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Section:

B

Deptt:

BE(C)

Subject:

Differential

Submitted to:

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## Question No 01:

The wave Equation

We generally ______

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

Where w ______

Show the following functions are all Solution of the wave equation by determining relevent Partial derivatives.

i. $w = \sin(x + ct) + \cos(2x + 2ct)$

## Solution:

$$w = \sin(x+ct) + \cos(2x+2ct$$

$$\frac{\partial w}{\partial t} = \left[\cos(x+ct)\cdot c\right] + \left[-\sin(2x+2ct).2c\right.$$

$$= c\cos(x+ct) - 2c\sin(2x+2ct)$$

$$\frac{\partial^2 w}{\partial t^2} = c - \sin(x+ct)\cdot c - 2c\cos(2x+2ct)\cdot 2c$$

$$\boxed{\frac{\partial^2 w}{\partial t^2} = -c^2 \sin(x+ct) - 4c^2\cos(2x+2ct)}$$

$\rightarrow$ (i)

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct)\cdot 2$$

$$\frac{\partial^2 w}{\partial^2 x} = -\sin(x+ct) - 2\,\overset{\cos}{\cancel{\sin}}\,(2x+2ct)\cdot 2$$

$$\boxed{\frac{\partial^2 w}{\partial^2 x} = -\sin(x+ct) - 4\,\overset{\cos}{\cancel{\sin}}\,(2x+2ct)}$$

From equ

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

$$= -c^2 \sin(x+ct) - 4c^2 \cos(2x+2ct) =$$

$$-c^2 \sin(x+ct) - 4c^2 \cos(2x+2ct)$$

So proved

(ii)

$w = \tan(2x+ct)$

**Sol:**

$$w = \tan(2x+ct)$$

$$\frac{\partial w}{\partial t} = \sec^2(2x+ct) \, \frac{\partial}{\partial t}(2x+ct)$$

$$= c \sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = c^2 \sec(2x+ct) \frac{\partial}{\partial x} \sec(2x+ct)$$

$$= c^2 \sec(2x+ct) \sec(2x+ct) \tan(2x+ct)$$

$$\boxed{\frac{\partial^2 w}{\partial^2 t} = 2c^2 \sec^2(2x+ct)\tan(2x+ct)}$$

$$\frac{\partial w}{\partial x} = \sec^2(2x+ct)\cdot 2$$

$$2\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial^2 x} = 2\cdot 2\sec(2x+ct)\cdot \sec(2x+ct)$$

$$\cdot\tan(2x+ct)\cdot 2$$

$$= 8\sec^2(2x+ct)\tan(2x+ct)\cdot 2$$

$$= 8\sec^2(2x+ct)\tan(2x+ct)$$

$$= 2c^2\sec^2(2x+ct)\tan(2x+ct) \neq$$

$$c^2 8\sec^2(2x+ct\cdot\tan(2x+ct)$$

From equ $\frac{\partial^2 w}{\partial t^2} = c^2\frac{\partial^2 w}{\partial^2 x}$ Not Satisfy

# Question No 02.

Expand The following function in a Fourier Series

$$Fx = x, \quad -\pi < x \leq 0$$
$$= 2x, \quad 0 \leq x \leq \pi$$

## Solution:

We have to find fourier Series Co-efficient, $a_0$, $a_n$ & $b_n$

Now

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$\boxed{a_0 = -\frac{\pi}{2} + \pi = \frac{\pi}{2}} \longrightarrow (i)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0} +$$

$$\frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right]_{0}^{\pi}\right.$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2} + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \right.\right.$$

$$\left.\frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right]$$

$$= \frac{(-1)^n - 1}{\pi n^2}$$

So

$$\boxed{a_n = \begin{cases} -\dfrac{2}{\pi n^2} & ; \text{ if } n \text{ is odd} \\ 0 & ; \text{ if } n \text{ is even} \end{cases}}$$ (11)

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} + \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right]$$

$$= -\frac{3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

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So the required fourier

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$\boxed{= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2x-1)}{(2x-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}}$$

# Question No 03:

$$y'' - 4y' + 13y = 8\sin 3x, \; y(0) = 1$$
$$\text{and } y'(0) = 2$$

## Solution:

$$y'' - 4y' + 13y = 8\sin 3x, \; y(0) = 1$$
$$\text{and } y'(0) = 2$$

Homogenous equation is

$$y'' - 4y' + 13y = 0 \longrightarrow (2)$$

$$8\sin 3x \longrightarrow (1)$$

Change (2) into Auxilary Eqn

Put $y = m$ in (2)

$$m^2 - 4m + 13 = 0$$

Now quadratic formula

$a = 1$, $b = -4$ ~~&~~ $c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm \sqrt{36}i}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$= 2 \pm 3i$$

$$\boxed{m_1 = 2 + 3i}$$
$$\boxed{m_2 = 2 - 3i}$$

$$y_c = e^{2x}(C_1 \cos 3x + C_2 \sin 3x) \text{ --- } (A)$$

Diff w.r.t "x"

$$y_p' = -3A\sin 3x + 3B\cos 3x$$

Again diff w.r.t x

$$y_p'' = -9A\cos 3x - 9B\sin 3x$$

Put in equ (1)

$$= (-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos^3 x) + 13(A\cos 3x + B\sin 3x) - 8\sin 3x$$

$$= -9A\cos 3x - 12B\cos 3x + 13A\cos 3x - 9B\sin 3x + 12A\sin 3x + 13B\sin 3x - 8\sin 3x$$

$$= (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x = 8\sin 3x$$

Comparing Co-efficient

$\sin 3x$, $4B + 12A = 8$ — (a)

$4A - 12B = 0 \Rightarrow 4A - 12B$

$\boxed{A = 3B}$ — (b)

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Put b in a

$4B + 12(3B) = 8$

$40B = 8$

$\boxed{B = 1/5}$ — 1

Put 1 in b

$\boxed{A = 3/5}$ — d

Put c & d in X

$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \longrightarrow$ B

the general solution is;

$y = y_c + y_p$

$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}$
$\sin 3x \longrightarrow$ C

Now we need to find the values of $c_1$ & $c_2$ for this

Put $x=0$ & $y=1$ in (C)

$$1 = e^{x(0)} \cdot (c_1 \cos 3(0) + c_2 \sin 3(0)) + \frac{3}{5} \cos 3(0) + \frac{1}{5} \sin 3(0)$$

$$1 = (c_1 (1)) + c_2 (0) + \frac{3}{5}(1) + \frac{1}{5}(0)$$

$$1 = c_1 + 3/5$$

$$\boxed{c_1 = 2/5} \longrightarrow \text{(XX)}$$

Differentiate c w.r.t "x"

$$y' = c_1 (2e^{2x} \cos 3x - 3e^{2x} \sin 3x) + c_2 (2e^{2x} \sin 3x + 3e^{2x} \cos 3x) - \frac{6}{5} \sin 3x + \frac{3}{5} \cos 3x \longrightarrow \text{(D)}$$

Put $y' = 2$, $x = 0$ in (D)

$\therefore y' = C_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) +$

$C_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x - 6/5$

$\sin 3x + 3/5 \cos 3x$

Put $y' = 2$ $x = 0$

$2 = C_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)) +$

$C_2(2e^{2(0)}\sin(0) + 3e^{2(0)}\cos 3(0))$

$2 = C_1(2) + C_2(3) - 0 + 3/5$

$2 = 2C_1 + 3C_2 + 3/5$

Put $C_1 = 2/5$

$2 = 4/5 + 3C_2 + 3/5$

$2 = 7/5 + 3C_2$

$3C_2 = 3/5$

$$\boxed{C_2 = 3/15} \text{ --- } (xxx)$$

Put (xx) & (xxx) in (c)

$$y = e^{3x}\left(\tfrac{2}{5}\cos 3x + \tfrac{3}{15}\sin 3x\right) + \tfrac{3}{5}\cos 3x + \tfrac{1}{5}\sin 3x$$

$$\boxed{y = \tfrac{2}{5}e^{2x}\cos 3x + \tfrac{3}{15}e^{2x}\sin 3x + \tfrac{3}{15}\cos 3x + \tfrac{1}{5}\sin 3x}$$

Required G.E

## Question No 04:

Solve

$$(D^2 - DD')Z = \cos x \cos 2y$$

## Solution:

$$(D^2 - DD')Z = \cos x \cos 2y$$

the given PDE can be rewrite as $D(D-D')Z = \cos x \cos 2y$ in CF is given by

$$CF = \phi_1(y) + \phi_2(y+x)$$

while its PI is given by

$$PI = \frac{1}{(D^2 - DD')} \cdot \frac{1}{2}\left[\cos(x+2y) + \cos(x-2y)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(-1+2)}\cos(x+2y) + \frac{1}{(-1-2)}\cos(x-2y)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(-1+2)}\cos(x+2y) + \frac{1}{(-1-2)}\cos(x-2y)\right.$$

$= \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$

Hence the Complete Solution of the PDE is :

$$\boxed{Z = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-y)}$$