Name # Farhan Shah

ID # 13180

Subject # signal and system

paper # signal and system

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Tecaher # Sir Engr-Mujtaba Ihan

Q: (a): Show with a help of an equation that the differentiation of a function in time domain results in the multiplication by $j\omega$ in frequency domain.

<u>Answer:-</u>

Let $x(t)$ be a continuous time signal with a fourier tranform of $X(j\omega)$.

i.e

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

Differentiating both sides

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, \frac{d}{dt}\{e^{j\omega t}\}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\{e^{j\omega t}\cdot j\omega\}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \{j\omega X(j\omega)\}\, e^{j\omega t}\, d\omega$$

$$\boxed{F\left\{\frac{d}{dt}x(t)\right\} = j\omega X(j\omega)}$$

Result:-

We conclude that If a fuction is differentiated in time domain it is multipled by Jw in frequency domain from the above result we also conclude that If a function is differented in time domain. it multiplied with Jw in frequency domain. Similarly If a fuction is Integrated in time domain then it is divided by Jw frequency domain.

We Known that differentiation in time domain corresponds to multiplication Jw frequency domain. form the property we might suspect that multiplication by jt. in the time domain correspond roughly to differentiation in frequency domain.

As we know that.

$$x(jw) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

D ——— both side

$$\frac{d}{d\omega} X(j\omega) = \int_{-\infty}^{\infty} -jt\, x(t)\, e^{-j\omega t}\, dt$$

$$= -jt \int_{-\infty}^{\infty} (x(t) e^{-j\omega t}\, dt$$

$$= -jt\, \mathcal{F}\{x(t)\}$$

$$= \boxed{-jt\, x(t) \xleftrightarrow{\mathcal{F}} \frac{d}{dt} X(j\omega)}$$

part (B)

If $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

Solution:-

$Y(z) = H(z)X(z)$

Find $y[n]$

$X(z) = 2 - 4z^{-2} + 2z^{-3}$

$H(z) = 3 + 1z^{-1} + 2z^{-2}$

Now

$Y(z) = H(z) * X(z)$

$= (2 - 4z^{-2} + 2z^{-3})(3 + z^{-1} + 2z^{-2})$

$= 6 + 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4} + 6z^{-3} + 2z^{-4} + 4z^{-5}$

$= 6 + 2z^{-1} - 8z^{-2} - 2z^{-3} + 6z^{-4} + 4z^{-5}$

To find $y[n]$ use the delay property

$$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] - 2\delta[n-3] + 6\delta[n-4] + 4\delta[n-5]$$

Q2:-

$$f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$$

Retrieve the fourier series for the given function.

<u>Answer:</u>

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\,dx + \int_{0}^{\pi} \frac{\pi}{2}\,dx\right.$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\int_{-\pi}^{0} 1\cdot dx + \frac{\pi}{2}\int_{0}^{\pi} 1\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi/2 \, [x] \Big|_{-\pi}^{0} + \pi/2 \, [x] \Big|_{0}^{\pi} \right.$$

$$= \frac{1}{\pi}\left[-\pi/2 \, (0) - (-\pi) \right] + \pi/2 \, (\pi) - (0)$$

$$= 1/\pi \left[-\pi/2^{2} + \pi^{2}/2 \right]$$

$$= 1/\pi \, [0] = 0$$

Determine the coefficients an:

$$an = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx =$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} \frac{-\pi}{2} \cos nx \, dx + \int_{0}^{\pi} \pi/2 \cos nx \, dx$$

$$= \frac{1}{\pi} \left(-\pi/2 \int_{-\pi}^{0} \cos nx \, dx + \pi/2 \int_{0}^{\pi} \cos nx \, dx \right.$$

$$= \frac{1}{\pi} \left[-\pi/2 \, (\sin x) \right] \Big|_{-\pi}^{0} + \pi/2 \, [\sin nx] \Big|_{0}^{-\pi}$$

$$= \frac{1}{\pi} \left[-\pi/2 \, [\sin n(0) - \sin n(-\pi) \right) + \pi/2 (\sin (\pi) - \sin n(0)$$

$= \frac{1}{\pi} [-\pi/2 (0) + \pi/2 (0)) = 0$

Now b_n

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$

$= \frac{1}{\pi} (\int_{-\pi}^{0} -\pi/2 \sin dx = \int_{0}^{-\pi} \pi/2 \sin x dx$

$= \frac{1}{\pi} [-\pi/2 (- \cos nx |_{x}^{0} + \pi/2 (-\cos \frac{x}{2} |_{0}^{n}$

$= \frac{1}{\pi} [- \pi/2 (- \cos_2 (0) - (-(\cos x(-\pi)))$

$+ \pi/2 (-\cos n(\pi) - (-\cos n (0))]$

$= \frac{1}{\pi} [- \frac{\pi}{2} (-2) + 2\pi/2$

$= \frac{1}{\pi} [2\pi/2 + 2\pi/2] = \frac{1}{\pi} [4\pi/2]$

$= \frac{4\pi}{n^2 \pi} \frac{1}{2\pi} = \frac{1}{2n}$

$= \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{1}{2n} & \text{if } n \text{ is old} \end{cases}$

$= 0 + 0 + 0 + \ldots \frac{1}{2} \sin x + \frac{1}{4} \sin 2x$

$+ \frac{1}{6} \sin 3x - \_\_\_$

Q3:- If $X(z) = 2z^2 + 2z / (z^2 + 2z - 3)$
Retrieve $x[n]$ using Inverse Z-transform method

<u>Solution:-</u>

$$X(z) = \frac{2z^2 + 2z}{z^2 + 2z - 3}$$

$$\frac{X(z)}{z} = \frac{2z^2 - 2z}{(z-1)(z+3)}$$

$$= \frac{2z^2 + 2z}{} = \frac{A}{(z-1)} + \frac{B}{(z+3)} \rightarrow (A)$$

$$2z^2 + 2 = A(z+3) + B(z-1) \rightarrow (B)$$

put $z = -3$ in eq (B)

$$2(-3)^2 + 2(-3) =$$

$$A(\cancel{-3} + \cancel{3}) + B(-3-1)$$

$$2(9) - 6 = B(-4)$$

$$18 - 6 = B(-4)$$

$$B = 12/4 = \boxed{B - 4}$$

Put $z=1$ in eq (B)

$$2(1)^2+2 = A(1+3)+B(\cancel{1-1})^0$$

$$2+2 = A(4)$$

$$4 = A(4) \quad 4/4$$

$$A = \boxed{1}$$

Now put the value of A and B in eq (A)

$$\frac{2z^2+2}{(z-1)(z+3)} = \frac{1}{(z-1)} - \frac{-3}{(z+3)}$$

$$X(z) = 1\,\frac{z}{z-1} - 3\,\frac{z}{z+3}$$

Inverse z- Transform

$$x[n] = 1u[n] - 3(-3)^k$$

Q4:- Express the tranfer funtion using the given Data.

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

Solution:-

$$G(s) = C(sI - A)^{-1}B + D$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} \cancel{s f 1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ 0-1 & s-0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s^2+2s+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} s & 2 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} 1 & 2 \end{bmatrix} \times \frac{1}{s^2+2s+1} \begin{bmatrix} s+0 \\ 1 \end{bmatrix}$$

$$H(s) = \frac{\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}}{s^2+2s+1}$$ ✓

$$\boxed{H(s) = \frac{s+2}{s^2+2s+1}}$$

Q5:- Apply fourier transform on the signal $x(t) = e^{-a(t)}\,u(t)$ where $u(t)$ is a unit step.

Solution:-

The fourier transform of the given function $x(t)$ is given by.

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-a(t)}\, e^{-j\omega t}\, dt$$

Note:-

$e^{-a|t|}$

$$= \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & t < 0 \end{cases}$$

$\therefore$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

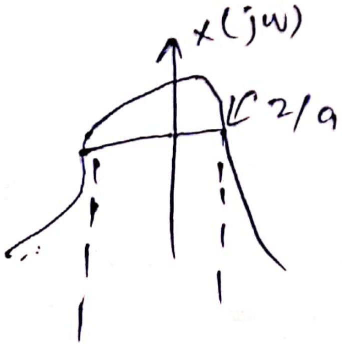
$$W = \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{e^{(a-j\omega)t}}{a-j\omega}\Bigg|_{-\infty}^{0} + \frac{e^{-(a+j\omega)t}}{-(a+j\omega}\Bigg|_{0}^{\infty}$$

$$= \frac{1}{(a-j\omega)}\left[e^{0} - e^{-\infty}\right] - \frac{1}{a+j\omega}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{(a-j\omega)}\left[1-0\right] - \frac{1}{(a+j\omega)}\left[0-1\right]$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$



$$= \frac{a+j\omega+a-j\omega}{a^2-(j\omega)^2} = (j\omega) = \frac{2a}{a^2+\omega^2}$$