Mid term Exam Summer 2020.

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Q NO # 01:

The function g (t) is denoted by g(t) = 0, t < 0

t^2 IIK10 $0 \leq t \leq 3$

$2t + 3$ $3 < t \leq 4$

12 $t > 4$

(a) State any point of discontinuity.

(b) Find if they Exist.

i) $\lim_{t \to 3} g$

Sol:-

a) TO check possibility of the discontinuity of the function is at $t = 0$ and 4

First at $t = 0$

$g(t) = t^2$

$g(0) = 0^2 = 0$

for R.H.L

$\lim g(1+h) = \lim (1+h)^2$

$= \lim_{h \to 0} 1 + h^2 + 2h$

apply limit.

$= 1 + 0^2 + (2(0)$

$= 1$

For L.H.L

$\lim_{h \to 0} g(1-h) = 2t+3$

$\lim_{h \to 0} 2(1-h)+3$

$= \lim_{h \to 0} 2-2h+3$

apply limit

$2-2(0)+3$

$\boxed{=5}$

R.H.L ≠ L.H.L = g(t) = 5

now at t=4

$g(4) = 2(4)+3$

$= 8+3 = \boxed{11}$

for R.H.L.

$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h)+3$

$= \lim 2+2h+3$

apply limits.

$= 2+2(0)+3 \Rightarrow 5$

for L.H.L

$\lim_{h \to 0} g(1-h) = 12$

$g(4) =$ R.H.L = L.H.L point of discontinuity is at $t=4$.

Q no# 02: Find the maclorin's series for

$y(x) = x^2 + \sin x$

Sol:- $f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$

let $y(x) = x^2 + \sin x$

$y'(x) = 2x + \cos x$

$y''(x) = 2 - \sin x$

$y'''(x) = -\cos x$

put $x=0$ in all function

$y(0) = 0$

$y'(0) = 1$

$y''(0) = 2$

$y'''(0) = -1$

putting value in formula.

$= 0 + x(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(-1)$

$= 0 + x + x^2 - \frac{x^3}{6}$

$x^2 + \sin x = x + x^2 - \frac{x^3}{6}$ Ans

Part (b) Find if they exist.

$\lim_{t \to 3} g$

for $g(t) = t^2$

R.H.L: $\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^3$

$\lim_{h \to 3} + h^2 + 2h$

Apply limit.

$\boxed{= 1 + 3^2 + 2(3) \Rightarrow 16}$

L.H.L

$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t + 3$

$= \lim_{h \to 3} 2(1-h) + 3$

$= \lim_{h \to 3} 2 - 2h + 3$

apply limit

$= 2 - 2(3) + 3$

$= 2 - 6 + 3 = \boxed{-1}$

R.H.L $\neq$ L.H.L (do not exist since L.H.L is -ve)

QNo# 03 Find y'' given

$$1 + xy = x^2 + y^2$$

Sol:-

$$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$0 + x\frac{d}{dx} + y\frac{dx}{dx} = 2x^{2-1} + 2y^{2-1}\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx}(x - 2y) = 2x - y$$

$$\frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

$$y' \quad \frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

(pto)

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{2x-y}{x-2y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$$

$$y'' = \frac{2x - xy' - 4y + 2yy' - (2x - 4xy' - y + 2yy')}{(x-2y)^2}$$

$$y'' = \frac{2x - xy' - 4y + 2yy' - 2x + 4xy' + y - 2yy'}{(x-2y)^2}$$

$$y'' = 4xy' - 3y - xy' \Big/ (x-2y)^2$$

$$y'' = 4x\left[\frac{2x-y}{x-2y}\right] - 3y - x\left[\frac{2x-y}{x-2y}\right] \Big/ (x-2y)^2$$

$$y'' = \frac{4x(2x-y) - 3y(x-2y) - x[2x-y)}{(x-2y)(x-2y)^2}$$

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$$y'' = \frac{8x^2 - 4xy - 3xy + 6y^2 - 2x^2 + xy}{(x-2y)^3}$$

$$\cancel{y'' = \frac{6x^2 + 6y^2 - 6xy}{(x-2y)^3}}$$

$$y'' = \frac{6x^2 + 6y^2 - 6xy}{(x-2y)^3}$$

Ans

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Q no# 03

part (B) Find y' by using logrithmic differentiation

$$y = x^3 (1+x)^9 e^{6x}$$

Sol:- taking $\ln$ on b/s

$$\ln y = \ln(x^3 (1+x)^9 \cdot e^{6x})$$

$$\ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

$$\ln y = 3\ln x + 9\ln(1+x) + \frac{d}{dx} 6x \times \ln e$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x} + \frac{1}{e^{6x}} \cdot 6$$

$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}}\right)$$

$$\frac{dy}{dx} = x^3(1+x)^9 e^{6x}\left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}}\right)$$

Solve