



Program: MBA (3.5)

ID#15610

Subject: Statistical inference

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PART A (10)

Choose the best answer.

1. The process of estimation in statistics is associated with
 - a. Population parameters

2. A range of values used to estimate the characteristic is referred to as
 - b. Interval estimate

3. When sample size is large , for estimation we normally us
 - b. z distribution

4. When confidence interval is at 99% , the value of $z_{\alpha/2}$ is
 - b. 2.58

5. Chi square distribution is
 - c. uniform

6. We cannot determine the sample size for mean , if
 - a. Std. deviation is unknown
 - b. confidence interval is unknown
 - c. Both are unknown

7. An economist is interested in studying the incomes of consumers in a particular region. The population standard deviation is known to be \$1,200. A random sample of 36 individuals resulted in an average income of \$15,500. What will be the 98% confidence interval for the mean income? (2)
 - b. \$15,966 and \$ 15,034

8. Suppose $z_{\alpha/2} = 1.96$, $\hat{p} = 0.68$, and error is 0.05 the sample size(n) would be
 - c. 334 (2)

PART B

Q1: (a) A production department wants to check the defects in a particular type of part. A random sample of 500 parts was taken out of which 125 were defected. Construct 75% confidence interval for the proportion of defective parts. (4)

Solution:

$$100 - 75 = 25\%$$

$$\alpha = 25\%$$

$$25/100 = 0.25$$

$$z_{\alpha/2} = 0.25/2 = 0.125$$

$$1 - \alpha/2 = 1 - 0.125 = 0.8749$$

when i search in the z table, I have under column 0.05 and side value is 1.1

so, $1.1 + 0.05 = 1.15$

(b) Differentiate between (3+3)

- Z test T test

Z test	T test
Data size is greater than 30. Data is normally distributed. Data was randomly selected from a broader population. Sample size should be equal if possible.	Data size is smaller than 30. Data is not normally distributed. Data is not randomly selected. Sample size are not equal.

- Mean and standard deviation

Mean	Standard deviation
A mean is basically the average of a set of two or more number. Simple average of data .	Standard deviation is used to measure the volatility of a stock. Standard deviation is basically used for the variability of data and frequently use to know the volatility of the stock.

Q2: (a) A dentist likes to know the average number of fillings for his patients. He took a sample of 90 patients that show an average of 5 fillings with a standard deviation of 1.2 fillings. Determine 90% confidence interval for mean fillings per patient. **(6)**

Solution :

Solution:-

$$n=90$$

$$\bar{x} = 5$$

$$\sigma = 1.2$$

Find 90%

$$\alpha = 1 - 0.90 = 0.10$$

$$\alpha/2 = 0.10/2 = 0.05$$

From table value

$$Z_{0.05} = 1.645$$

Confidence Interval Estimates of the population mean, μ .
 The $(1-\alpha)100\%$ Confidence Interval for μ for Large Samples ($n \geq 30$).
 (i) $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ if σ is known and normally distributed population

$$5 \pm 1.645 \times 1.2/\sqrt{90}$$

$$5 \pm 0.2081$$

$$(4.7919, 5.2081)$$

(b) The diameter of a two years old Sentang tree is normally distributed with a Standard deviation of 8 cm. how many trees should be sampled if it is required to estimate the mean diameter within ± 1.5 cm with 99% confidence interval.

Solution:-

$$\sigma = 8\text{cm}$$

$$E = \pm 1.5$$

$$\text{C.I.} = 99\%$$

$$n = ?$$

$$1-\alpha = 0.99$$

$$\alpha = 1-0.99 = 0.01$$

$$\alpha/2 = 0.01/2 = 0.005$$

From table value

$$Z_{0.005} = 2.58$$

$$n = \frac{(Z_{\alpha/2})^2 \sigma^2}{E^2}$$

$$n = \frac{(2.58)^2 \times (8)^2}{(1.5)^2}$$

$$n = 189.34 \text{ Ans}$$

Hence, 189.34 or 189 trees should be sampled with 99%

Q3: (a) Variability of Carbonated content present in cans was estimated. Find 90% confidence interval for the variance of all such cans if a sample of 21 cans is selected

And it shows a standard deviation of 1.3ml. (5)

Solution:-

$$\text{Sample Size} = 21$$

Standard Deviation = 1.3

Find 90%

$$1-\alpha=0.90$$

$$\alpha=1-0.90=0.10$$

$$\alpha/2=0.10/2=0.05$$

$$1-\alpha/2=1-0.05=0.95$$

Now

$$X^2 \alpha/2 \text{ at df} = 21-1=20$$

$$\text{In the table value} = 31.41$$

$$X^2 1-\alpha/2 \text{ at df} = 21-1=20$$

$$\text{In the table value} = 10.85$$

Putting in formula

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{(1-\alpha/2)}}$$

$$\frac{20(1.3)^2 \leq \sigma^2 \leq 20(1.3)^2}{31.41 \quad 10.85}$$

$$1.076 \leq \sigma^2 \leq 3.115$$

(b) Find the tabulated value of (5)

Chi square

C.I 95% , df = 18

$$1-\alpha=0.95$$

$$\alpha=1-0.95=0.05$$

$$\alpha/2=0.05/2=0.025$$

$$1-\alpha/2=1-0.025=0.975$$

Tabulated value of $X^2 \alpha/2$ with df = 18 is (31.53)

And tabulated value of

$X^2 1-\alpha/2$ with df = 18 is (8.23)

T-Value

C.I 50% , df = 22

$$1-\alpha=50\%$$

$$\alpha=1-0.50=0.5$$

$$\alpha/2=0.5/2=0.25$$

so

$$t_{0.25}(22) = 0.6858$$

Z-Value

C.I 70%

$$1-\alpha = 0.70$$

$$\alpha = 1-0.70 = 0.30$$

$$\alpha/2 = 0.30/2 = 0.15$$

So,

$$Z_{0.15} = 1.036$$

Z-value

When $z = -1.78$

$$\begin{aligned} P(Z \leq -1.78) &= 0.5 - p(-1.78 \leq Z \leq 0) \\ &= 0.5 - 0.462 \\ &= 0.0375 \end{aligned}$$

Q4:(a) Write a brief analysis of the subject “*statistical inference*” in your own words.

The process of analysing the result and drawing conclusions from data subject to random variation is called statistical inference. Hypothesis testing and confidence intervals are the applications of the statistical inference. Statistical inference is a method of making decisions about the parameters of a population, based on random sampling. It helps to assess the relationship between the dependent and independent variables. The purpose of statistical inference to estimate the uncertainty or sample to sample variation.

The following elements are used for making statistical inference:

- Sample Size
- Variability in the sample
- Size of the observed differences.

Importance of Statistical Inference

Inferential Statistics is important to examine the data properly. To make an accurate conclusion, proper data analysis is important to interpret the research results. It is majorly used in the future prediction for various observations in different fields. It helps us to make inference about the data. The statistical inference has a wide range of application in different fields such as:

- Business Analysis
- Artificial Intelligence
- Financial Analysis
- Fraud Detection
- Machine Learning
- Share Market

- Pharmaceutical Sector

Types of Statistical Inference

There are different types of statistical inferences that are extensively used for making conclusions. They are:

- One sample hypothesis testing
- Confidence Interval
- Pearson Correlation
- Bi-variate regression
- Multi-variate regression
- Chi-square statistics and contingency table
- ANOVA or T-test

(b) The brightness of a television picture tube can be evaluated by measuring the amount of current required to achieve a particular brightness level. A random sample of 10 tubes indicated a sample mean 317.2microamps and a sample standard deviation is 15.7microamps. Find 99% confidence interval estimate for mean current required to achieve a particular brightness level. (6)

Solution:

$$n = 10$$

$$\text{sample mean } \bar{X} = 317.2$$

$$s = 15.7$$

$$\text{confidence interval} = 99\%$$

Formula:

The $(1-\alpha)100\%$ Confidence Interval for μ for small Samples ($n < 30$).

(i) $\bar{x} \pm t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$ if σ is not known, normally distributed population

$$= 317.2 \pm 3.250 \times 15 / \sqrt{10}$$

$$= 317.2 + 16.14 = 333.34 \text{ (upper)}$$

$$= 317.2 - 16.14 = 301 \text{ (lower)}$$