Course title:-

Electro Magnetic Field Theory

Name:-

Ali Darvish Kayani

ID:-

15243

Instrutor:-

Dr. Rafiq Mansoor

Signature:-

Date:-

15 April - 2020

Question No 1

Short questions

(a) Transfer the vector $B = yi(x+z)j$ located at point $(-2, 6, 3)$ into cylindrical coordinates.

Solution:-

$$B = yi(x+z)j$$

Given points are $(-2, 6, 3)$

$$B = yi(xj + zj)$$

$$B = yxij + yzij$$

$$\rightarrow \rho = \sqrt{x^2 + y^2}$$

$$\rho = \sqrt{(-2)^2 + (6)^2}$$

$$\rho = \sqrt{40}$$

$$\rho = 6.32$$

As we know that

$z = z$

So $\boxed{z = 3}$

As we know that

$\phi = \tan^{-1}(y/x)$

$\phi = \tan^{-1}(6/-2)$

$= \tan^{-1}(-3)$

$\boxed{\phi = -71.56}$

So $\boxed{B = 6.32, -71.56, 3}$

(b) Convert the point (3, 4, 5) from Cartesian to spherical coordinates.

Solution:-

$P(3, 4, 5)$

$x = 3, \quad y = 4, \quad z = 5$

In spherical coordinates system-

$r, \theta, \phi.$

$r = \sqrt{x^2 + y^2 + z^2}$

$r = \sqrt{3^2 + 4^2 + 5^2}$

$r = \sqrt{9 + 16 + 25}$

$r = \sqrt{50}$

$\boxed{r = 7.07}$

Now,- $\theta = \tan^{-1}(y/x)$

$\theta = \frac{4}{3} = \tan^{-1}(1.33)$

$\theta = 53.1°$

$\therefore \quad \phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{3^2+4^2}}{5}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{9+16}}{5}\right) = \tan^{-1}\left(\frac{\sqrt{25}}{5}\right)$

$\phi = \tan^{-1}\left(\frac{5}{5}\right) \Rightarrow \tan^{-1}(1)$

$\phi = 45$

$r = 7.07, \theta = 53.1°, \phi = 45$

c) Spherical coordinates of A(2, 3, -1)

Solution:- r, θ, ϕ

$r = \sqrt{x^2 + y^2 + z^2}$

$r = \sqrt{2^2 + 3^2 + (-1)^2}$

$r = \sqrt{14} \Rightarrow r = 3.74$

$\theta = \tan^{-1}(y/x)$

$\theta = \tan^{-1}(\frac{3}{2})$

$\theta = \tan^{-1}(1.5)$

$\theta = 56.3^\circ$

$\therefore \phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{2^2+3^2}}{-1}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{4+9}}{-1}\right) \Rightarrow \left(\frac{\sqrt{13}}{-1}\right)$

$\phi = \tan^{-1}(-3.60)$

$\phi = 74.4$

$r = 3.74$, $\theta 56.3^\circ$, $\phi = 74.4$.

(d) cartesian coordinates of:-

B (4, 25, 120)

Solution:-

B(4, 25, 120) is in spherical coordinates (r, θ, ϕ) we will find (x, y, z):.

P.T.O

$x = r \sin\theta \cdot \cos\phi$

$x = 4\sin(25) \cdot \cos(120)$

$x = 4(0.42)(-0.5)$

$\boxed{x = -0.84}$

As:- $y = r\sin\theta\sin\phi$

$y = 4\sin(25) \cdot \sin(120)$

$y = 4(0.42)(0.86)$

$\boxed{y = 1.45}$

:-

$z = r\cos\theta$

$z = 4\cos(25)$

$z = 4(0.90)$

$\boxed{z = 3.62}$

$(x, y, z) = (-0.84, 1.45, 3.62)$

e) Find the force b/w the charge?

Solution:-

$q_1 = 2nc$, $q_2 = 1nc$

$d = 4cm$.

To find $F = ?$

Where,

$$F = K \frac{q_1 q_2}{r^2}$$

$$K = \frac{1}{4\pi\varepsilon_0}$$

$$F = \frac{2\times10^{-9} \times 1 - 1\times10^{-9}}{4(3.14)\times 8.8\times10^{-12} \times (4\times10^{-2})^2}$$

$$F = -1.124 \times 10^{-5}$$

$$\boxed{F = -11.24 \text{-} 4N}$$

7) Find electric field intensity of two charges.

Solution:-

$q_1 = -2C \quad q_2 = -1C$

$d = 1m$

To find $E = ?$

we have :-

$E_1 = \frac{kq_1}{d^2}$ $k = 9\times10^9$

$E_1 = \frac{9\times10^9 \times -2}{(1)^2}$

$E_1 = -18\times10^9 \frac{V}{m}$

$E_2 = \frac{k\,q_2}{d^2}$

$E_2 = \frac{9\times10^9 \times (-1)}{(1)^2}$

$E_2 = -9\times10^9 \frac{V}{m}$

As :-

$E_T = E_1 + E_2$

$E_T = -18\times10^9 + (-9\times10^9)$

$E_T = -18\times10^9 + (-9\times10^9)$

$ET = -18\times10^9 - 9\times10^9$

$ET = -27\times10^9 \frac{V}{m}$.

9) To Determine strength produce by electric field

Solution

$E = 40 \frac{V}{cm}$, $d = 30cm$

To find $Q = ?$

where

$$E = \frac{KQ}{d^2}$$

$$Ed^2 = KQ$$

$$\frac{Ed^2}{K} = Q \rightarrow (1)$$

$$Q = \frac{Ed^2}{K} \Rightarrow Q = \frac{40 \times (30)^2}{9 \times 10^9}$$

$$Q = \frac{40 \times 900}{9 \times 10^9}$$

$$Q = 4 \times 10^{-6} C$$

$$\boxed{Q = 4 \mu C}$$

h) A charge of 2×10^{-7} is acted upon by a force of 0.1N. Determine the distance to the other charge of $4.5\times10^{-7}C$. both the charges are in vacum.

Solution:-

$q_1 = 2\times10^{-7}C$ $F = 0.1N$

$q_2 = 4.5\times10^{-7}C$ $K = 9\times10^{9}$

Find $d = ?$

$$F = K\frac{q_1 q_2}{d^2}$$

$$d^2 = \frac{Kq_1q_2}{F}$$

$$d^2 = \frac{9\times10^{9}(2\times10^{-7})(4.5\times10^{-7})}{0.1}$$

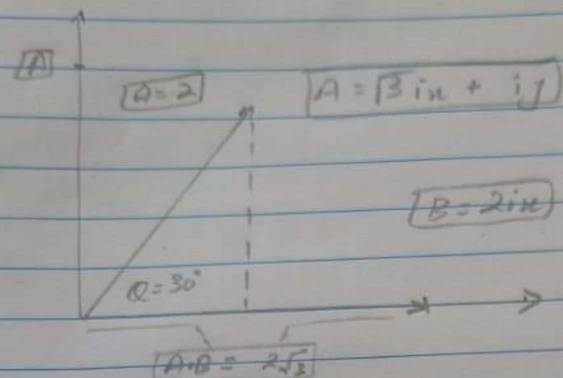
$$d^2 = 8.1\times10^{-3}$$

$$d^2 = 0.0081$$

$$\sqrt{d^2} = \sqrt{0.0081}$$

$$d = 9\,cm$$

Q2:- Find the ~~vector~~ angle between the vector show in Figure.



Ans :-

$$A \cdot B = |A||B| \cos \theta \rightarrow (eq)$$

$$A \cdot B = 2\sqrt{3}$$

$$|A| = \sqrt{2^2} \qquad |B| = \sqrt{2^2}$$

$$|A| = 2 \qquad |B| = 2$$

So eq becomes

$$2\sqrt{3} = 2 \times 2 \cos \theta$$

$$2\sqrt{3} = 4 \cos \theta$$

$$\frac{\cancel{2}\sqrt{3}}{\cancel{4}} = \cos \theta$$

$\frac{\sqrt{3}}{2} = \cos\alpha$

$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\alpha = \cos^{-1}\left(\frac{1.73}{2}\right)$

$\alpha = \cos^{-1}(0.866)$

$\alpha = 30^\circ.$

(b) Find the gradient of each of the following functions where a and b are constant.

(i) $z = ax^3 + by^3 z$

Solution-

$z = ax^2 + by^3 z$

$\frac{dF}{dy} = 2ax$

$\frac{dz}{dy} = \frac{d}{dz}(ax^2 + by^3 z)$

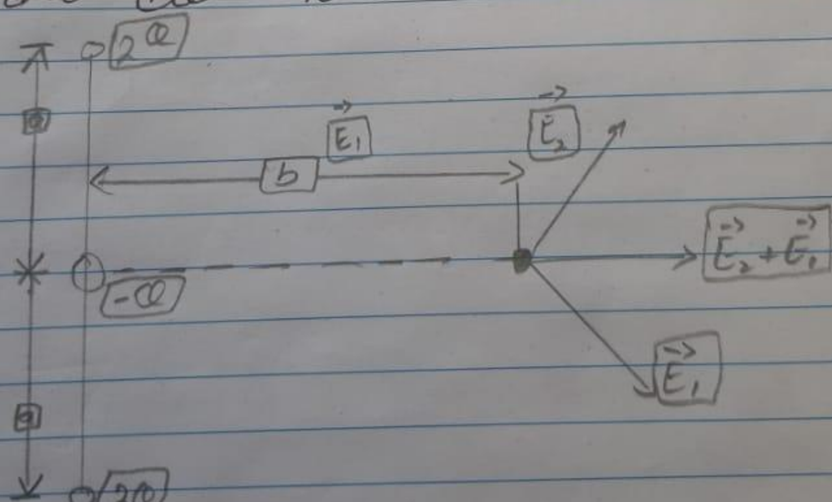
$\frac{dz}{dy} = 3by^2 z$

$$\frac{dt}{dz} = \frac{d}{dz}\left(ax^2 + by^3 z\right)$$

$$\frac{dt}{dz} = by^3$$

$$\nabla t(x, y, z) = (2ax + 3by^2 z, by^3)$$

Q3:- Three pointer charges are placed on the y-axis as shown. Find the electric field at point P on the x-axis.



Q3 Solution:-

The distance between the charge 2Q and point 'p' is

$$r^2 = b^2 + a^2$$

$$r = \sqrt{b^2 + a^2}$$

let assume that charge 2Q make angle (α) and $(-\alpha)$ with x-axis.

Magnitude of $|\vec{E_1}| = |\vec{E_2}| = \frac{Kq}{r^2}$

$$= \frac{K(2Q)}{r^2}$$

$$= \frac{K(2Q)}{b^2 + a^2}$$

So Resultant of $\vec{E_1}$ and $\vec{E_2}$ is

$$\vec{E}_{1+2} = \vec{E_1} + \vec{E_2} = \vec{E}_{1x} + \vec{E}_{2x}$$

(Y - component will be cancel).

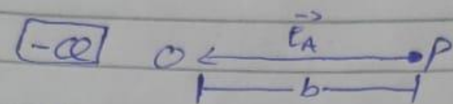
$$= \frac{K(2Q)}{b^2 + a^2}(\cos(\alpha) + \cos(-\alpha)$$

$$= \frac{K(2Q)}{b^2 + a^2}(2\cos(\alpha) \quad \because \cos(\alpha) = \cos(-\alpha)$$

$$\vec{E}_{1+2} = \frac{4KQ\cos(\alpha)}{b^2 + a^2} \longrightarrow (1)$$

Now electric field at point 'p' due to charge "-Q":

As charge is Negative Electric field at point will to directed towards charge "-Q".



$$\vec{E_A} = -\frac{K(Q)}{b^2}$$

Net electric field at point 'p' will be $\vec{E}_{net} = \vec{E}_A + (\vec{E}_1 + \vec{E}_2)$

$$= -\frac{K(Q)}{b^2} + \frac{4KQ\cos\alpha}{b^2 + a^2}$$

$$= \frac{KQ(a^2+b^2) + 4KQb^2\cos\alpha}{b^2(a^2+b^2)}$$

$$= \frac{KQ}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

where $K = 9 \times 10^9\ Nm^2/C^2$.

$$\vec{E}_{net} = \frac{9\times10^9 Q}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

Now $\alpha = \tan^{-1}\left(\frac{a}{b}\right)$

$$E_{net} = \frac{9\times10^9 Q}{b^2(a^2+b^2)}\left[4b^2\cos\left[\tan^{-1}\left(\frac{a}{b}\right)\right] - (a^2+b^2)\right]$$