NAME M. Afnan

ID 7895

Section "A"

Assignment No 03

Department Civil Engineering

Submitted to Mam Shomaila

Subject Differential equation

Application of Partial differential Equation:

Many engineering problem are governed by differents types of Partial differential equations and some of the more important types are given below

Tricomi Equation:

$$y\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0\begin{cases}y>0: \text{elliptic}\\ y<0: \text{hyperbolic}\end{cases}$$

Laplace Equation. (or variants): $\partial_2\varphi\,\partial x_2+\partial_2\varphi\,\partial y^2$

$\nabla_2\varphi=0$

Poissons Equation:

$$\partial_2\varphi\,\partial x_2+\partial_2\varphi\,\partial y^2=f(x,y)$$

Helmholtz Equation:

$$\partial_2\varphi\,\partial x^2+\partial_2\varphi\,\partial y^2+C_2\varphi=0$$

Plate bending:

$$\nabla\cdot\nabla_2 w=\nabla_4 w=q/D$$

Wave Equation:

$$\frac{\partial^2 u}{\partial t^2}-c^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=0$$

Fourier Equation:

$$\partial T\,\partial t=a(\partial_2 T x^2)$$

Separable differential Equation:

For equation which can be expressed in separable form as shown below the solution can be obtained easily as

$$\frac{dy}{du} = f(u,y) \quad \frac{dy}{n(y)} = f(u)\,du \quad \int \frac{dy}{n(y)} = \int f(u)\,du + C$$

$$M(u,y)\,du + N(u,y)\,dy = 0 \quad M(u)\,du = N(y)\,dy$$

then $\int M(u)\,du = -\int N(y)\,dy + C$

Separable differential Equation:

For equation which can be expressed in seperable from as shown below the solution can be obtained easily as

$$\frac{dy}{du} = f(u,y) \quad \frac{dy}{n(y)} = f(u)\,du \quad \int \frac{dy}{n(y)} = \int f$$

$(u)\,du + C$

$$M(u,y)\,du + N(u,y)\,dy = 0 \quad M(u)\,du = -N(y)\,dy$$

Example:

$$\frac{dy}{du} = u^3(y^2+1) \Rightarrow \frac{dy}{y^2+1} = u^3\,du$$

$$\int \frac{dy}{y^2+1} \quad \int u^3\,du + C \Rightarrow \tan^{-1}y = \frac{1}{4}u^4 + C$$

$$\Rightarrow \quad y = \tan\left(\tfrac{1}{4}u^4 + C\right)$$

Example

$$\frac{dy}{du} = \frac{3u^2 + 4u + 2}{2(y-1)} \quad \text{subject to } y(0) = -1$$

Since is a separable function the problem can be solved as

$$2(y-1)\,dy = (3x^2+4x+2)\,dx$$

$$y^2 - 2y = x^3+2x^2+2x+c$$

Example:

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)} \text{ subject to } y(0)=-1$$

Since this is a separable function the problem can be solved as

$$2(y-1)\,dy = (3x^2+4x+2)\,dx$$

$$y^2 - 2y = x^3+2x^2+2x+C$$

Based on the boundary condition $C=3$

hence, $y^2-2y = x^3+2x^2+2x+3$

This quadratic equation in y^2 can be solved with two solution by the quadratic equation as

$$y = 1 \sqrt{x^3+2x^2+2x+4} \text{ and } y = 1 \pm \sqrt{x^3+2x^2+2x+4}$$

Since the second solution does not satisfy the boundary condiction it will not be accepted hence, the solution to this differential equation is obtained

Variation of Parameters:

For the following equation form it is possible to solve it by variation of Parameters.

For

$$\frac{dy}{dx} = P(x)y + Q(x)$$

and $y = c(x)e^{\int P(x)dx}$ by differentiating it given.

$$\frac{dy}{dx} = \frac{dc(x)}{dx}e^{\int P(x)dx} + \frac{c(x)P(x)e^{\int P(x)dx}}{P(x)y}$$

Substitute it to the original ODE

$$\frac{dc(x)}{dx} = Q(x)e^{-\int P(x)dx}$$ comparing the terms it given.

$$c(x) = \int Q(x)e^{-\int P(x)dx}dx + C$$

Example:

$$(x+1)\frac{dy}{dx} - ny = e^x(x+1)^{n+1}$$

This equation is now expressed as

$$\frac{dy}{dx} = P(x)y + Q(x)$$

$$\frac{dy}{dx} = \frac{n}{x+1}y + \frac{e^x(x+1)^{n}}{dx}$$

For $x \neq -1$

solving the homogeneous part of the ODE

$$\frac{dy}{dx} = \frac{n}{x+1}y$$ then $$\frac{dy}{y} = \frac{n}{x+1}dx$$

Scanned with CamScanner

$\ln(y) = \ln\ln(|x+1|) + c_1$

$y = c(x+1)^n$

Look for solution $y = c(x)(x+1)^n$

where $c(x)$ is the variation of Parameters

Substitute it to the ODE

$$\frac{dc(x)}{dx}(x+1)^n + nc(x)(x+1)^{n-1} = nc(x)$$

$$(x+1)^{n-1} + nc(x)(x+1)^{n-1} + e^x(x+1)^n$$

$$\frac{dy}{dx} = \frac{n}{x+1}y + e^x(x+1)^n$$

Comparison givens $\frac{dc(x)}{dx} = e^x$

Integration of this equation gives

$$c(x) = e^x + \tilde{c}$$

General solution is hence given by

$$y = (x+1)^n(e^x + \tilde{c})$$

This Bernoulli equation is an important equation type which can be solved in a similar way by variation of Parameters.

Consider the following form of equation.

$$\frac{dy}{dx} = P(x)y + Q(x)y^n$$

step 1 - Put $z = y^{1-n}$

step 2 - Then $\frac{dz}{dx} = (1-n)y^{-n}\,\frac{dy}{dx}$

$$\frac{dz}{dx} = (1-n)P(x)z + (1-n)Q(x)$$

The non linear ODE now become linear ODE It can be solved by formula

Step 3: $n = -1, z, y^2$ Inverting z to get y

$$\frac{dy}{du} = \frac{y}{du} + \frac{u^2}{\partial y}$$

$$\frac{d^2}{du} - \frac{1}{u}^2 + u^2$$

$$z = e^{\int \frac{1}{u} du} \left(\int u^2 e^{-\int 1/u \, du} du + c \right) = cu + \frac{1}{2} u^3$$

Back substitution of $z = y^2$ of $z = y^2$

$$y^2 = cu + \frac{1}{2} u^2$$

Homogeneous Equation:

For equation of the following type, where all the cofficients constant, it can be evaluated according different

Laplace Equation

Laplace equation form an important governing condiation for many types of Problems some of the more common forms are given by

Three dimensional Laplace Equation.

two dimensional heat conditions

$a^2 (u_{xx} + u_{yy}) u_t$

two dimensional seepage Problem

$(k_x u_{xx} + k_y u_{yy}) = 0$

There a two major types of boundary condition to this problem

Dirichlet Problem:

Boundary condition

Prescribed as u.