

Name # Ikramullah

ID # 7976

Subject # MDS(II)

Submitted To # Eng Usama Ali

Iqra National University
Peshawar.

①

Q #1

Given Data

The state of stresses which are given as;

$$\sigma_x = -40 \text{ MPa}$$

$$\sigma_y = 70 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

Required Data

⇒ Determine the state of stress at point rotated at 15° in clock wise direction -

Solution

The given stresses are

$$\sigma_x = -40 \text{ MPa}$$

$$\sigma_y = 70 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

(2)

When we rotate the element of beam at 15° in clockwise direction then we find σ_{x_1} , σ_{y_1} and $\tau_{x_1 y_1}$.

First we find σ_{x_1} ,

$$= \sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \sigma_{x_1} = \frac{-40 + 70}{2} + \left(\frac{-40 - 70}{2} \right) \cos 2(-15^\circ) + (-50) \sin 2(-15^\circ)$$

$$= \boxed{\sigma_{x_1} = -7.63 \text{ MPa}}$$

\Rightarrow Now we have to find σ_{y_1} ,

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\sigma_{y_1} = \frac{-40 + 70}{2} - \left(\frac{-40 - 70}{2} \right) \cos 2(-15^\circ) - (-50) \sin 2(-15^\circ)$$

$$\boxed{\sigma_{y_1} = 37.63 \text{ MPa}}$$

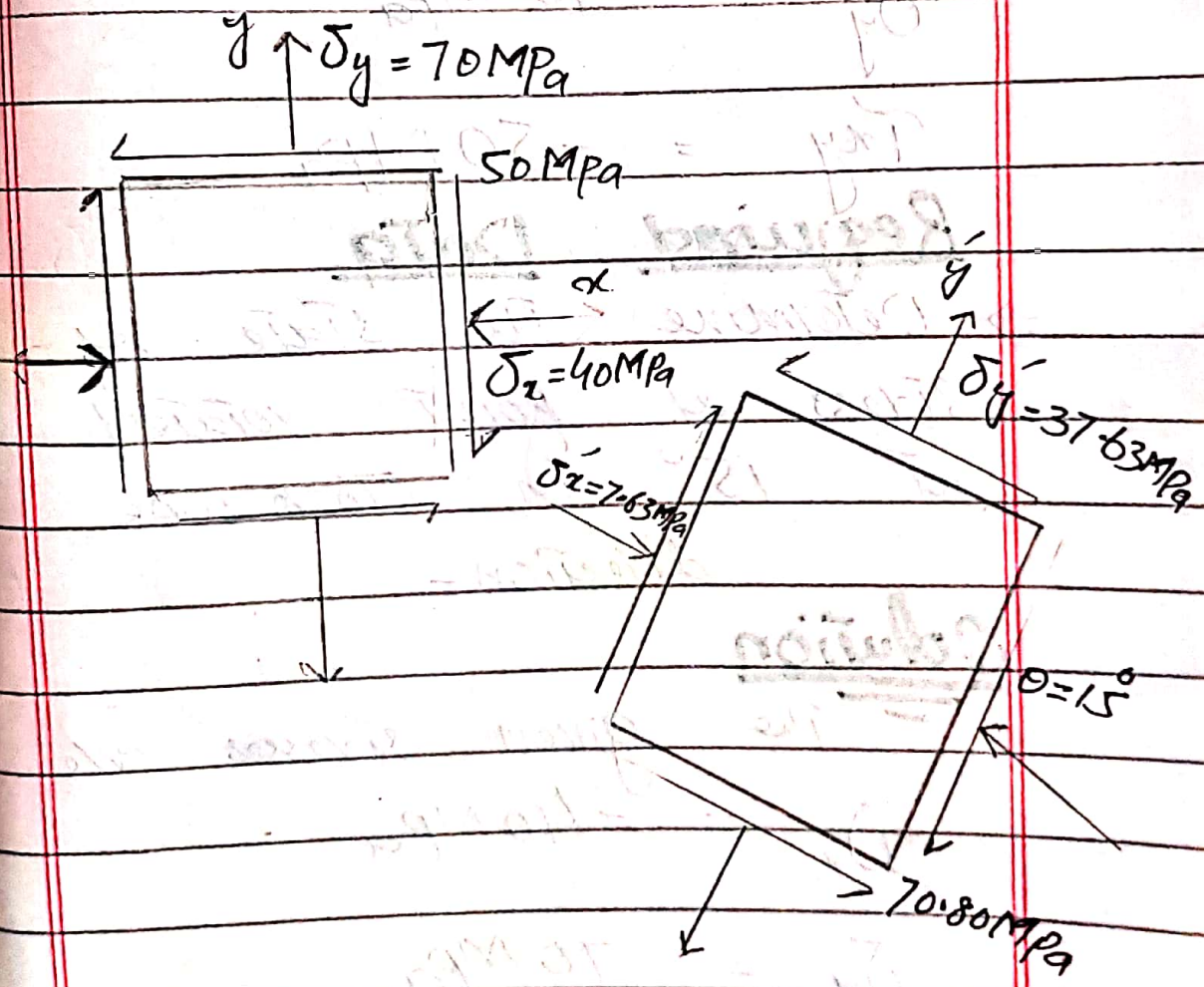
③

Also we have to find $\tau_{x_1 y_1}$

$$\tau_{x_1 y_1} = \frac{-(\sigma_x - \sigma_y) \sin 2\theta + \tau_{xy} \cos 2\theta}{2}$$

$$\tau_{x_1 y_1} = \frac{-(-40 - 70) \sin 2(-15^\circ) + (-50) \cos 2(-15^\circ)}{2}$$

$$\tau_{x_1 y_1} = -70.80 \text{ MPa}$$



(4)

Question : 02

Part: a

Given Data

→ The State of stresses which are given as;

$$\sigma_x = -40 \text{ MPa}$$

$$\sigma_y = -70 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

Required Data

principal plane = ?

principal stresses = ?

Solution

→ First we find plane

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2(-50)}{-40 - 70}$$

$$\tan 2\theta_p = 0.9091$$

$$2\theta_p = \tan^{-1}(0.9091) = 42.27$$

$$\theta_{p_2} = \frac{42.27}{2}$$

$$\theta_{p_2} = 21.14^\circ$$

5

Also

$$2\theta_{P_1} = 42 \cdot 27 + 180$$

$$2\theta_{P_1} = 222 \cdot 27$$

$$\theta_{P_1} = \frac{222 \cdot 27}{2}$$

$$\theta_{P_1} = 111.135^\circ \cdot Y\text{-axis}$$

Now we have to find principal stresses are;

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-40 + 70}{2} + \sqrt{\left(\frac{-40 - 70}{2}\right)^2 + (-50)^2}$$

$$\sigma_1 = 89.33 \text{ MPa}$$

Also

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{-40 + 70}{2} - \sqrt{\left(\frac{-40 - 70}{2}\right)^2 + (-50)^2}$$

$$\sigma_2 = -59.33 \text{ MPa}$$

(6)

Hence

$$\sigma_1 = 89.33 \text{ MPa with } \theta_{p_1} = 111.135^\circ$$
$$\sigma_2 = -59.33 \text{ MPa with } \theta_{p_2} = 21.14^\circ$$

Part: b;

Given data:

$$\sigma_x = -40 \text{ MPa}$$
$$\sigma_y = 70 \text{ MPa}$$
$$\tau_{xy} = -50 \text{ MPa}$$

Required data

Max In-plane shear stress = ?

Max: Shear plane = ?

Solution

First we find max in plane shear

$$|\tau_{\text{max in plane}}| = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-40 - 70}{2}\right)^2 + (-50)^2}$$

$$|\tau_{\text{max in plane}}| = 74.3 \text{ MPa}$$

(7)

$$\bar{\sigma}_{avg} = \frac{\bar{\sigma}_x + \bar{\sigma}_y}{2} = \frac{-40 + 70}{2}$$

$$\bar{\sigma}_{avg} = 15$$

now we have to find max
shear plane

$$\tan 2\theta_s = \frac{-(\bar{\sigma}_x - \bar{\sigma}_y)}{2\tau_{xy}}$$

$$\tan 2\theta_s = \frac{-(-40 - 70)}{-50}$$

$$\tan 2\theta_s = -1.1$$

$$\theta_s = \frac{1}{2} \tan^{-1}(-1.1)$$

$$\theta_s = -23.9^\circ$$