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I.D : 15385.

Subject :

Statistics

&

Probability.

Final Exam

Q1:

Sol:

The sample space S for this experiment is:

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$$
$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$$
$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$$
$$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

A = {Sum is 7}

B = {Sum is even}

C = {Sum is greater than 8}

D = {Both dice had same outcome}

$\Rightarrow A = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$\Rightarrow B = \{(1,1),(1,3),(1,5),(2,2),(2,4),(2,6) \ldots (6,6)\}$

$\Rightarrow C = \{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}$

$\Rightarrow D = \{(1,1), (2,2), (3,3), (4,4),$
$(5,5), (6,6)\}$

probability of $A = \frac{6}{36} = \frac{1}{6}$.

" " $B = \frac{18}{36} = \frac{3}{6} = \frac{1}{2}$

" " $C = \frac{10}{36} = \frac{5}{18}$.

" " $D = \frac{6}{36} = \frac{1}{6}$.!

Q2:

Sol:

Sum of 2 has 1 way (1,1)
Sum of 3 has 2 ways (1,2) & (2,1)
sum of 4 has 3 ways (1,3,2) & (2,3,1),
Sum of 5 has 4 ways
6 has 5 ways
8 has 5 ways
9 has 4 ways
10 has 3 ways
11 has 2 ways
12 has 1 way

Those are 15/36 for each side with a sum of 30/36.
That leaves a 6/36 = 1/6 probability for a sum of 7.

Q3.

Solution:-

Given that $p = 2/3$ $n = 8$

$q = 1 - p$

$= 1 - \frac{2}{3}$

$q = \frac{1}{3}$

Let "x" denotes the number of games won by A, Then

i) $P(x = 4)$

$= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$

$= \frac{1120}{6561}$

$= 0.1707$

ii) $P(X \geq 4)$

$1 - P(X < 4)$

$= 1 - \sum_{x=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$

$= 1 - \left[\left(\frac{1}{3}\right)^8 + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^7 + 28\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6 + 56\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5\right]$

$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$

$= 1 - \frac{577}{6561}$

$= \frac{6561 - 577}{6561}$

$= 5984/6561$

$= 0.9121$

iii) $P(3 \le X \le 6)$

$$\sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5 + \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4 + \binom{8}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^3 +$$

$$\binom{8}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^2$$

$$= \frac{8}{(3)^8}\{56+140+224+224)$$

$$= \frac{8 \times 664}{6561} = \frac{5152}{6561} = 0.7852$$

Q No: 4

Proof:-

Since the C_i's from a partition of the sample space we can apply the law of total probability for $A \cap B$

$$P(A \cap B) = \sum_{i=1}^{m} P(A \cap B / C_i)\, P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i)\, P(B/C_i)\, P(C_i)$$

$\because$ (A and B are conditionally independent)

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i)\, P(B)\, P(C_i)$$

$\because$ (B is independent of all C_i

$$P(A \cap B) = P(B) \sum_{i=1}^{m} P(A/C_i)\, P(C_i)$$

$$P(A \cap B) = P(B)\, P(A)$$

$\because$ Law of total probability

Hence A & B are independent.

Q 5

Sol:

Binomial distribution:

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}.$$

for $x = 0, 1, 2, \ldots, n$.

$\mu = np$ // mean.

$\sigma^2 = np(1-p)$ // variance.

A binomial random variable can be thought of as the sum of n independent Bernoulli random variables, each with mean p and variable $p(1-p$

Let $U_1, \ldots, U_n$ be independent Bernoulli random variables.

$E(U_i) = p$ & $Var(U_i) = p(1-p)$.

$X = U_1 + \cdots + U_n$

$Var(X) = Var(U_1) + \cdots + Var(U_n)$

Date: / /

The binomial theorem:

$$(a+b)^m = \sum_{y=0}^{m} \binom{m}{y} a^y b^{m-y}.$$

$$E(X) = \sum_{x} x\, p(x).$$

$$= \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}.$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}.$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!((n-1)-(x-1))!} \cdot p^{x-1} (1-p)^{(n-1)-(x-1)}$$

$$m = (n-1), \quad y = (x-1).$$

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}.$$

$$= np \sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}.$$

Now,

$$Var(X) = E[(X-\mu)^2]$$

$$= \sum_x (x-\mu)^2 p(x)$$

$$E[(x-\mu)^2] = E(X^2) - [E(X)]^2$$

$$E[X(X-1)] = \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} \cdot p^x (1-p)^{n-x}$$

$$E[X(X-1)] = n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{(n-2)-(x-2)}$$

By binomial theorem:

$$E(X(X-1)] = n(n-1)p^2$$

$$E(X^2 - X) = n(n-1)p^2$$

$$E(X^2) - E(X) = n(n-1)p^2$$

since $E(X) = np$, which is mean of binomial

$$E(X^2) = n(n-1)p^2$$

$$E(x^2) = n(n-1)p^2 + np$$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$Var(x) = np\,[(n-1)p + 1 - np].$$

This is variance of binomial distribution.

Q:6

Ans:

The Binomial distribution is denoted and formulated by

$$f(x) = P(X=x) = \binom{n}{x} p^x q^{n-x}.$$

where;

$x = 0, 1, 2, \ldots, n.$

It shows only the probability of an Individual.

while,

Binomial frequency distribution, if the binomial probability distributed is multiplied by N, the number of experiments, the resulting distribution is known as the binomial frequency distribution. Thus the expected frequency of x successes in N experiments is $N\binom{n}{x} p^x q^{n-x}$. It should be noted the n independent trails constitute one experiment.

Q7:

Solution:-

Measure	Data set A	B	C	D
Co-efficient of variation	$CV = \frac{3}{45} \times 100$	$CV = \frac{11}{60} \times 100$	$CV = \frac{5}{50} \times 100$	$CV = \frac{15}{25} \times 100$
	$CV = 6.7$	$CV = 18.3$	$CV = 10$	$CV = 60$