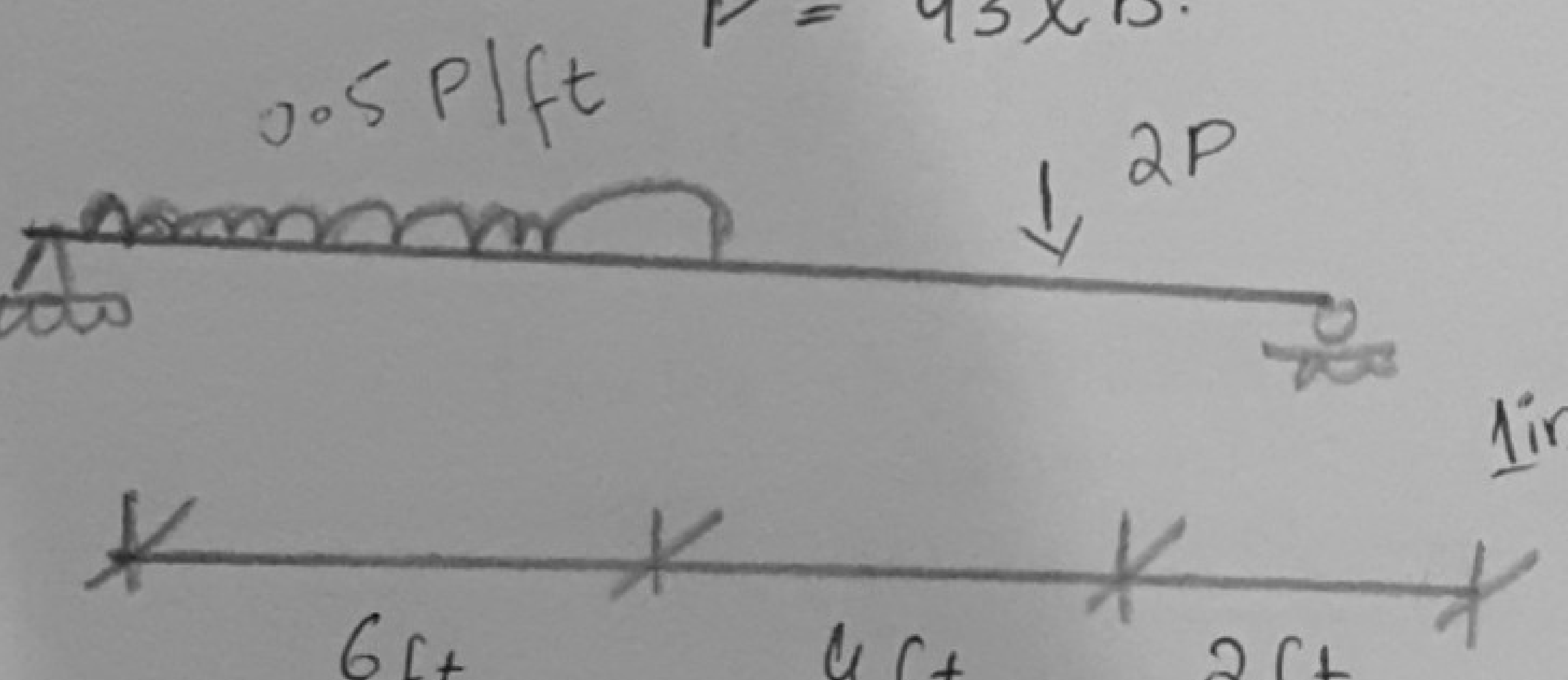
(1)

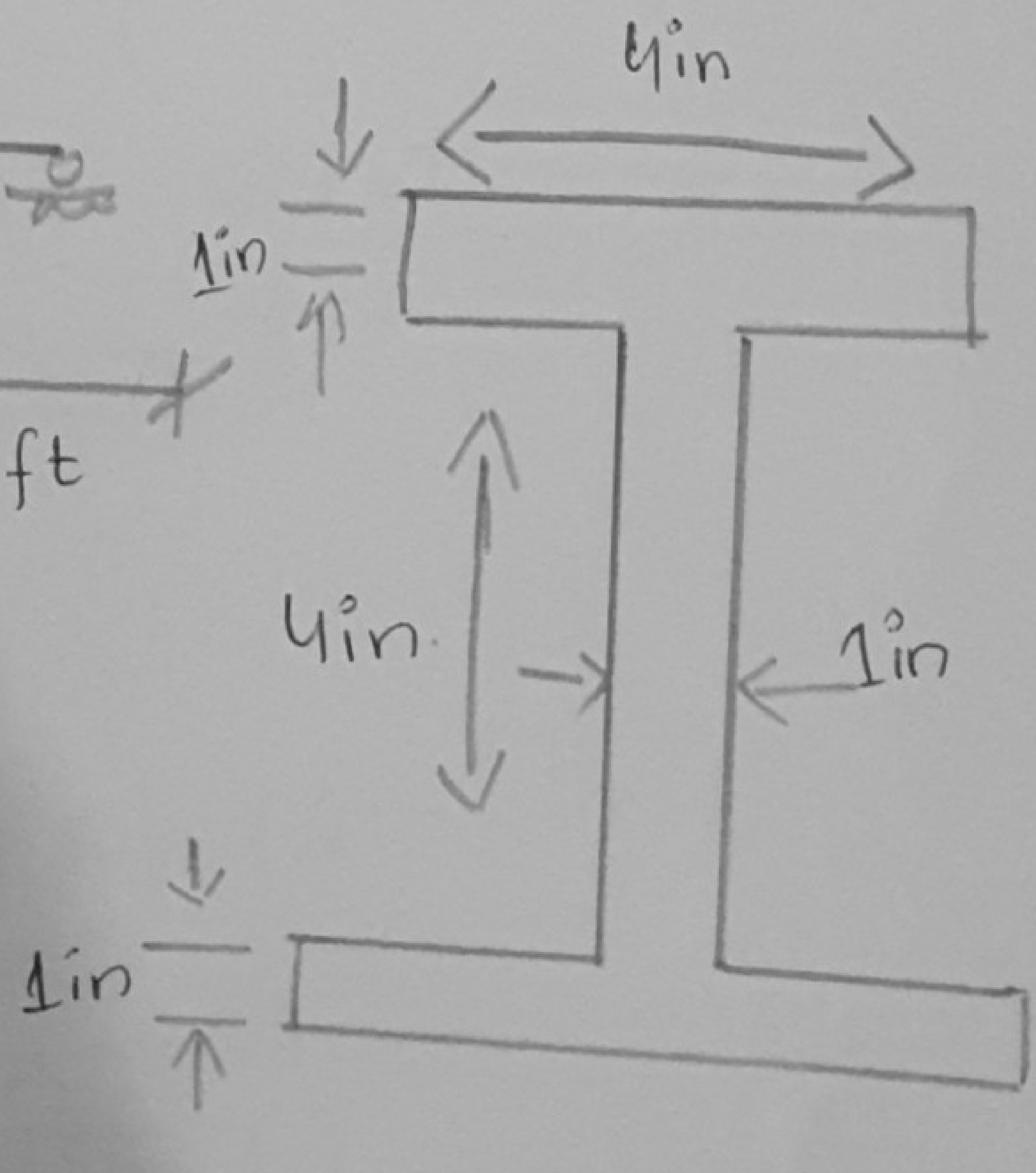
* Construct the morhr's Circle diagram
* find the principle stress
* Max in plane shear stress.

For a stress state of a point c located at the Center of uniformly distributed load and 1' below the top fiber of beam X- Section.

* Draw shear stress variation diagram.
* Draw flexural stress variation diagram.
* Compare the result obtained from the mohr's Circle with stress transformation equation

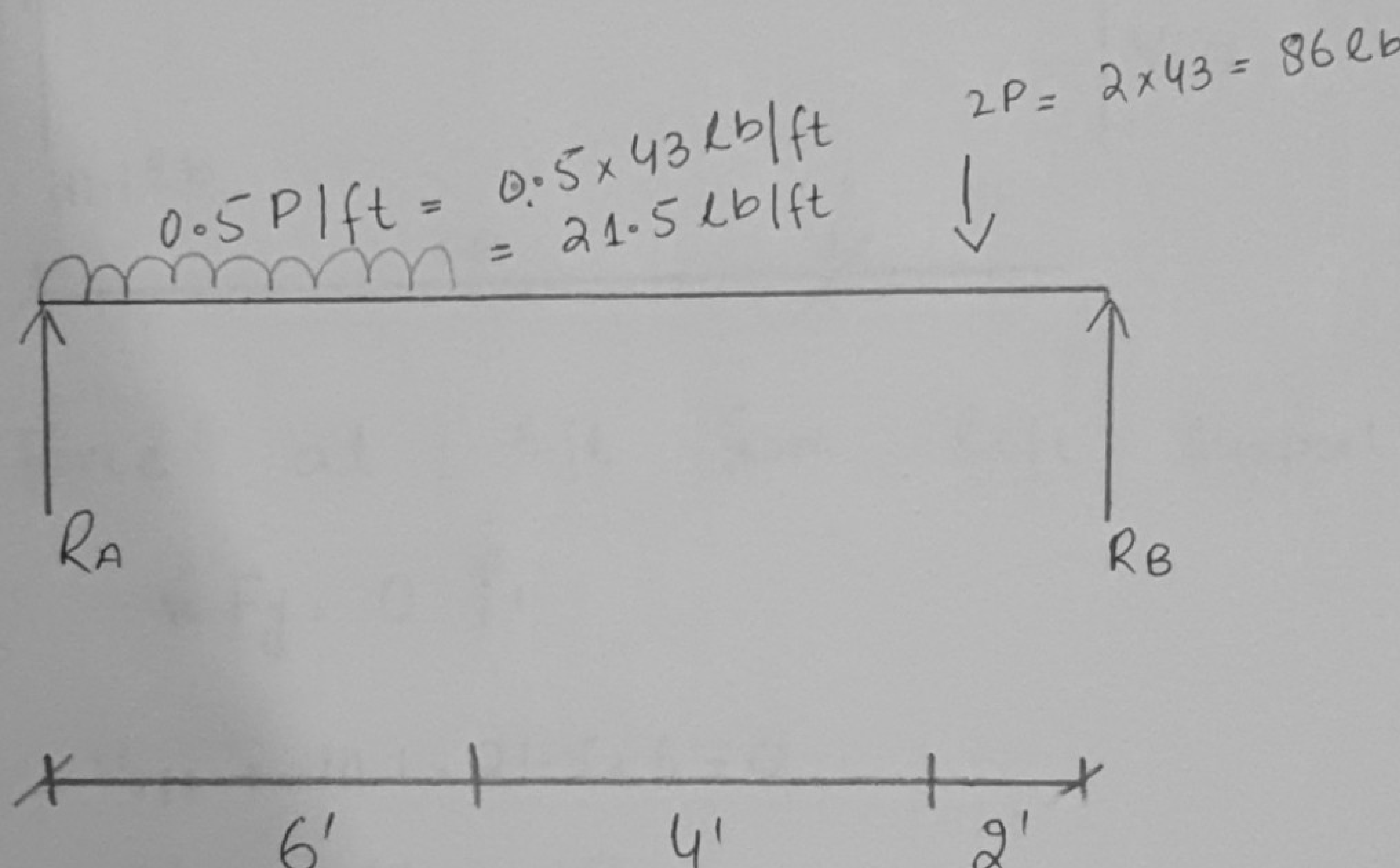
P = 43 lb.





Solution:-

First of all we find the reactions.



$\Sigma F_y = 0$ ($\uparrow +$)

$R_A + R_B - (21.5 \times 6) - (86) = 0$

$\boxed{R_A + R_B = 215 \text{ lb}}$

$\Sigma M_A = 0$ (+)

$(R_B \times 12) - (86 \times 10) - (21.5 \times 6 \times 6/2) = 0$

$12 R_B = 860 + 387$

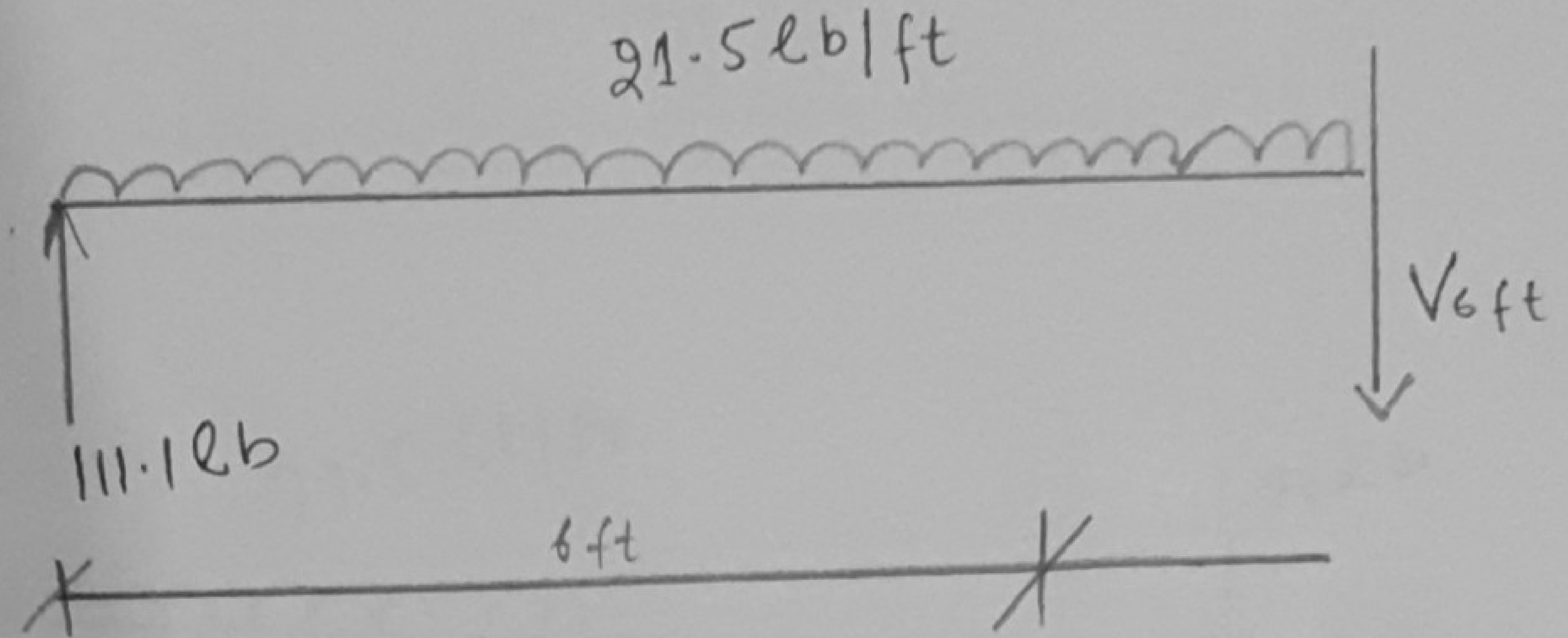
$\boxed{R_B = 103.91 \text{ lb}}$

$R_A = 215 - 103.91 \text{ lb}$

$\boxed{R_A = 111.1 \text{ lb}}$

Now we will draw shear force and bending moment diagram

Now shear force at change point or beam



Shear force at 6ft from left support

$$\sum F_y = 0 \uparrow +$$

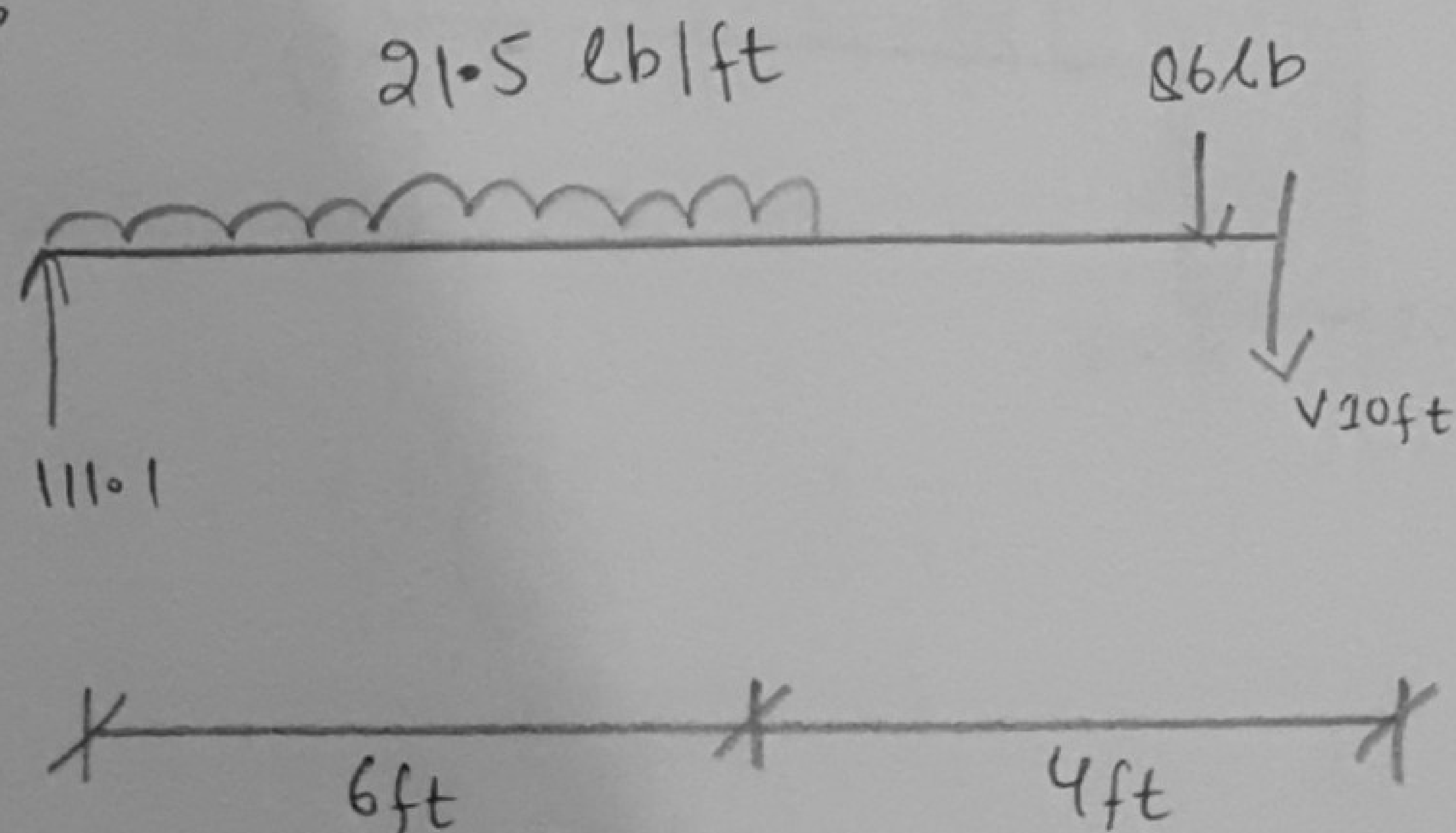
$$-V_{6ft} + 111.1 - 21.5 \times 6 = 0$$

$$-V_{6ft} - 17.9 = 0$$

$$\boxed{V_{6ft} = -17.9\ lb}$$

Now shear force at 10ft from left end

$V_{10} = ?$

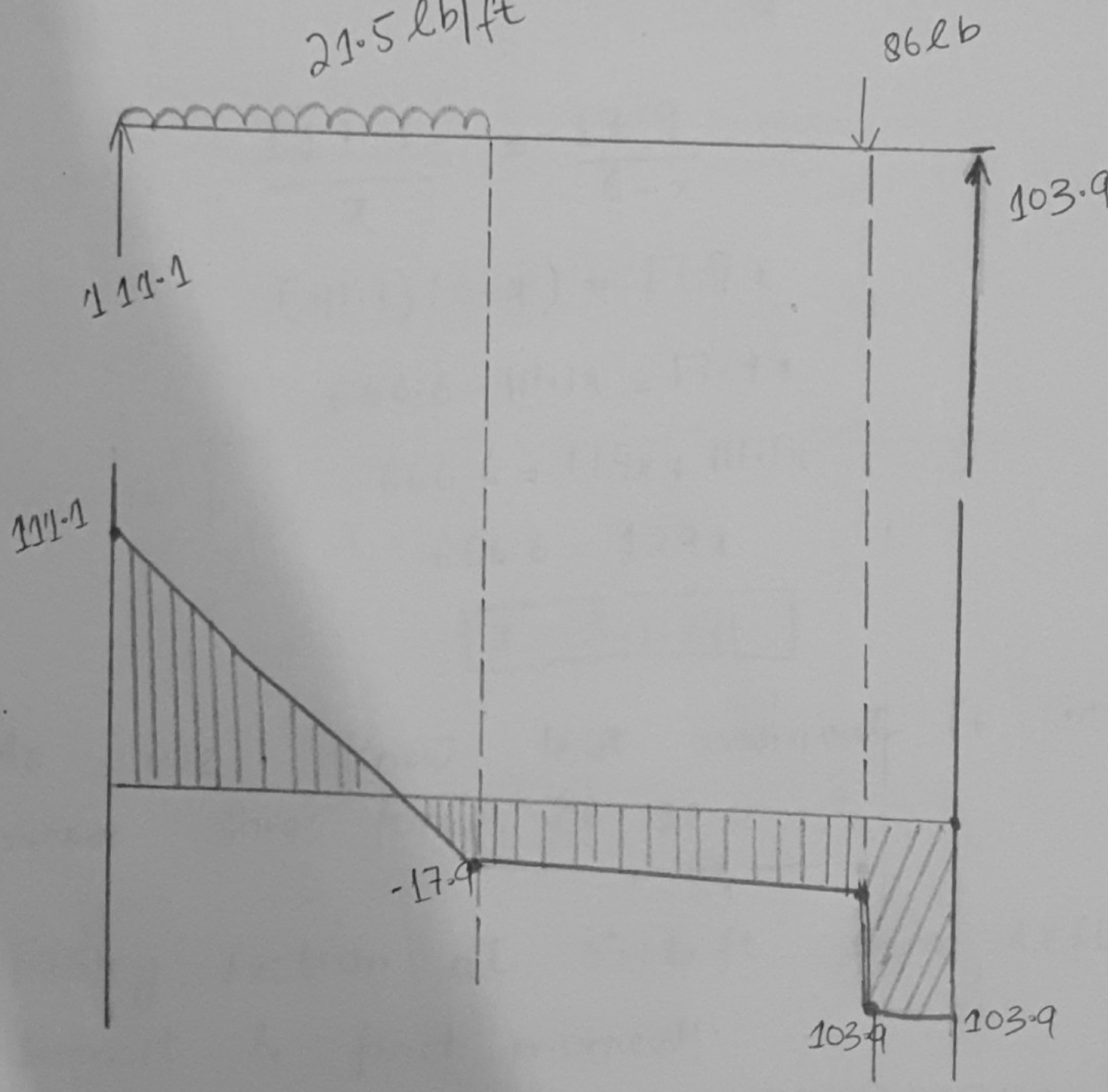


$\Sigma f_y = 0 \uparrow (+)$

$111.1\ (21.5 \times 6) - 86 - V_{10ft} = 0$

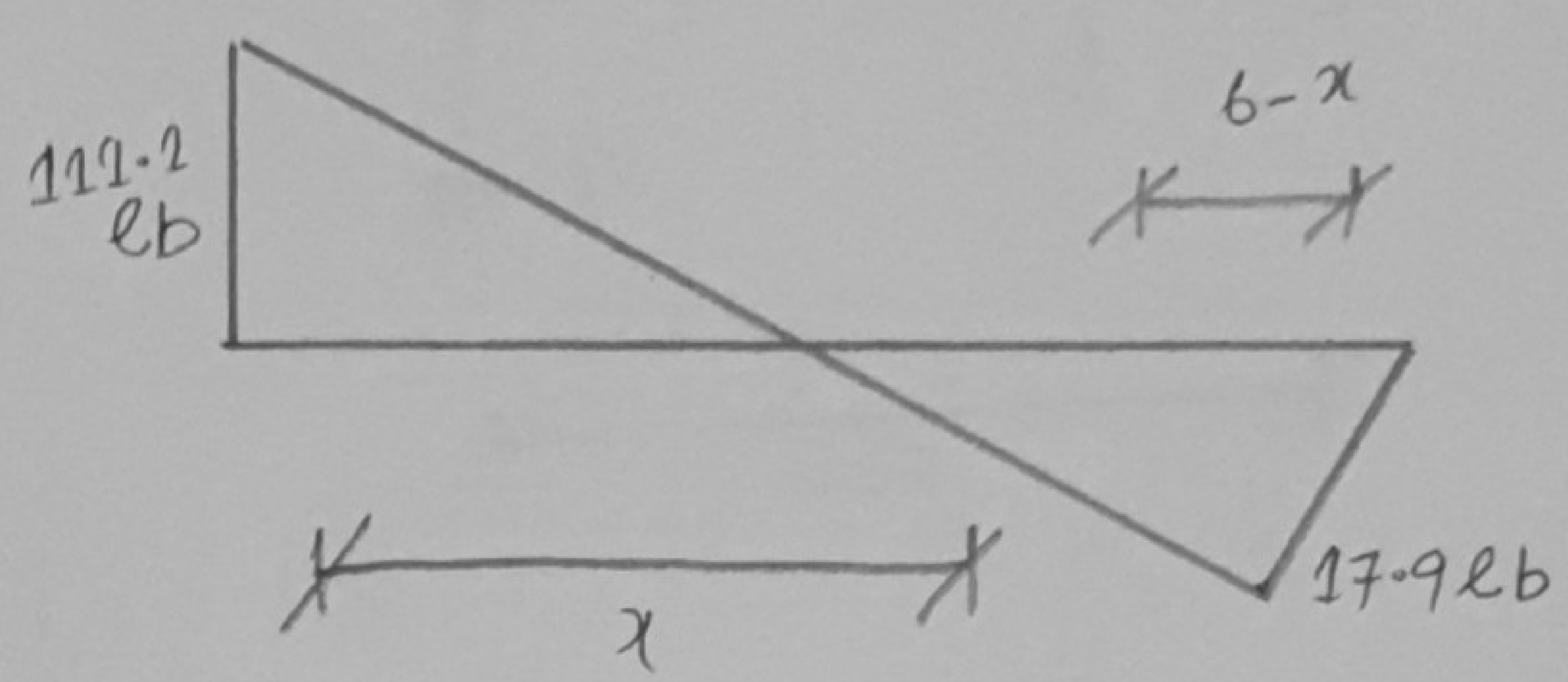
$-103.9 - V_{10ft} = 0$

$\boxed{V_{10ft} = -103.9\ lb}$



Now for moment diagram, we find moment at change point

First, finding moment at zero shear point



$$\frac{111.1}{x} = \frac{17.9}{6-x}$$

$$(111.1)(6-x) = 17.9x$$

$$666.6 - 111.1x = 17.9x$$

$$666.6 = 17.9x + 111.1x$$

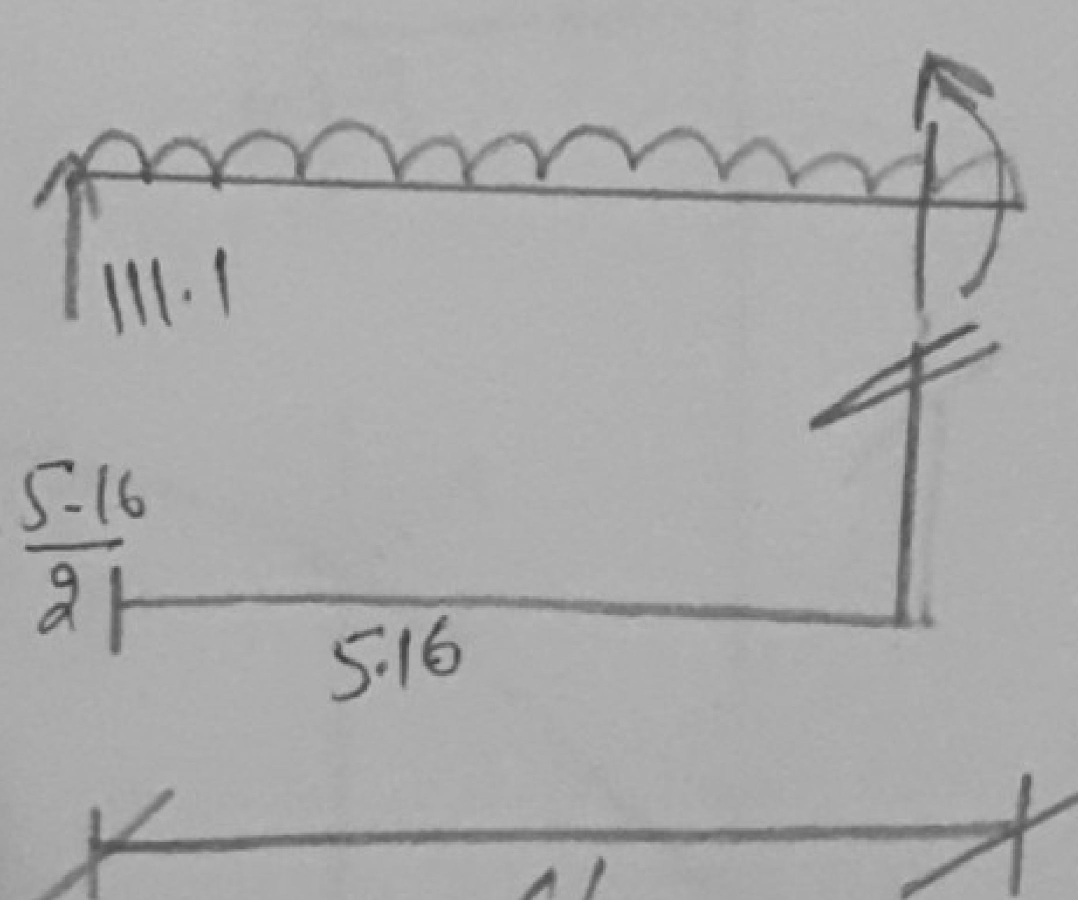
$$666.6 = 129x$$

$$\boxed{x = 5.167 ft}$$

As we know that moment is max, where shear force is zero.

Taking section at 5.16 ft from left support & find moment.

$$\Sigma M_{5.16} = 0$$



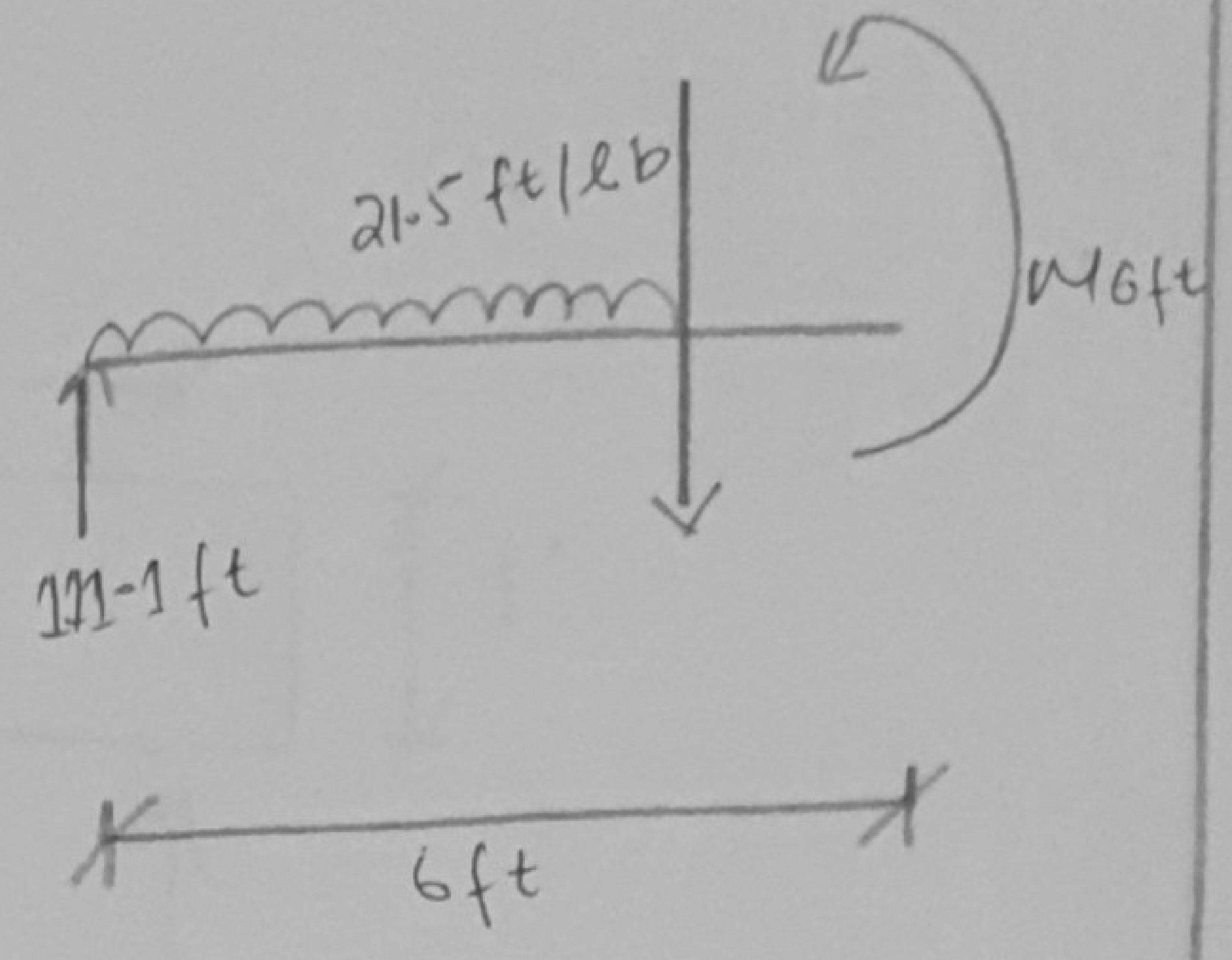
$$0 = M_{5.16} - 111.1 \times 5.15 + 21.5 \times 5.15 \times \frac{5.16}{2}$$

$$M_{5.16} - 572.165 + 286.22 = 0$$

$$\boxed{M_{5.16} = 285.93 \text{ lb-ft}}$$

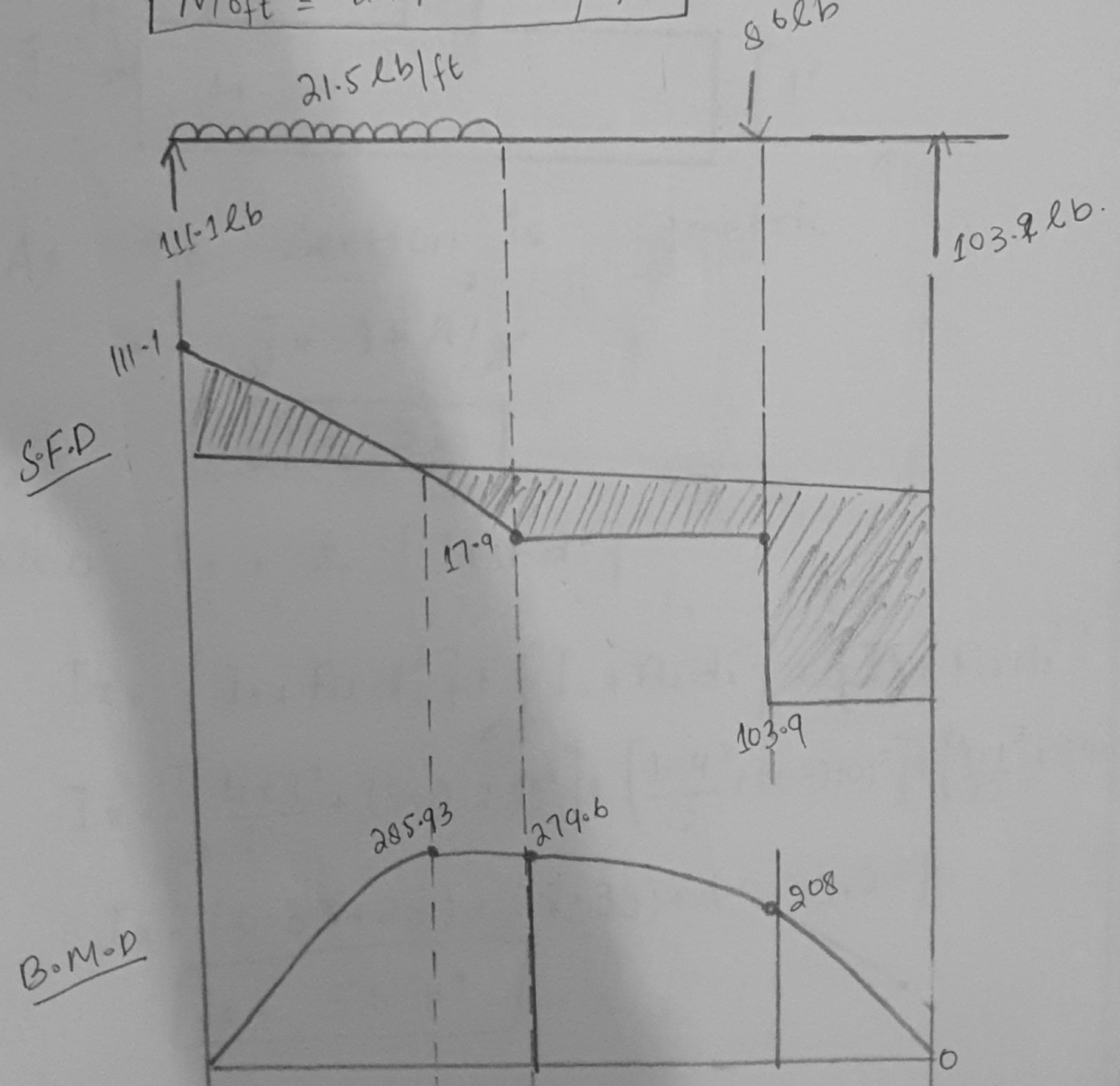
Moment at 6ft from left side.

$\Sigma M_{6ft} = 0$ (+)

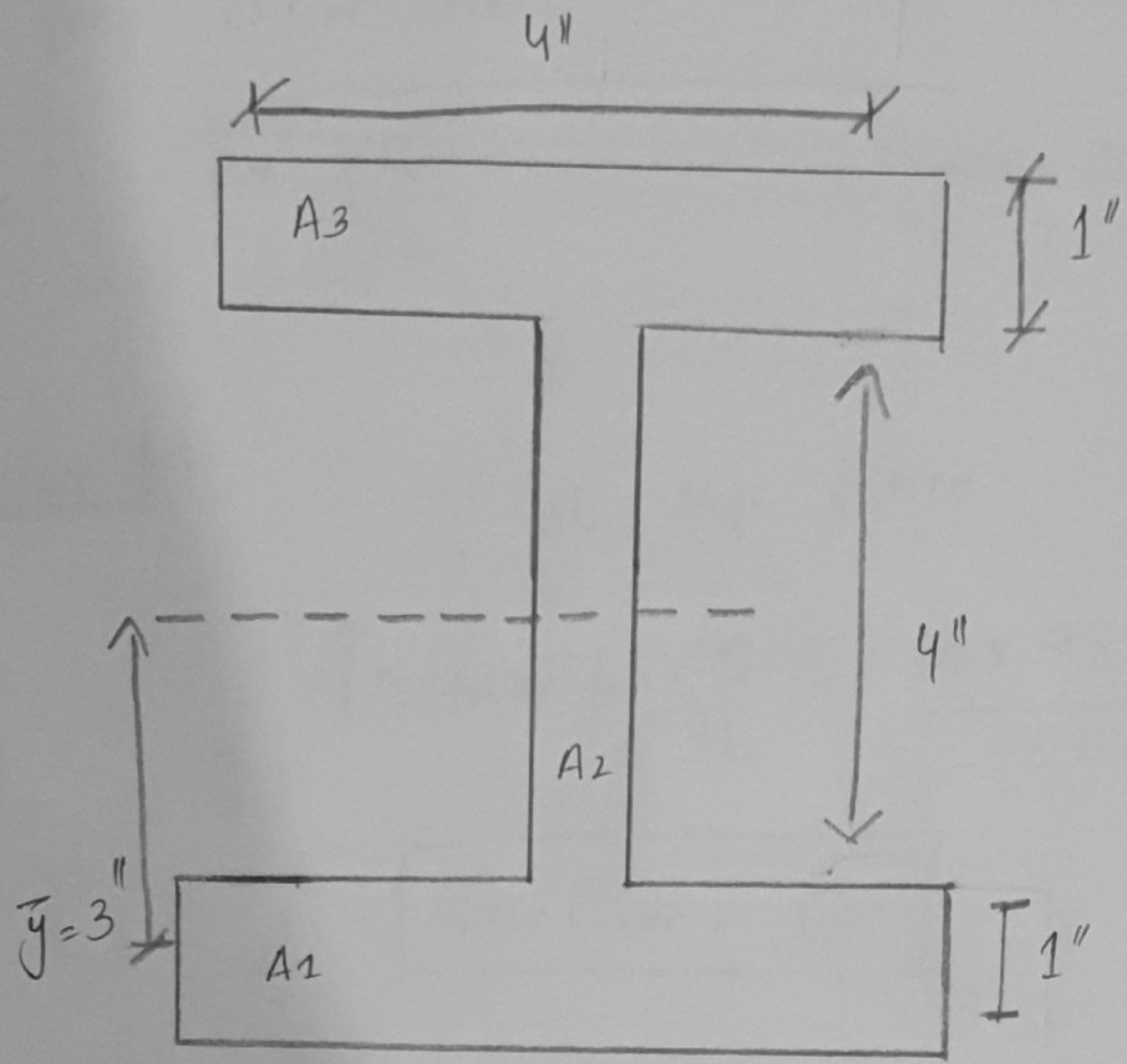


$M_{6ft} - (111.1 \times 6) + (21.5 \times 6 \times 6/2) = 0$

$\boxed{M_{6ft} = 279.6 \text{ lb/ft}}$



Now we find moment of intertia of the section.



As the section is symetric

so $\bar{y} = 1 + \cancel{4}^{2}/\cancel{2}$

$\boxed{\bar{y} = 3 \text{ in}}$

Now $I_x = \Sigma \left[I + Ad^2 \right]$

$I_x = \left[I_1 + A_1 d_1^2 \right] + \left[I_2 + A_2 d_2^2 \right] + \left[I_3 + A_3 d_3^2 \right]$

$I_x = \left[\frac{4 \times 1^3}{12} + (4\times1)(2.5)^2 \right] + \left[\frac{1 \times 4^3}{12} + (1\times4)(0)^2 \right] + \left\{ \frac{4 \times 1^3}{12} + (4\times1)(2.5)^2 \right\}$

$I_x = (0.33 + 25) + (5.33) + (0.33 + 25)$

$\boxed{I_x = 56 \text{ in}^4}$

Now we Calculate shear stresses and flexural stresses at different points in the beam.

$\Rightarrow$ for shear stress

$$\tau = \frac{VQ}{Ib}$$

$V_{max} = 103.9\,lb$

$I = 56\,in^4$

Case 1:-

τ at top fibre

$$\tau_{Top\,fiber} = \frac{VQ}{Ib} = \frac{103.9 \times (0)}{56 \times 4}$$

$$\boxed{\tau_{Top\,fiber} = 0}$$

Case 2:-

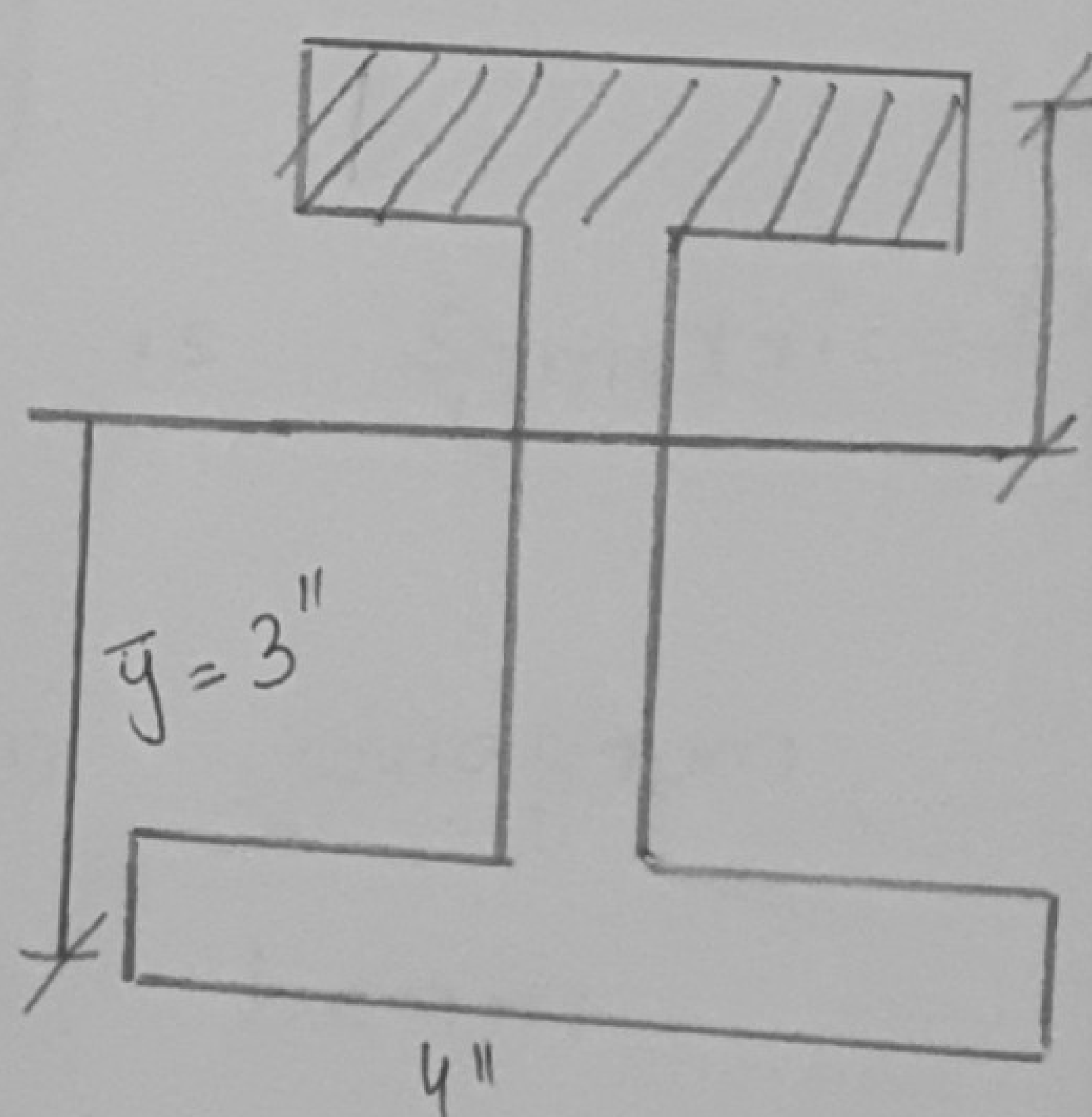
τ at 1 in below the top fibre

Two Cases A & B.

$$\tau_A = \frac{VQ}{Ib}$$

$$\tau_A = \frac{103.9 \times (4 \times 1)(2.5)}{56 \times 4}$$

$$\boxed{\tau_A = 4.64\,psi}$$



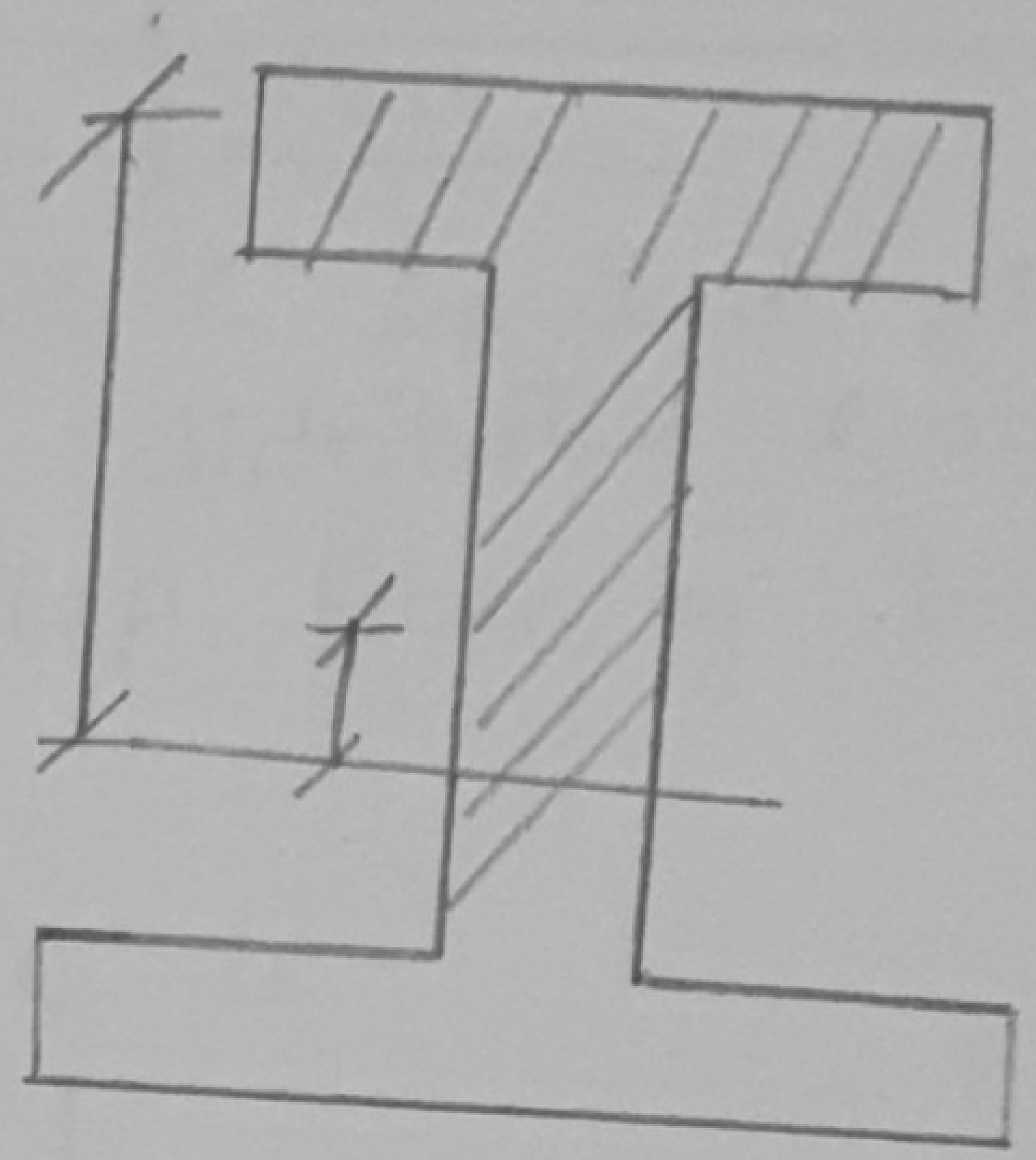
$$\tau_B = \frac{103.9 \times 4 \times 1 \times 2.5}{56 \times 1''}$$

$$\tau_B = 18.56 \text{ psi}$$

Case # 3 :-

"τ" at Centroidal axis of the section which will be the max. shear stress

Here $Q = Q_1 + Q_2$



$Q = A_1 \bar{y}_1 + A_2 y_2$

$(4\times1)(2.5) + (2\times1)(1)$

$Q = 10 + 2$

$\boxed{Q = 12}$

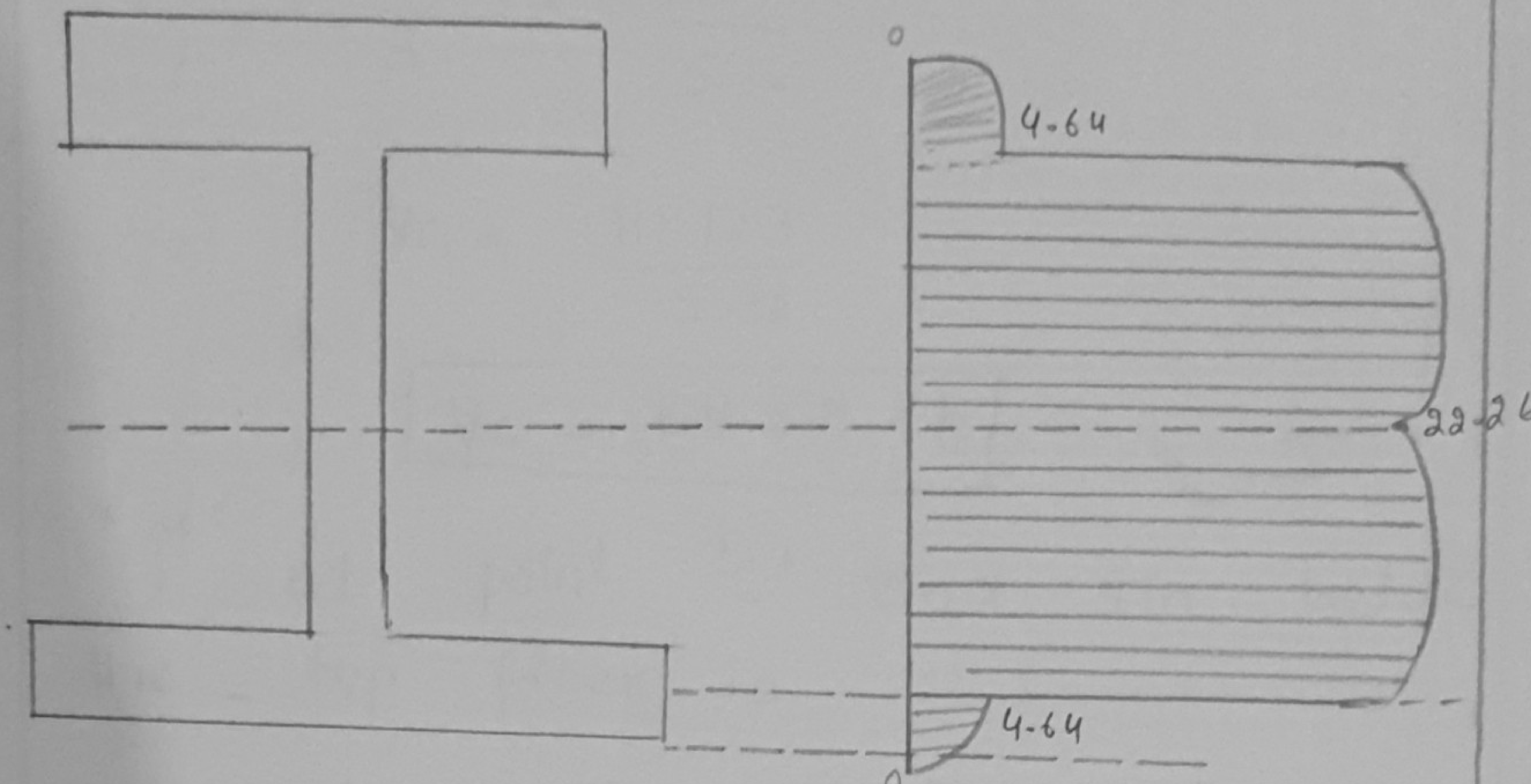
Now

$$\tau_{max} = \frac{103.9 \times 12}{56 \times 1}$$

$$\boxed{\tau_{max} = 22.26 \text{ psi}}$$

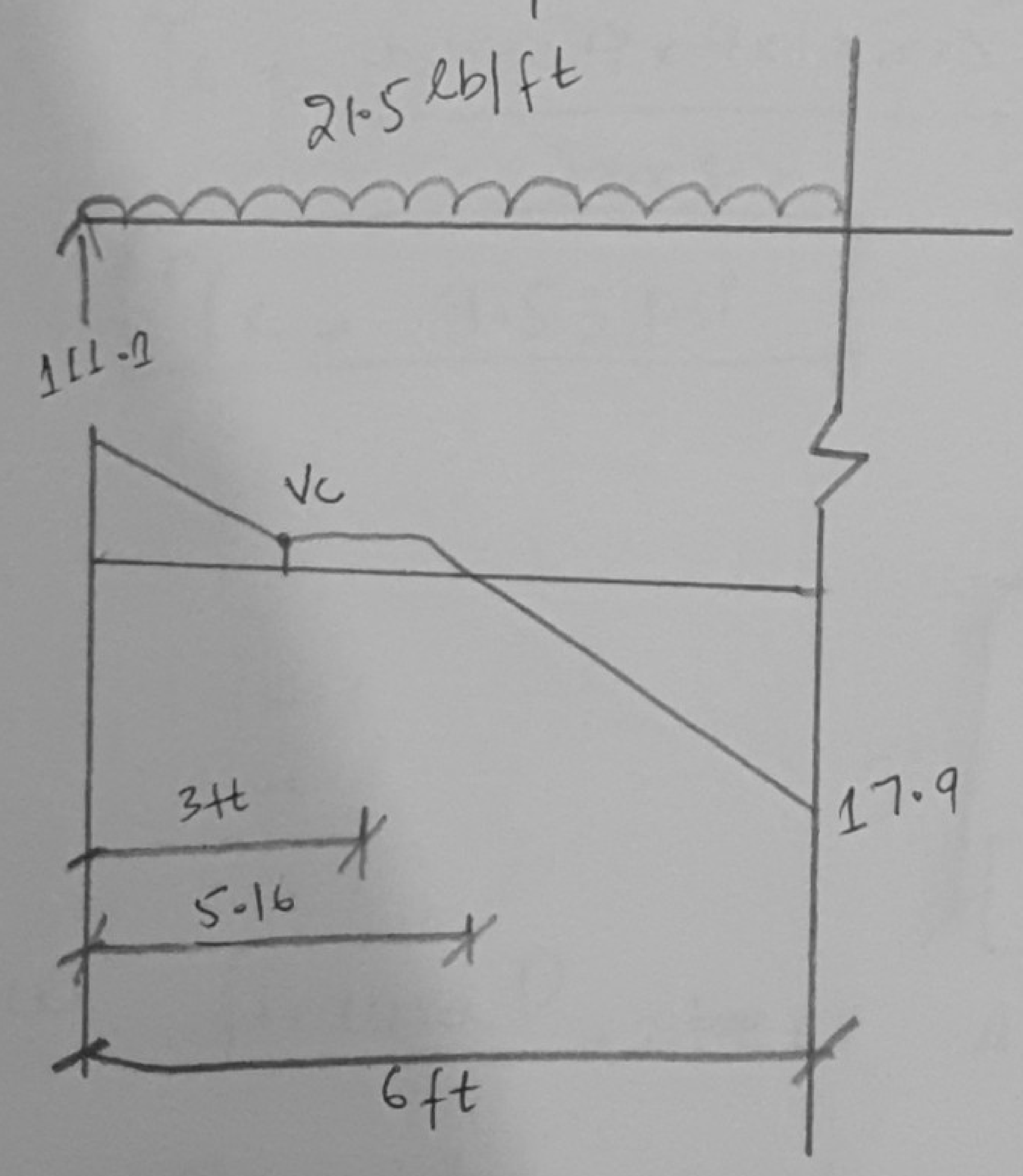
Rest of the section is symetrical as above.

shear stress variation diagram along the depth of beam.



Now shear stress at point "C" located at the Center of uniformly distributed load and 1in below the top fiber of beam Cross Section

Now "V" at point "C" is.



$$\frac{V_c}{3} = \frac{111.1}{5.16}$$

$$V_c = \frac{111.1 \times 3}{5.16}$$

$$\boxed{V_c = 64.59\ lb}$$

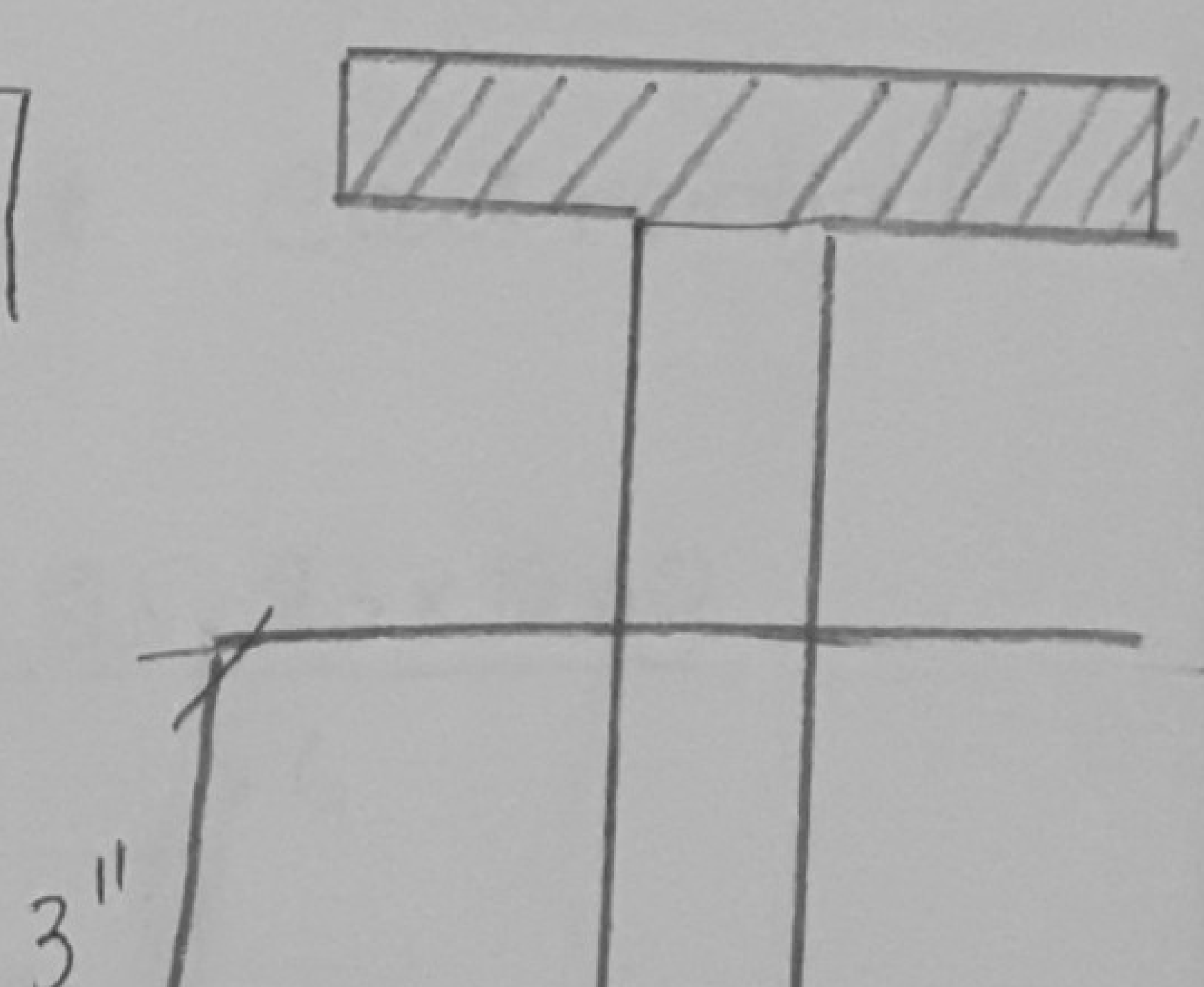
"τ" at point "c" and 1 in below the top fiber is.

$$\tau_c = \frac{V_c Q}{I b}$$

we will take b = 1" to get higher value of τ_c

$$\tau_c = \frac{64.59 \times 4 \times 1 \times 2.5}{56 \times 1}$$

$$\boxed{\tau_c = 11.53\ psi}$$



Now flexural stress Analysis:-

we consider maximum moment from B.M.D

Max. moment = 385.93 lb-ft

flexural stress $\sigma = \frac{M \cdot y}{I}$

Case # 1

stress at top fiber

$$\sigma_{TOP} = \frac{285.93 \times 12 \times 3}{56}$$

∴ 12 is multiply to convert lb-ft into lb-in for unit constency

$$\boxed{\sigma_{TOP} = 183.81\ psi}$$

Case # 2 :

stress at 1 below Top fiber

$$\sigma_1 = \frac{285.93 \times 12 \times 2}{56}$$

$$\boxed{\sigma = 122.54\ psi}$$

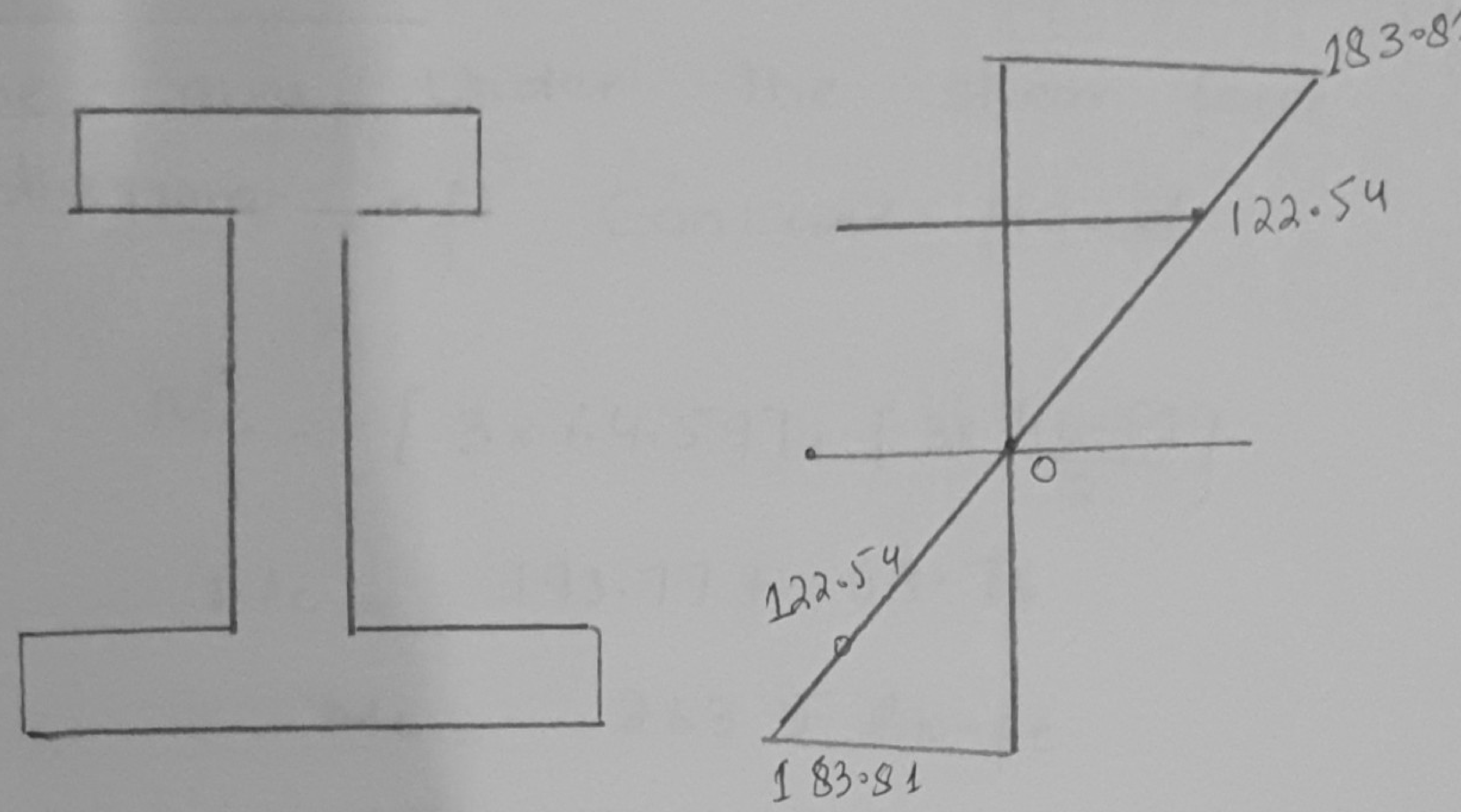
Case # 3 :- stresses at Centroidial axis

$$\sigma_{center} = \frac{285.93 \times 12 \times 0}{56}$$

$$\boxed{\sigma_{center} = 0}$$

Rest of the section is symetry of above.

Flexural stress variation diagram.



Stress state of a point element :-

Now the stress state of a point element located at the Center of uniformly distributed load and 1" below the top fiber of beam Cross section.

All applied stresses are required at point C

we have formd the shear stress at the required point which is

$$\tau_c = 11.53\ psi$$

Flexural stress at required point is

$$\sigma_c = \frac{M_c Y}{I}$$

Moment at C is approximately the area under the shear force diagram of Conlesed placed

$$M_C = \left[3 \times 64.59\right] + \left[3 \times \frac{46.51}{2}\right]$$

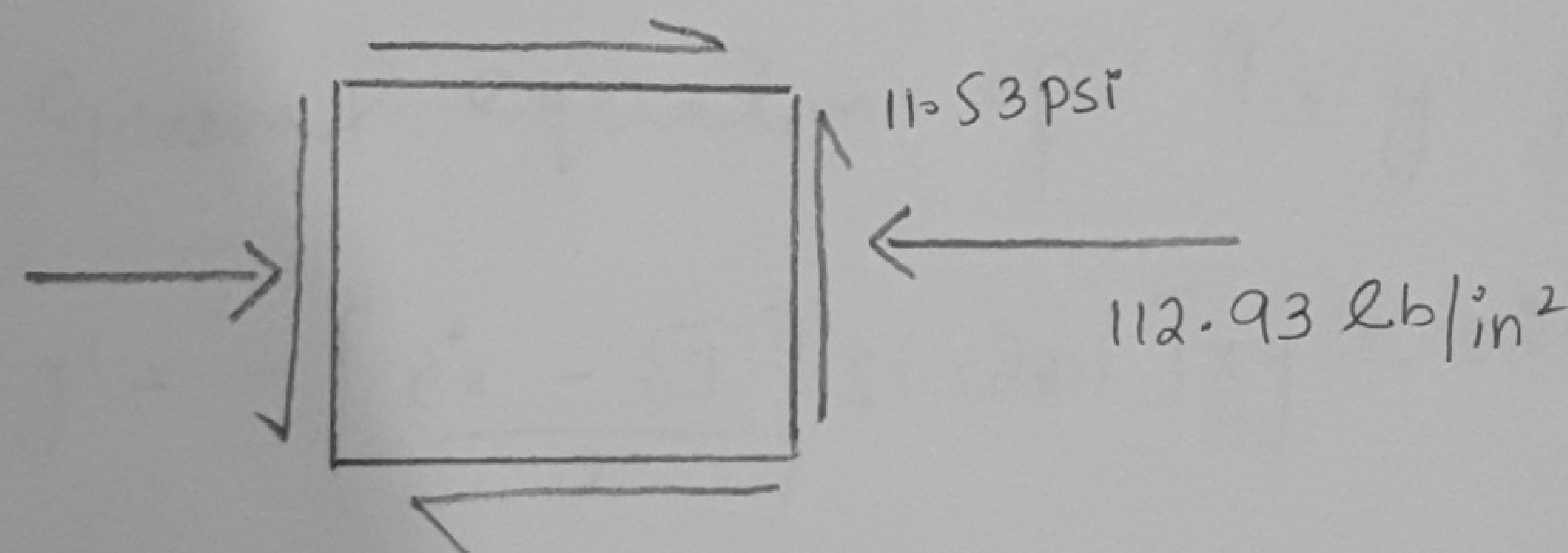
$$M_C = 193.77 + 69.76$$

$$M_C = 263.5 \text{ lb-ft.}$$

$$\sigma_C = \frac{263.5 \times 2 \times 12}{56}$$

$$\boxed{\sigma_C = 112.93 \text{ lb/in}^2}$$

Combine stress on 2D element.



principle stresses :-

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-112.93 + 0}{2} \pm \sqrt{\left(\frac{112.93 - 0}{2}\right)^2 + (11.53)^2}$$

$$-56.46 \pm \sqrt{3188.3 + 132.94}$$

$$= 56.46 \pm 57.63.$$

$\sigma_y = \sigma_1 = 1.17$ psi

$\sigma_x = \sigma_2 = -114.1$ psi.

Max In plane shear stress

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)}{\tau_{xy}} / 2$$

$$\tan 2\theta_s = \frac{-(-112.93-0)}{11.53} / 2$$

$$\boxed{\theta_s = 39.23^\circ}$$

As General equation of $\tau_{x'y'}$ is

$$\tau_{x'y'} = -\left[\frac{\sigma_x - \sigma_y}{2}\right] \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_{x'y'} = -\left(\frac{-112.93-0}{2}\right) \sin(2 \times 39.23)$$
$$+ 11.53 \cos 2(39.23)$$

$$\tau_{x'y'} = +56.46(0.98) + 2.3$$

$$\tau_{x'y'} = 57.63 \text{ psi}$$

max in plane shear stress.

Mohr's Circles.

Center Coordinate

$$(h, K) = \left[\frac{-112.93+0}{2}, 0\right]$$

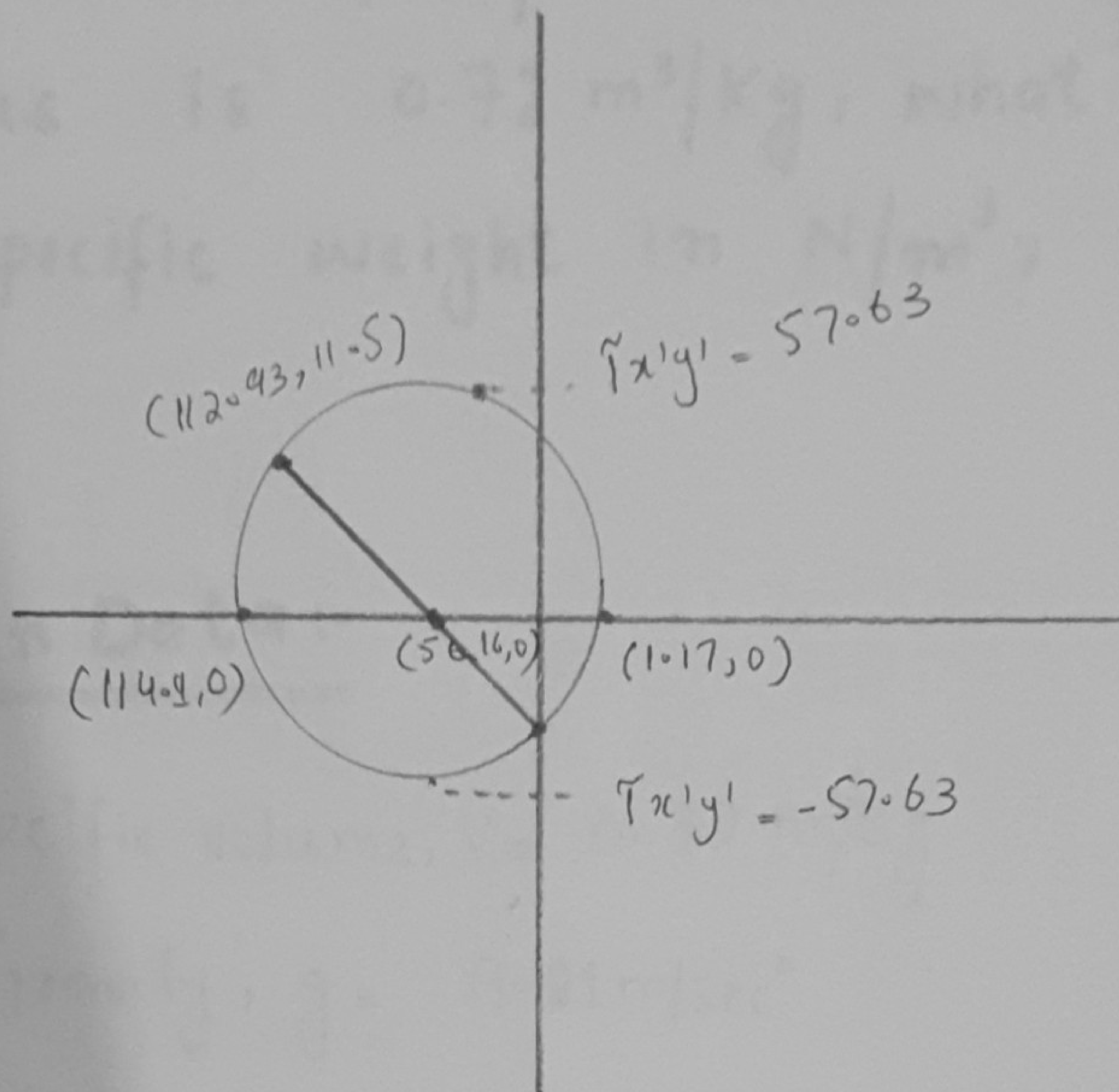
$$= [56.46, 0]$$

Radius of Mohr's Circle is

$$r = \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau x^2 y}$$

$$r = \sqrt{\left(\frac{-112.93-0}{2}\right)^2 + (11.53)^2}$$

$$\boxed{r = 57.63 \text{ psi.}}$$


(112.93, 11.5)
$\tau_{x'y'} = 57.63$
(114.9,0)
(1.17,0)
$\tau_{x'y'} = -57.63$