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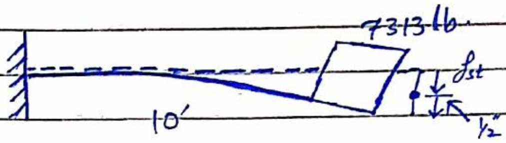
Paper: Intro to Structure dynamics & Earthquake Engineering.

QNo1:- A beam shown . . . .
. . . . froce with time.

$\omega_n = ?$

$I = 150\ in^4$

δ_{st} = due to 7313 lb.



Critical eq. of motion.

$$ku + c\dot{u} + m\ddot{u} = P(t) \quad \text{---} \ (1).$$

As the system vibrates freely, genny damping effects. the eq(1) becoms.

$$ku + m\ddot{u} = 0.$$

As:

$$K = \frac{3EI}{L^3}$$

$$\frac{3\ (2900\ k/in^2 \times 150\ in^4}{(10 \times 12\ in)^3}$$

$$= \frac{1305\,0,000.}{1728000.}$$

$$\boxed{k = 7.55 \text{ k/in.}}$$

$$7.55 \times 1000 \times 12.$$

$$90600 \text{ lb/ft.}$$

$$m = \frac{7313 \text{ lb sec}^2}{32.2 \text{ ft.}}$$

$$m = 227.11 \text{ slug.}$$

$$\omega_n = \sqrt{k/m}.$$

$$\omega_n = \sqrt{\frac{90600}{227.11}}$$

$$\omega_n = 19.97 \text{ rad/sec.}$$

Time Period.

$$T_n = \frac{2\pi}{\omega_n}.$$

$$T_n = \frac{2(3.14)}{19.97}.$$

$$T_n = 0.3144 \text{ sec.}$$

By subtituting values in (I).

$$90600u + 227.11\ddot{u} = 0.$$

General Sol. of Equation of Motion for undamped free vibration.

$$u(t) = u(0)\cos(\omega_n t) + \dot{u}(0)/\omega_n \sin(\omega_n t).$$

$$U(0) = \frac{1}{2}'$$

$$\frac{1}{2''} \quad \& \quad \dot{u}(0) = 0.$$

$$u(t) = \left(\frac{1}{24}\right)\cos(19.97t) + 0.$$

$$u(t) = \frac{1}{24}\cos(19.97t).$$

Equivalent static force at "t".

$$f_s(t) = K \cdot u(t) = \frac{90600 \times \cos(19.97}{24.}$$

$$f_s(t) = 3775(\cos(19.97t).$$

Now for u_o = Amplitude

Amplitude of equivalent static force.

$$Ku_o = 90600 \times \frac{1}{24}$$

$$Ku_o = 3775 \text{ lb}.$$

QNo:2: For the beam . . . .
. . . . force with time.

Equation of motion for damped free vibration:

$$ku + c\dot{u} + m\ddot{u} = 0.$$

As $k = 90600$ lb/ft & $m = 227.11$ lbsec²/ft.

$C = 2 \times \xi m\omega_n$. damping ratio 2.5% $= 0.025$.

$(0.025) \times 2 \times (227.11)(19.97)$.

$c = 226.76$ lbsec/ft.

Putting values in eq.

$$90600u + 226.76\dot{u} + 227.11\ddot{u} = 0.$$

Solution of equation of motion for damped free vibration.

$$U(t) = e^{-\xi\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0) - \xi\omega_n\right]\sin(\omega_D t)\right].$$

As:

$\omega_n = 19.97$ rad/sec.

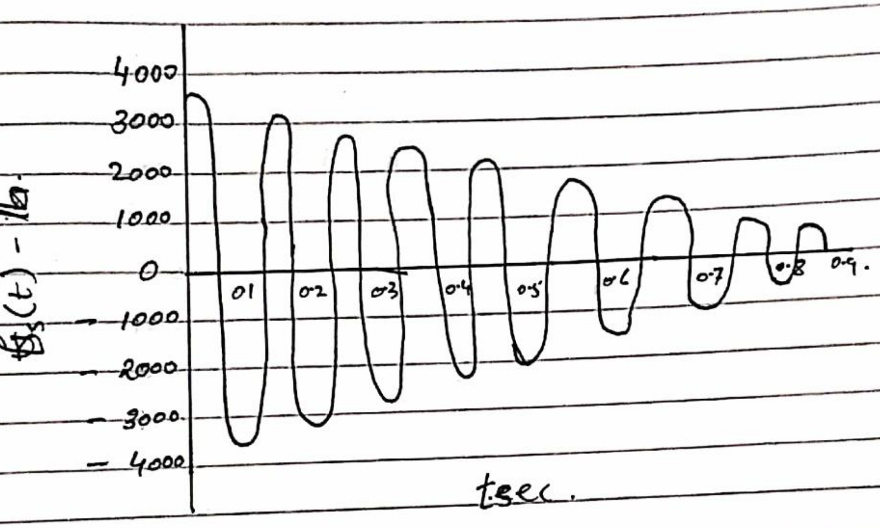
$$U(t) = e^{-0.025 \times 19.97}\left[u\frac{1}{24}\cos(29.97t) + \frac{1}{19.97}\left[0 + \frac{1}{24}s(0.025)(19.97t)\right) \times \sin(29.97t)\right].$$

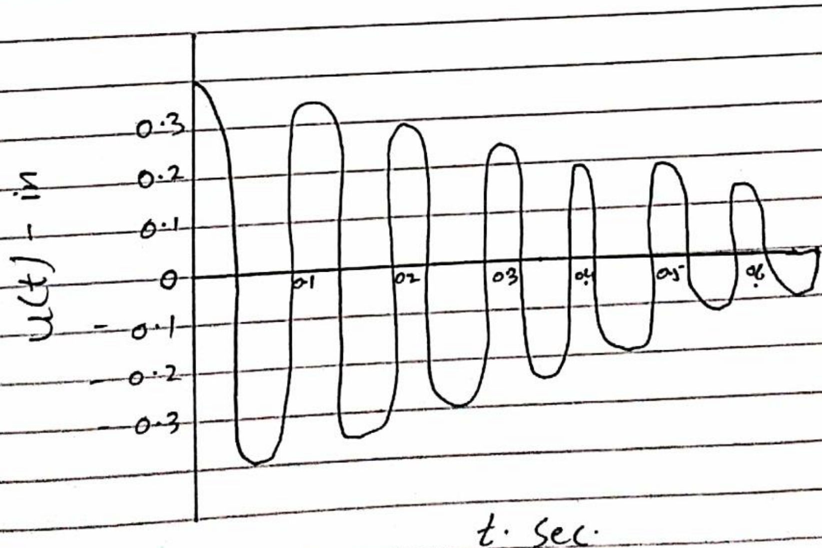
$u(t) = e^{-0.49t}\left(0.041 \cos(19.97t) + 0.00125 \times \sin(19.97t)\right).$

$f_s(t) = k \cdot u(t) \Rightarrow 90600 \times u(t).$

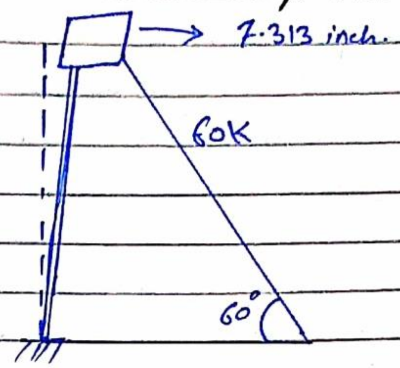
$f_{st} = e^{-0.49t}\,(90600 \times 0.041) \cos(19.97 + (90600 \times 0.00125)$
$\sin(19.97t)$

$f_{st} = e^{-0.49t}\left(3714.6 \cos(19.97) + 113.25 \sin(19.97t)\right).$





QNo:3:- A free vibration test........
.... amplitude to 0.5".



Cycles = 7.
T = 3.57 sec
Amplitude of displacement = 0.9"

Damping ratio.

$U_1 = 7.313$ inch.

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{U_1}{U_{j+1}}\right]$$

$$7 = \frac{1}{2(3.14)\zeta} \ln\left[\frac{7.313}{0.9}\right]$$

$$7(6.28)\zeta = \ln(8.125).$$

$$43.96\zeta = 0.90$$

$$\zeta = 0.90/43.96$$

$\zeta = 0.020$ x.ing by 100.

$\zeta = 2.04\%$.

(b) $T_n = ?$

if cycles of vibrations are completed in 3.57 sec

Time required to complete one cycle $\frac{3.57}{7} = T_D$.

$T_D = 0.51$ sec.

Now:- $\omega_D = \omega_n \sqrt{(1-\zeta^2)}$

$\frac{2\pi}{\omega_D} = 2\pi / \omega_n \sqrt{(1-\zeta^2)}$

$T_D = \frac{T_n}{\sqrt{(1-\zeta^2)}}$

$T_n = T_D \sqrt{(1-\zeta^2)}$

$= 0.51 \sqrt{1-(0.020)^2}$

$= 0.51 \sqrt{0.99958}$.

$= T_n = 0.5099$ sec.

(c) $K = ?$

$K = ?$

$K = \frac{60 (\cos 60^\circ)}{7.313}$

$K = 4.1022$ k/in.

49220 lb/ft.

(D): $w_n = \sqrt{k/m} = \sqrt{k/(w/g)}$

$\sqrt{\frac{k g}{w}}$

$w = Mg/w_n^2$

$w_n = 2\pi/T_n$

$w = kg \frac{T_n^2}{4\pi^2}$

$w = \frac{49220}{ft} \times \frac{32.2}{sec} = \frac{(0.5094)}{4\pi^2}$

$w = 1584884(0.01291)$

$w = 20460.0900$ lb.

$w = 20.46$ kl.

(C):- $c = ?$

$\zeta = \frac{c}{2mw_n}$

$c = \zeta\, 2m w_n$

$\zeta\, 2m\,(2\pi/T_n)$

$$c = \frac{(0.020)\,4(3.14)\left(\frac{2046.90}{32.2}\right)}{0.5}$$

$c = 384.33$ lb.sec/ft

(f) No. of cycle to reduce displacement Amplitude from

7.313 to 0.5 in $j = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2\times(0.020)} \ln\left(\frac{7.313}{0.5}\right)$$

$$j = \frac{1}{0.04} \ln 1.165.$$

$j = 8.96$ or 9 cycles.