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Subject A.Calculs

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Q: No: 1 :-

Find PQ where P is the point in three-dimentional Space with coordinates (4, 1, 3) and the point Q with coordinate (1, 2, 4).

i)- Find Distance b/w P and Q.

ii)- Find position vector of the point dividing PQ in the ratio 1:3

Solution :-

i. coordinate of P = (4, 1, 3)

In vector form: $\vec{OP} = 4i + 1j + 3k$

$\vec{OQ} = i + 2j + 4k$

Here

$OQ = \vec{OQ} - \vec{OP}$

putting above vectors

$OQ = (i + 2j + 4k) - (4i + 1j + 3k)$

$= -3i + 1j + 1k$

Now distance between P and Q = $|PQ|$

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9 + 1 + 1}$

$= \sqrt{11} \longrightarrow$ Required solution ii)

Let M be the point which divided PQ in ratio 1:3, then by the ratio theorm position vector of M = $\vec{OM}$

$$= \frac{3(\vec{OP}) + 1(\vec{OQ})}{1+3}$$

putting values of "$\vec{OP}$" and "$\vec{OQ}$"

$$= \frac{3(4i + 1j + 3K) + 1(i + 2j + 4K)}{1+3}$$

$$= \frac{12i + 3j + 9K + i + 2j + 4K}{4}$$

$$= \frac{13i + 5j + 13K}{4}$$ — Required Solution

Q: No: 2:

Evaluate

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x}\,dx$$

Solution :-

Denominator have 2 variables i.e "x^2" & 'x'
so first we should rearrange
whole equation

Now,

$$\begin{array}{r|l} & 2x - 1 \\ \hline 2x^2 + x & 4x^3 + 10x + 4 \\ & \pm 4x^3 \quad \pm 2x^2 \\ \hline & -2x^2 + 10x + 4 \\ & \mp 2x^2 \mp x \\ \hline & 11x + 4 \end{array}$$

So, the new equation

$$2x - 1 + \frac{11x + 4}{2x^2 + x} = \frac{4x^3 + 10x + 4}{2x^2 + x}$$

$\int \frac{4x^3 + 10x + 4}{2x^2 + x}\,dx$ becomes,

$$= \int \left(2x - 1 + \frac{11x + 4}{2x^2 + x} \right) dx$$

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = \int 2x-1 + \int \frac{11x+4}{2x^2+x} \quad \text{--- (i}$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \longrightarrow \text{(ii}$$

First we are going to find
" $\int \frac{11x+4}{x(2x+1)}\,dx$ ".

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \quad \text{--- } (\star)$$

$$\frac{11x+4}{x(2x+1)} = \frac{A(2x+1)+Bx}{x(2x+1)}$$

Denominator cancle on both sides

$$11x+4 = A(2x+1) + Bx \quad \text{--- (iii}$$

put $x=0$ in eq (iii)

$\lfloor 4 = A \rfloor$

Now put $x = -1/2$ in (iii

$$11(-1/2) + 4 = B(-1/2)$$

$$-\frac{11}{2} + 4 = -B/2$$

$$-\frac{11+8}{2} = -\frac{B}{2}$$

$$-3 = -B \Rightarrow$$

$$B = 3$$

putting values of A and B in equation (*)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Now taking Integral on Both sides

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$\because \int \frac{1}{x} = \ln(x)$

$$= 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

Now, putting values in Equation (ii

$$= x^2 - x + 4\ln|x| + \frac{3}{2}\ln(2x+1)$$

Now put these value in (i

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + c$$

Required Answer

Q : No : 3 :-

Evaluate :

a) $\int_0^2 x^2 e^x \, dx$

b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}} \, dx$

Solutions:-

a) $\int_0^2 x^2 e^x \, dx$

There are two variables "x^2" & "e^x".
So,
We use Integration by Parts.

$$A = x^2 \int_0^2 e^x - \int_0^2 \int_0^2 e^x \frac{d}{dx}(x^2) \, dx$$

$$= |x^2 e^x|_0^2 - \int_0^2 2x e^x \, dx$$

Here, taking Integration by Parts again,

$$= 4e^2 - 2\left[|x \cdot e^x|_0^2 - \int_0^2 e^x \, dx \right]$$

$$= 4e^2 - 2\left[2e^2 - |e^x|_0^2 \right]$$

$$= 4e^2 - 2\left[2e^2 - (e^2 - 1) \right]$$

$$A = 4e^2 - 4e^2 + 2(e^2 - 1)$$

$$A = 2(e^2 - 1) \quad \text{— Required Answer}$$

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b)- $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

suppose:

$$B = \int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx \quad \text{— (i}$$

substitution:

put $\sqrt{x} = u$

$$\frac{1}{2\sqrt{x}}\, dx = du$$

$$\frac{dx}{\sqrt{x}} = 2\, du$$

At $x \to 1,\ u \to 1$
$x \to 2,\ u \to \sqrt{2}$

put the substitution values in eq (i

$$B = \int_1^{\sqrt{2}} \sin u \cdot 2\, du$$

$\because \int \sin x = \cos x$

$$B = -2 \left| \cos u \right|_1^{\sqrt{2}}$$

$$B = -2(\cos\sqrt{2} - \cos(1))$$

or

$$B = 2(\cos(1) - \cos\sqrt{2}) \quad \text{— Required Answer}$$

Q: No: 4 :-

Verify that

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies three-dimensional Laplace's Equation.

Solution:-

As we know
Laplace three-dimensional laplace eq is;

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial^2 y} + \frac{\partial^2 u}{\partial z^2} = 0 \longrightarrow (i$$

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

re-arrange

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

For x :-

Taking partial differentiation w.r.t "x"

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

Again take partial differentiation w.r.t "x"

$$\frac{\partial^2 u}{\partial^2 x} = \frac{\partial^2 u}{\partial^2 x}\left(-x \cdot (x^2+y^2+z^2)^{-3/2}\right)$$

Using differentiation by parts

$$\frac{\partial^2 u}{\partial^2 x} = -\left[x\,(-3/2)\,(x^2+y^2+z^2)^{-5/2}\,(2x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial^2 x} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \longrightarrow (a$$

For y :-

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

Taking partial differentiation w.r.t "y"

$$\frac{\partial u}{\partial y} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2y)$$

$$\frac{\partial u}{\partial y} = -y\,(x^2+y^2+z^2)^{-3/2}$$

Again take partial differentiation w.r.t y"

$$\frac{\partial^2 u}{\partial^2 y} = -\left[y\,(-3/2)\,(x^2+y^2+z^2)^{-5/2}\,(2y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \longrightarrow (b$$

For 'z' :-

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

Taking partial differentiation w.r.t 'z'

$$\frac{\partial u}{\partial z} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z.(x^2+y^2+z^2)^{-3/2}$$

Now, Again taking partial diff w.r.t z

$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \longrightarrow (c)$$

putting Equation "a", "b" and "c" in Equation (i)

$$\Rightarrow 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2 - x^2 - y^2 - z^2 + 3y^2 - x^2 - y^2 - z^2 + 3z^2 - x^2 - y^2 - z^2\right]$$

cutting opposite values here.

$$\Rightarrow (x^2+y^2+z^2)^{-5/2}\left[3x^2 + 3y^2 + 3z^2 - 3x^2 - 3y^2 - 3z^2 \right]$$

$$\Rightarrow (x^2+y^2+z^2)^{-5/2} \cdot (0)$$

$$\Rightarrow 0$$

Hence ; $\frac{\partial^2 u}{\partial^2 x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

Now, Proved that given equation " $u(x,y,z)$ is Laplace Equation

x ——— x ——— x