Question No 1

Part 1

$$w = \sin(x+ct) + \cos(2x+2ct)$$

$$\boxed{\frac{\partial^2 w}{\partial t^2} = \frac{c^2 \partial^2 w}{\partial x^2}}$$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct)$$
$$+4c^2 \text{ ---- } (i)$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct) + 2$$

$$\frac{\partial^2 w}{\partial x} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= [-\sin(x+ct) - 4\cos(x+2ct)$$

$$\frac{\partial^2 w}{\partial t^2} = +c^2[-\sin(x+ct) - 4(\cos(2x+2ct)]$$

$$\boxed{c^2 \frac{\partial^2 w}{\partial x^2}}$$

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Question No 1
Part ii

$w = \tan(2x + ct)$

Now: $\frac{\partial w}{\partial t} = c\sec^2(2x+ct)$

$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left(c\sec^2(2x+ct)\right)$

$= c^2\, 2\sec^2(2x+ct)\tan(2x+ct)$

Now $\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$

$\frac{\partial^2 w}{\partial t^2} = 4\sec^2(2x+ct)\tan(2x+ct)$

$\Rightarrow \cancel{4c^2\sec^2(2x+ct)\tan(2x+ct)} =$

$\cancel{4c^2\sec^2(2x+ct)\tan(2x+ct)}$

$0 = 0$ (Satisfied)

Question No 2

Given function is

$$F(x) = \begin{cases} x; & -\pi < x \le 0 \\ 2x; & 0 \le x \le \pi \end{cases}$$

We have to find the Fourier co-efficients a_0, a_n & b_n

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\pi^2}{\cancel{2}} - 0\right]$$

$$\boxed{a_0 = \frac{-\pi}{2} + \pi = \frac{\pi}{2}} \rightarrow (1)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} (x \cos nx \, dx) + \frac{1}{\pi} \int_{0}^{\pi} (2x \cos nx) dx$$

$$= \frac{1}{\pi} \left[x \left(\frac{\sin nx}{x} \right) - \left(- \frac{\cos nx}{n^2} \right) \right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi} \left[x \left(\frac{\sin nx}{n} \right) - \left(- \frac{\cos nx}{n^2} \right) \right]_{0}^{-\pi}$$

$$a_n = \frac{1}{\pi} \left[\frac{\cos(0)}{n^2} - \frac{\cos nx}{n^2} \right] + \frac{2}{\pi} \left[\frac{\cos nx}{n^2} - \frac{\cos(0)}{n^2} \right]$$

$$= \frac{1}{\pi} \left[\frac{1 - (-1)^n + 2(-1)^n - 2}{n^2} \right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$\boxed{a_n = \begin{cases} \frac{-2}{\pi n^2} ; & \text{if } n \text{ is odd} \\ 0 ; & \text{if } n \text{ is even} \end{cases}} \rightarrow ①$$

Question No 3

Solution

Associated Homogenous Eq of (1) is $y'' - 4y' + 13y = 0 \rightarrow (2)$

change (2) into Auxiliary equation

put $y = m$ in eq (2)

$m^2 - 4m + 13 = 0$

use Quadratic Formula

$a = 1 \quad b = -4 \quad c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$

$= 4 \pm \sqrt{+36}$

$= 4 \pm \sqrt{36i}$

$= \dfrac{4 \pm \sqrt{36}\, i}{2}$

$= \dfrac{4 \pm 6i}{2}$

$= 2 \pm 3i$

$$\boxed{\begin{aligned} m_1 &= 2 + 3i \\ m_2 &= 2 - 3i \end{aligned}}$$

$y_c = e^{2x}(C_1 \cos 3x + C_2 \sin 3x) \to (A)$

Let $=$ $y_p = A\cos 3x + B \sin 3x \to (*)$

Diff w.r.to x

$y'_p = -3A \sin 3x + 3B \cos 3x$

Again Diff w.r.to x

$$y_p'' = -9A\cos 3x - 9B\sin 3x$$

put in (i)

$$\Rightarrow (-9A\cos 3x - 9B\sin 3x) - 4(-3x\sin 3x + 3B\cos 3x) + B(A\cos 3x + B\sin 3x) + 8\sin 3x$$

$$\Rightarrow -9A\cos 3x - 12B\cos 3x + 13A\cos 3x - 9B\sin 3x + 12A\sin 3x + 13B\sin 3x - 8\sin 3x$$

$$\Rightarrow (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x = 8\sin 3x$$

$$\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)\sin 2x = 8\sin 3x$$

comparing Co-efficients

$\sin 3x$ $\quad 4B + 12A = 8 \rightarrow$ (a)

$\cos 3x$ $\quad 4A - 12B = 0 \Rightarrow 4A = 12B$

$$\boxed{A = 3B} \rightarrow (b)$$

put (a) in (b)

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$\boxed{B = 1/5} \to (c)$

put (c) in (b)

$\boxed{A = 3/5} \to (d)$

put (c) & (d) in (*)

$y_p = \frac{3}{5} \cos 3x + 1/5 \sin 3x \to (B)$

The G sol is

$y = y_c + y_p$

$$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (c)$$

Now we need to find the values of c_1 & c_2 for this

put $x = 0$ & $y = 1$ in (c)

$$1 = e^{2(0)}(c_1 \cos 3(0) + c_2 \sin(3(0)) + \frac{3}{5}\cos 3(0) + \frac{1}{5}\sin(3)(0)$$

$$1 = c_1(1) + c_2(0)) + \frac{3}{5}(1) + \frac{1}{5}(0)$$

$$1 = c_1 + \frac{3}{5}$$

$$c_1 = 1 - \frac{3}{5}$$

$$\boxed{c_1 = \frac{2}{5}} \rightarrow (**)$$

Diff (c) w.r.to "x"

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$$

$- 6/5 \sin 3x + 3/5 \cos 3x \rightarrow (D)$

put $y' = 2,\quad x = 0$ in (D)

$$y' = c_1 (2e^{2x} \cos 3x - 3e^{2x} \sin 3x) + c_2 (2e^{2x} \sin 3x + 3e^{2x} \cos 3x) - 6/5 \sin 3x + 3/5 \cos 3x$$

put $= y' = 2 \qquad x = 0$

$$2 = c_1 (2e^{2(0)} \cos 3(0) - 3e^{2(0)} \sin 3(0)) + c_2 (2e^{2(0)} \sin 3(0) + 3e^{2(0)} \cos 3(0)) - 6/5 \sin 3(0) + 3/5 \cos 3(0)$$

$$2 = c_1 (2) + c_2 (3) - 0 + 3/5$$

$$2 = 2c_1 + 3c_2 + 3/5$$

put $c_1 = 2/5$

$$2 = 4/5 + 3c_2 + 3/5$$

$$2 = 7/5 + 3c_2$$

$3C_2 = 2 - 7/5$

$3C_2 = 3/5$

$\boxed{C_2 = 3/25} \rightarrow \boxed{***}$

put $**$ & $***$ in (c)

$y = e^{2x}(2/5 \cos 3x + 3/25 \sin 3x) + 3/5 \cos 3x + 1/5 \sin 3x$

$$\boxed{y = 2/5 e^{2x} \cos 3x + 3/25 e^{2x} \sin 3x + 3/5 \cos 3x + 1/5 \sin 3x}$$

Required General

Solution.

Question No 4

Solution

It is already is symbolic form

$$(D^2 - DD')Z = \cos x \cos 2y \text{ --- (a)}$$

put A.E $D^2 - DD' = 0$

As we know

$\frac{D}{D'} = m$ i.e $D = m, D' = 1$

$m^2 - m = 0$

Therefore $C.F = f_1(y) + f_2(y+x)$

from eq (a)

$$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y$$

$$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} 2\cos x \cos 2y$$

As

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$CF = f_1(y-x) + x f_2(y-x)$$

$$PI = \frac{1}{D^2 + 2DD' + D'^2}\left[2(y-x) + \sin(x-y)\right]$$

$$= \frac{1}{(D+D')^2}\left[2(y-x) + \sin(x-y)\right]$$

By General Method

$$m = -1; \quad y - x = c$$

$$= \frac{1}{D+D'}\int\left[2c + \sin(-c)\right]dx$$

$$\frac{1}{D+D'}\left[2cx - (\sin c)x\right]$$

Replacing c by $y-x$

$$= \frac{1}{D+D'}\left[2x(y-x) - x\sin(y-x)\right]$$

Again put $y-x=0$

$$\int (2xc - x\sin c)\,dx \Rightarrow (x^2c - \frac{x^2}{2}\sin c$$

Replacing c by $y-x$

$$x^2(y-x) - \frac{x^2}{2}\sin(y-x) = x^2y - x^3 + \frac{x^2}{2}$$
$\sin(x-y)$

Hence the required Solution is

$$Z = C.F + P.I = f_1(y-x) + x f_2(y-x)$$
$$+ x^2y - x^3 + \frac{1}{2}x^2\sin(x-y)$$

—x—x—

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