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7820

Section:

A

Subject:

Probability & Statistics

Question No. 2

Solution:

Estimated Regression

$$a = \bar{y} - b\bar{x}$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{y} = \frac{\sum y}{n}$$

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

y on x

$$\bar{y} = a + bx$$

$$\sum y = na + b \sum x$$

$$\sum xy = a \sum x + b \sum x^2$$

$$a = \bar{y} - b\bar{x}$$

$$b = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

P.T.O

X	Y	XY	X ²	Y ²
53	20	1060	2809	400
62	32	1984	3844	1024
57	45	2565	3249	2025
71	60	4260	5041	3600
78	80	6240	6084	6400
20	100	2000	400	10000
86	120	10320	7396	14400
87	140	12180	7569	19600
96	180	15360	9216	25600
91	180	16380	8281	32400
94	200	18800	8836	40000
94	210	19740	8836	44100

~~705~~ 1067
 110889
~~110889~~
 70661
~~70661~~
 $\Sigma Y^2 = 1995$

Now $\bar{x} = \frac{\sum x}{n} = \frac{889}{12}$
 $= 74.08$

Now $\bar{y} = \frac{\sum y}{n}$
 $= \frac{1067}{12}$
 $= 88.91$

$$b = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

$$b = \frac{12(110889) - (889)(1067)}{12(70661) - (889)^2}$$

$$= \frac{1330668 - 948583}{847932 - 790321}$$

$$= \frac{382105}{57611} = \boxed{6.99}$$

$$a = \bar{y} - b\bar{x}$$

$$= 88.91 - (6.99)(74.08)$$

$$= 88.91 - 517.81$$

$$a = -428.9$$

Coefficient of Correlation

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2 (n \sum y^2 - (\sum y)^2)}}$$

$$= \frac{12(110889) - (889)(1067)}{\sqrt{12(70661) - (889)^2(12)(199549) - 13661869}}$$

$$r = \frac{1330,668 - 948,583}{\sqrt{847932 - 790321(12)(199549) - 13661869}}$$

$$= \frac{382105}{\sqrt{168540.81}}$$

$$r = 181.25$$

Question No. 2

(a) A box contains 4 red, 4 white and 5 green balls. Three balls are drawn from the box together. Find the probability that may be (i) all of different colours. (ii) all of the same colours.

Solution:

$$n(S) = \binom{13}{3} = 286$$

Let $A =$ Event all balls are of different colours.

$$n(A) = \binom{4}{1} \binom{4}{1} \binom{5}{1} = 4 \times 4 \times 5 = 80$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{80}{286} = 0.28$$

\Rightarrow There are 28% chances that all balls are of different colours.

(ii) Let $B =$ Event all balls of same colours.

$$|B| = \binom{4}{3} \text{ (Red)} \text{ or } \binom{4}{3} \text{ (White)} \text{ or } \binom{5}{3} \text{ (Green)}$$

$$= \binom{4}{3} + \binom{4}{3} + \binom{5}{3}$$

$$= 4 + 4 + 10$$

$$= 18$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{18}{286} = 0.063$$

\Rightarrow There are 6.3% chances that all balls of same colours.

Q.No. 2

Part (b): Of 12 eggs in a refrigerator, 2 are bad. From these, 4 eggs are chosen at random to make a cake. What are the probabilities that

- exactly one is bad?
- At least one is bad?

Solution:

$$n(S) = \binom{12}{4} = \frac{12!}{4!8!} = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} = 495$$

Let A = denote the event that exactly one egg is bad.

$$n(A) = \frac{n(A)}{n(S)} = \frac{240}{495} = 0.48$$

\Rightarrow There are 48% chances that exactly one egg is bad.

(ii) let $B =$ be the event that atleast one bad egg is selected.

$$n(B) = \binom{2}{1} \binom{10}{3} + \binom{2}{2} \binom{10}{2}$$

$$= 2 \times 120 + 1 \times 45$$

$$= 240 + 45$$

$$= 285$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{285}{495}$$

$$= 0.58$$

\Rightarrow There are 58% chances that atleast one bad egg is selected.

Question No. 3..

The following are the scores made by three batsmen A, B & C in a series of innings.

A	12	15	6	73	7	20	199	36	84	29
B	47	12	76	48	4	20	37	48	13	3
C	15	23	52	4	24	20	74	52	13	4

- Find the range of batsmen A, B & C?
- Who is more consistent player?
- Compare A with B, B with C & A with C

SOLUTION:

	A	B	C
	12	47	15
	15	12	23
	6	76	52
	73	48	4
	7	4	24
	20	20	20
	199	37	74
	36	48	52
	84	13	18
	29	3	4

$$\text{Range} = X_m - X_0$$

$$\begin{aligned}\text{Range of A} &= X_m - X_0 \\ &= 199 - 6 \\ &= 193\end{aligned}$$

$$\begin{aligned}\text{Range of B} &= X_m - X_0 \\ &= 76 - 3 \\ &= 73\end{aligned}$$

$$\begin{aligned} \text{Range of } C &= X_m - X_0 \\ &= 74 - 4 \\ &= 70 \end{aligned}$$

Batsman A		Batsman B		Batsman C	
X	X ²	Y	Y ²	Z	Z ²
12	144	47	2209	15	225
15	225	12	144	23	529
6	36	76	5776	52	2704
73	5329	48	2304	4	16
7	49	4	16	24	576
20	400	20	400	20	400
199	39601	37	1369	74	5476
36	1296	48	2304	52	2704
84	7056	13	169	13	169
29	841	3	9	4	16

$$\begin{aligned} \Sigma X &= 481 & \Sigma X^2 &= 54,977 \\ \Sigma Y &= 308 & \Sigma Y^2 &= 14,700 \\ \Sigma Z &= 281 & \Sigma Z^2 &= 12,815 \end{aligned}$$

OF Variable is smallest.

Batsman A :-

$$\bar{x} = \frac{\sum x}{n} \quad \therefore n = 10$$

$$= \frac{481}{10} = 48.1$$

$$s_x = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$= \sqrt{\frac{54977}{10} - \left(\frac{481}{10}\right)^2}$$

$$= \sqrt{\frac{54977}{10} - \frac{231361}{100}}$$

$$= \sqrt{5497.7 - 2313.61}$$

$$= \sqrt{3184.09}$$

$$s_x = 56.42$$

$$C.V = \frac{56.42}{\bar{x}} \times 100 = \frac{56.42}{48.1}$$

$$C.V = 117.01$$

Batsman B :-

$$y = \frac{\Sigma y}{n} = \frac{308}{10} = 30.8$$

$$S_y = \sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}$$

$$= \sqrt{\frac{14700}{10} - \left(\frac{308}{10}\right)^2}$$

$$= \sqrt{1470 - 948.64}$$

$$= \sqrt{521.36}$$

$$= 22.83$$

$$C.V = \frac{22.83}{30.8} \times 100$$

$$= 0.741 \times 100$$

$$= 74.1 \%$$

Batsman C:

$$\bar{z} = \frac{\sum z}{n} = \frac{281}{10} = 28.1$$

$$s_z = \sqrt{\frac{\sum z^2}{n} - \left(\frac{\sum z}{n}\right)^2}$$

$$= \sqrt{\frac{12815}{10} - \left(\frac{281}{10}\right)^2}$$

$$= \sqrt{1281.5 - 789.61}$$

$$= \sqrt{491.89}$$

$$= 22.17$$

$$C.V = \frac{s_z}{\bar{z}} = \frac{22.17}{28.1} \times 100$$

$$= 78.8 \%$$

Batsman B is more consistent as its value of coefficient of variance is smallest.

3) 7)

Compare A with B

B is consistent

Compare B with A

B is more consistent

Compare A with C

C is more consistent.

