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Question #1  $\rightarrow$  Find the root of the equation ①

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2$$

~~Ans~~

use newton raphson method

with  $x_0 = 0.02$

Solution  $\rightarrow$  Rearranging the equation

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.02 - \frac{0.00033}{-0.0054}$$

$$x_1 = 0.081$$

$$f(x_0) = 0.00033$$

$$f'(x_0) = -0.0054$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$0.081 - \frac{(-0.00016)}{-0.0070}$$

$$f(x_1) = -0.00016$$

$$f'(x_1) = -0.0070$$

(2)

$$x_2 = 0.058$$

$$\rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$0.058 - \frac{0.000030}{(-0.0090)}$$

$$x_3 = 0.061$$

$$f(x_2) = 0.000030$$

$$f'(x_2) = 0.0090$$

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Question # 2 :  $\rightarrow x_0 = 2, y_0 = 0.5$  (3)

Answer :  $\rightarrow$   
 $x_1 = 2.75, y_1 = 0.36$   
 $x_2 = 4, y_2 = 0.25$

As we know that

Lagrange Interpolation formula

$$y = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0$$

$x_0 = 2$	$y_0 = 0.5$
$x_1 = 2.75$	$y_1 = 0.36$
$x_2 = 4$	$y_2 = 0.25$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

Putting value

$$y = \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)} (0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} (0.36) + \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)} (0.25)$$

(P-T-O)  $\Rightarrow$

4

$$y = (-0.833) + 0.384 + 0.025$$

$$y = -0.424$$

% =————— %

# Question #3: →

(5)

Complete the divided difference table the given and construct the interpolating polynomial that uses all this data.

$X =$	1.0	1.3	1.6	1.9	2.2
$Y =$	0.7651977	0.6200860	0.4554022	<del>0.41135</del> 0.2818186	0.1103623

$x_i$	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}, x_i)$	$f(x_{i-4}, \dots, x_i)$
$x_0$	1.0				
	0.7651977				
		-0.4837056			
$x_1$	1.3				
	0.6200860				
		-0.108734			
				0.6587805	
$x_2$	1.6				
	0.4554022				
		-0.548946			
			-0.0494433		
		-0.578612			
		<del>0.546012</del>		0.06251255	
$x_3$	1.9				
	0.2818186				
		-0.571521			
		<del>0.572612</del>	0.006818		
$x_4$	2.2				
	0.1103623				

(P-T-O) ⇒

⑥

$$\begin{aligned} \textcircled{1} \quad f(x_0, x_1) &= \frac{f(x_1) - f(x_0)}{x_1 - x_0} \\ &= \frac{0.6200860 - 0.7651977}{1.3 - 1} \end{aligned}$$

$$f(x_0, x_1) = 0.4837056$$

$$\begin{aligned} \textcircled{2} \quad f(x_1, x_2) &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ &= \frac{0.4554022 - 0.6200860}{1.6 - 1.3} \end{aligned}$$

$$f(x_1, x_2) = -0.548946$$

$$\begin{aligned} \textcircled{3} \quad f(x_2, x_3) &= \frac{f(x_3) - f(x_2)}{x_3 - x_2} \\ &= \frac{0.2818186 - 0.4554022}{1.9 - 1.6} \end{aligned}$$

$$f(x_2, x_3) = -0.57612$$

$\Rightarrow$   
(P-T-O)

④

$$f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3}$$

$$= \frac{0.1103623 - 0.2818186}{2.2 - 1.9}$$

$$f(x_3, x_4) = -0.571521$$

Second divided difference

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$$

$$= -0.108734$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{-0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$f(x_1, x_2, x_3) = -0.0494433$$

P-T-O →



$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$= 0.006818$$

Third divided difference.

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{0.0494433 - (-0.108734)}{1.9 - 1}$$

$$= 0.0658785$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-0.049443)}{2.2 - 1.3}$$

$$= 0.06251255$$

9

4th divided difference.

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$= \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$= 0.0028049$$

$$f(x) = f(x_0) + (x-x_0)f(x_0, x_1) + (x-x_0)(x-x_1)f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2)f(x_0, x_1, x_2, x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)f(x_0, x_1, x_2, x_3, x_4).$$

$$= 0.7651977 + (x-1) - 0.4837056 + (x-1)(x-1.3) - (-0.108734) + (x-1)(x-1.3)(x-1.6)(0.0658785) + (x-1)(x-1.3)(x-1.6)(x-1.9)(-0.0028049).$$

0% 0%