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Subject Advanced Fluid Mechanics

Summer Final term Exam

Q 1)

a) Drag:-

A body which is wholly immersed in a homogenous fluid may be subjected to two kind of forces arising from relative motion between body and fluid these forces are termed as drag and lift. If the forces parallel to the motion then it is termed as drag force.

There are two components

=> Pressure Drag (F_p):-

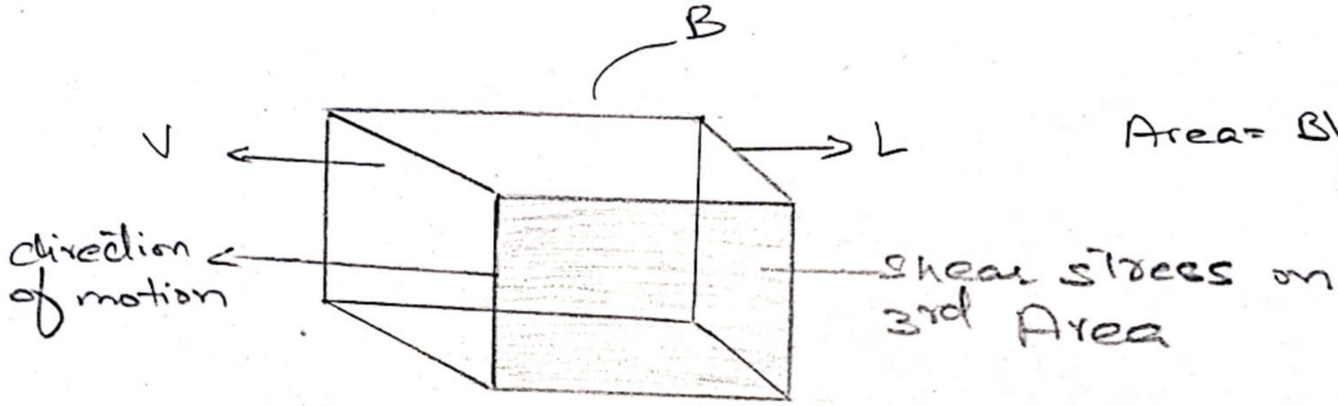
It is equal to integration of ~~components~~ components in direction of motion of all pressure forces exerted on surface of body

$$F_p = C_p \rho \frac{V^2}{2} A$$ where "cp" depends on shape.

=> Friction Drag (F_f):-

It is equal to integration of components of shear stress along surface of body in direction of motion

$$F_f = C_f \, \rho \frac{V^2}{2} BL$$ where "C_f" depends on velocity



Area = BVL

→ Friction Drag of Boundary layer :-

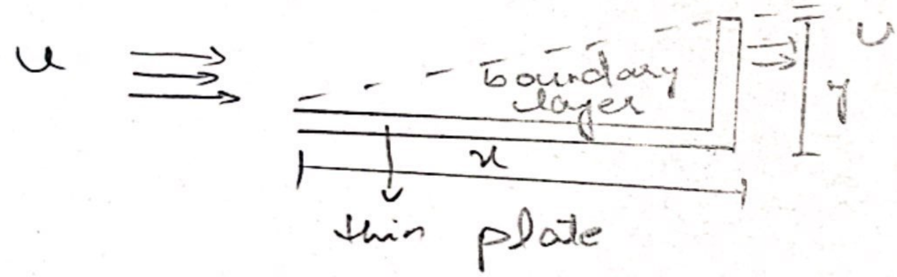
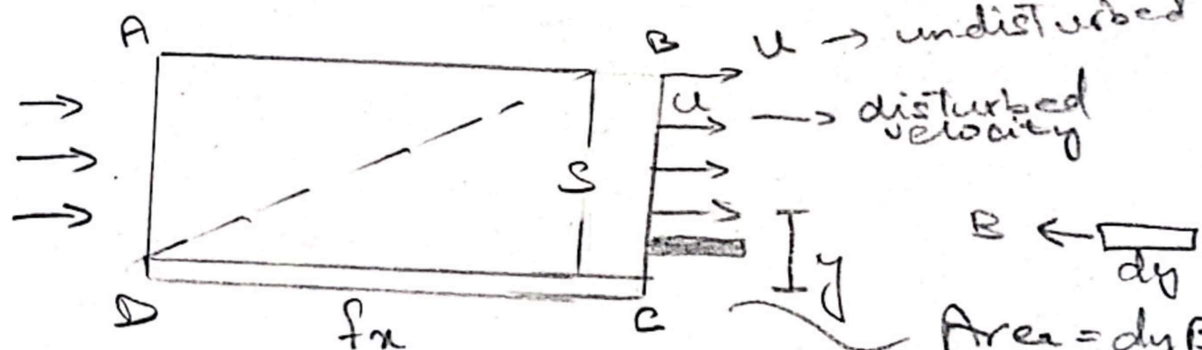


Fig shows growth of boundary layer along one side of smooth plate inside the fluid

Now consider a control volume



Area = dy B

Where δ is thickness of boundary layer and U is undisturbed velocity

Thus $-F_x$ = drag = (rate in momentum in x-direction) Leaving through BC + rate of momentum through AB) - rate of momentum entering through DA)

$$\Delta P = P_{out} - P_{in}$$

Thus according to momentum

$$\Sigma F = \frac{d(p)}{dt} = \frac{dmv}{dt}$$

where

$$\frac{dm}{dt} = \rho Q$$

$$F = \rho Q V$$

or

$$F = \rho A \cdot V \cdot V$$

$$F = \rho A V^2$$

$$DA \rightarrow \rho U (U B \delta)$$

$$BC \rightarrow \rho B \int_{0}^{\delta} u^2 \cdot dy$$

$$AB \rightarrow \rho U \left(U B \delta - B \int_{0}^{\delta} u \cdot dy\right)$$

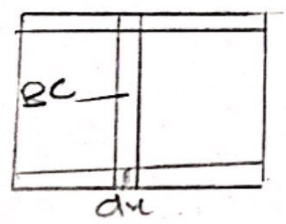
Putting Value

$$Fx = \rho B \int_0^{\delta} u(U-u)\,dy$$

$Fx = \rho B U^2 \delta \alpha$ where α is function of boundary layer

Now to find local wall shear stress

$$\tau_0 = \frac{dfx}{B.dx - \text{area}}$$

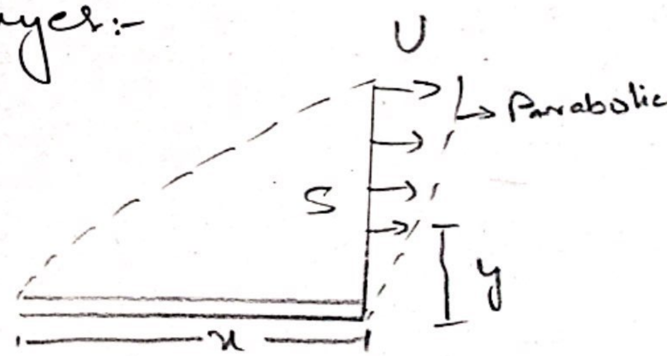


$$Fx = \rho B U^2 \delta \alpha$$

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$ in general equation of shear stress

⇒ Laminar Boundary layer:-

$$\frac{u}{U} = F\left(\frac{y}{\delta}\right)$$



Assume

$$\eta = \frac{y}{\delta} \text{ or } y = \eta\delta$$

Thus $\frac{u}{U} = f(\eta)$ or $u = Uf(\eta)$

in case of laminar flow

$$\tau_0 = \mu\left(\frac{du}{dy}\right)$$

$$= \frac{w}{\delta}\left(\frac{du}{dn}\right) = \frac{\mu U}{\delta}\left[\frac{df(\eta)}{dn}\right]$$

Solving the equation

$$\tau_0 = \frac{\mu U B}{\delta} \quad -①$$

As general equation is $\tau_0 = \rho U^2 \alpha \frac{ds}{dx}$

equating both equations

$$\frac{\mu U B}{\delta} = \rho U^2 \alpha \frac{ds}{dx}$$

or

$$\delta\, ds = \frac{\mu B}{\rho U \alpha} dx$$

integrating the equation

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x + C$$

Now at $x = 0$, $\delta = 0$ Thus $C = 0$

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x$$

or

$$\delta = \sqrt{\frac{2\mu B}{\rho U \alpha} x} \quad \text{or} \quad \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

xing or ÷ing by "x"

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}} \cdot \frac{x}{\sqrt{x} \cdot \sqrt{x}}$$

where $\alpha = 0.135$

$B = 1.63$

$R_x = \frac{\rho U x}{\mu}$

$$\delta = \frac{4.91}{\sqrt{R_x}} \cdot x \quad \text{or} \quad \frac{\delta}{x} = \frac{4.91}{\sqrt{R_x}}$$

Now

$$\tau_0 = \frac{\mu U B}{\delta}$$

thus putting value

$$\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

where

R_x is local Reynold's Number

Now

$$F_f = B \int_0^2 \underbrace{\tau_0}_{\text{stress}} dx$$

Putting values

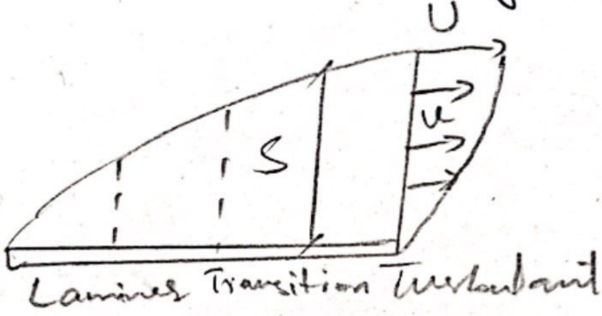
$$F_f = 0.664 B \sqrt{\rho \mu L U^3}$$

As general equation is

$$F_f = C_f \frac{\rho V^2}{2} BL \rightarrow \text{equating both equations}$$

$$C_f = 1.328\sqrt{\frac{\nu}{\rho L V}} = \frac{1.328}{\sqrt{R}}$$

⇒ Turbulant boundary layer :-



resistance is less so curve becomes straight

Fig shows that velocity distribution in turbulant boundary layer shows a much steeper gradient near wall and flatter through out reoccuring layer.

The shear stress is greater in turbulant than in laminar

As we have

$$\tau_o = f\frac{\rho V^2}{8}$$

where V denotes avg velocity of pipe

Now we have obtained an approximate relation between V and U by using pipe factor equation of

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{f}}$$

using friction factor of 0.028 from chart which is middle critical value

so

$$\boxed{U = 1.235 V}$$

Now we have

$$\tau_o = f \frac{\rho v^2}{8}$$

As we have

$$f = \frac{0.316}{R^{0.25}}$$

Thus

$$\tau_o = \frac{0.316}{\left(\frac{DV}{\nu}\right)^{1/4}} \frac{\rho V^2}{8}$$

where $V = \frac{U}{1.235}$ thus

$$\tau_o = \left(\frac{0.316}{\left(\frac{D}{\nu}\left(\frac{U}{1.235}\right)\right)}\right)^{1/4} \cdot \frac{\rho}{8}\left(\frac{V}{1.235}\right)^2$$

& $D = 2\delta$

thus

$$\tau_o = \frac{0.023 \rho U^2}{\left(\frac{\delta U}{\nu}\right)^{1/4}}$$

As we have

$$\tau_o = \rho U^2 \alpha \frac{d\delta}{dx}$$

Equating both and integrating for boundary condition of

$$x = 0 \ , \ \delta = 0$$

thus

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{Ux}\right)^{1/5} x$$

For $\alpha = 0.0972$

$$\boxed{\frac{\delta}{x} = \frac{0.377}{(R_x)^{1/5}}}$$

Putting values in equation

$$\tau_o = 0.0587 \frac{\rho U^2}{2} \left(\frac{\nu}{Ux}\right)^{1/5}$$

Now

$$F_f = B \int_0^2 \tau_o \, dx$$

$$F_f = 0.0735 \rho \frac{U^2}{2} \left(\frac{\nu}{UL}\right) BL$$

As

$$F_f = C_f \rho \frac{V^2}{2} BL$$

equating both

$$Cf = \frac{0.0785}{R^{1/5}}$$

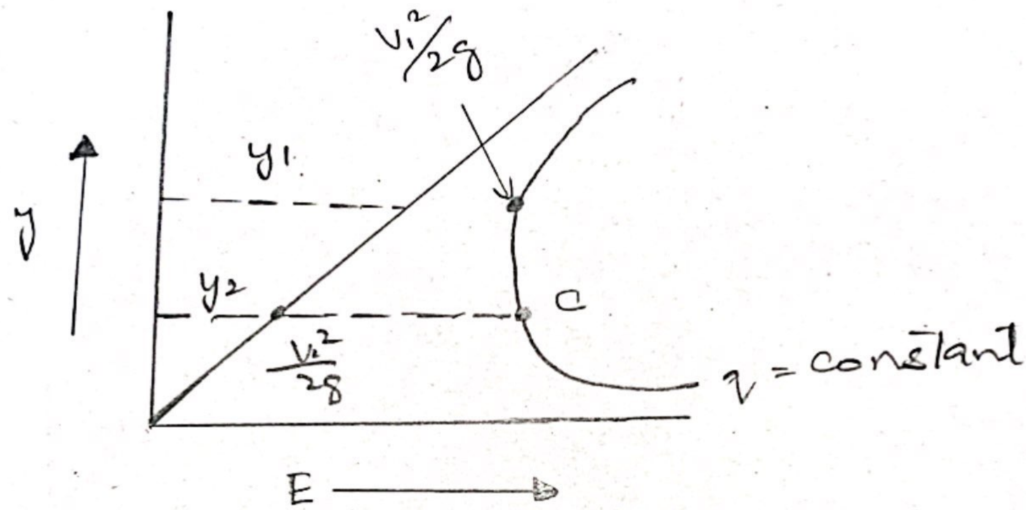
R is less than 10^7
for $500,000 < R < 10^7$

For $R > 10^7$

$$\boxed{Cf = \frac{0.455}{(\log R)^{2.58}}}$$ Ans

Q1)

b) Derive equation for critical depth Critical velocity of rectangular section of a channel



This is specific energy eq :-

For particular q, there will be two kind of Possible values of y for given E. The equation is cubic with three roots with third being negative giving no values thus two alternative depths represents two totally different flow regimes - slow & deep on upper portion & fast and shallow on lower portion

Point represent dividing point between two regime of flow.

Thus for given "q", value of E is minimum & flow at this point is critical flow Depth of flow at this Point is critical depth y_c & velocity at this point is critical velocity

Thus relation of critical depth can be found as

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

For minimum specific energy

$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = 1 - \frac{\cancel{2}}{\cancel{2}g}\left(\frac{q^2}{y^3}\right)$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3}$$

$$1 - \frac{q^2}{gy^3} = q^2 = gy^3$$

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

As $= q = Vy$, $V_c^2 = gy^3$

OR

$$V_c = \sqrt{gy_c}$$

$$y_c = \frac{V_c^2}{g}$$

Now

$$\frac{y_c}{2} = \frac{V_c^2}{2g}$$

$$E_{min} = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2}$$

$$\frac{3}{2} y_c \quad \text{OR} \quad y_{cr} = \frac{2}{3} \text{ constant}$$

	Subcritical	Critical	Supercritical
Depth of flow	$y > y_c$	$y = y_c$	$y < y_c$
Velocity slope	$V < V_c$ mild slope $S_o < S_c$	$V = V_c$ critical slope	$V > V_c$

Q2)

Given:-

Depth of Rectangular Channel (d) = ?

Flowrate (Q) = 3.5 m³/sec

Slope of bed (S_0) = 0.0008

n = 0.0219

width of Bed = 7480 mm

= 7.48

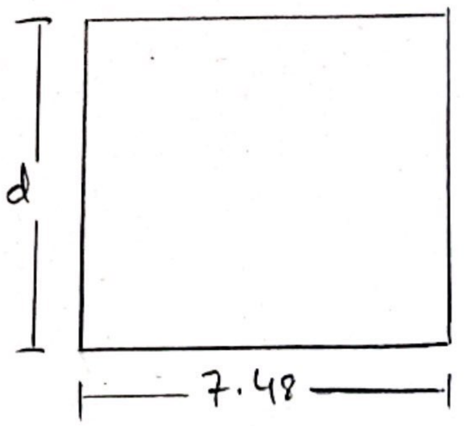
Critical depth = ?

Flow subcritical or super critical = ?

Solution:-

Area = 7.48 × d

= 7.48d



Parameter = d + 7.48 + d

= 7.48 + 2d

Hydraulic Radius $(R_h) = A/P$

$$= \frac{7.48d}{7.48+2d}$$

By using Manning's Equation

$$Q = \frac{1}{n} A R_h^{2/3} \cdot S_o^{1/2}$$

Putting Values

$$3.5 = \frac{1}{0.0219} \times 7.48d \times \left(\frac{7.48d}{2d+7.48}\right)^{2/3} \times (0.0008)^{1/2}$$

$$\boxed{d = 0.575}$$

$$\text{Area} = 7.48(0.575)$$

$$\boxed{= 4.301 m^2}$$

$$\text{Parameter} = 7.48 + 2(0.575)$$

$$\boxed{= 8.63 m}$$

Hydraulic Radius $(R_h) = \frac{4.301}{8.63}$

$$\boxed{= 0.4983m}$$

Finding critical depth

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

As $= q = \frac{Q}{B}$

$$= \frac{3.5}{7.48}$$

$$= 0.467 m^2/sec$$

$$\Rightarrow y_{cr} = \left(\frac{0.467^2}{9.81}\right)^{1/3}$$

$$y_{cr} = 0.281$$

As

$$y > y_{cr}$$

$$0.575 > 0.281$$

So the flow is sub-critical.

Q3)

Given :-

Friction Drag (FD) = ?

Width (B) = 200mm = 0.2m

Length (L) = 800mm = 0.8m

Specific Gravity (S) = 0.89

Undistributed Velocity (U) = 5m/sec

Kinematic Viscosity (ν) = 0.93 × 10⁻⁴ m²/sec

Solution:-

Checking whether Flow is laminar or not

By Reynold Number,

$$R = \frac{DV}{\nu}$$

For smooth flat plate

$D = L$, $V = U$

so

$$R = \frac{LU}{\nu}$$

$$= \frac{0.8 \times 5}{0.93 \times 10^{-4}} = 43010$$

$43010 < 500{,}000 \longrightarrow$ Laminar

Now,

Using formula

$$F_f = C_f \cdot \rho \cdot \frac{V^2}{2} \cdot BL$$

where

$$C_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43010}}$$

$$= 0.0064$$

$$S = \frac{\rho_{oil}}{\rho_{water}} \Rightarrow 0.89 = \frac{\rho_{oil}}{1000}$$

$$\rho_{oil} = 0.89 \times 1000$$

$$\rho_{oil} = 890 \text{ kg/m}^3$$

$$\Rightarrow F_f = C_f \cdot \rho \cdot \frac{U^2}{2} \cdot BL$$

$$= 0.0064 \times 890 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$\boxed{F_f = 11.39 N}$$