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Paper : Differential Equation
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x ———— x ———— x

Q No : 1

$$f(t) = 1 + t, \; -\pi \le t \le \pi$$

Here we use formula.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(\frac{-\pi^2}{2} \right) \right)$$

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$$a_0 = \frac{1}{2\pi}\left(2\pi + \frac{2\pi^2}{2}\right)$$

$$\boxed{a_0 = \frac{1}{2\pi}\left(2\pi + \pi^2\right)}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)(\cos nt)\,dt$$

$$a_n = \frac{1}{\pi}\left((1+t)\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right)$$

$$a_n = \frac{1}{\pi}\left((1+t)\cancel{\frac{\sin nt}{n}}^{0} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1 - (-1)\right)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\,dt$$

$$b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\int\sin nt\ \frac{d}{dt}(1+t)\,dt\right)\right)$$

$$bn\ \frac{1}{\pi}\left(\frac{(1+t)(-\cos nt)}{n}\Big|_{-\pi}^{\pi} - \int\left(-\frac{\cos nt}{n}(1)\right)\right)$$

$$bn = \frac{1}{\pi}\left(\frac{-(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \left(\frac{\sin n\pi}{n^2}\Big|_{-\pi}^{\pi}\right)\right)$$

$$bn = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - \left((1+(\pi)(\cos nt)\right)\right)$$

$$bn = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos\pi - \cancel{\cos n\pi} + \pi\cos\pi\right)$$

$$bn = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi}\cos n\pi\right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$bn = \frac{2}{n}(-1)^{n+1}$$

eq become;

$$\boxed{f(x) = \frac{1}{2\pi}(2\pi + \pi^2) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n}\sin t}$$

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Q No: 2

sol:-

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen values = ?

Step # 01

we have:

$(A - \lambda I)x = 0$ A = Given Matrix

Step # 02

we have; The characteristic equation is given by

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

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Step # 03

$$\lambda^3 - \left|\begin{matrix}\text{Sum of} \\ \text{Diagonal elem}\end{matrix}\right| \lambda^2 + \left|\begin{matrix}\text{Sum of} \\ \text{Diagonal minor}\end{matrix}\right| \lambda - |A| = 0 \longrightarrow (B)$$

Sum of Diagonal elements $= 1+1+2 = 4$

Sum of Diagonal minors $=$

$$= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$$

$$= (-6) + (2) + (1)$$

$$= -6 + 2 + 1$$

$$= -3$$

By Putting values in eq (B);

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \longrightarrow (C)$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 1(2-8) - 0 + 1(6-0)$$

$$= -6 + 6$$

$$= 0$$

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By Putting values in (C)

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

using Quadratic formula;

$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad a = 1$, $b = -4$, $c = -3$

$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$

$= \frac{4 \pm \sqrt{16 + 12}}{2} = \frac{4 \pm \sqrt{28}}{2}$

$\lambda = \frac{4 + \sqrt{23}}{2}$, $\lambda = \frac{4 - \sqrt{28}}{2}$

we have eigenvalues

$\lambda = \left(0, \frac{4 + \sqrt{28}}{2}, \frac{4 - \sqrt{28}}{2}\right)$

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Q No : 3

Sol:- $\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$ $R_4 - R_2$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right]$$ $5 \times R_3$ & $5 \times R_4$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right]$$ $5R_3$ and $5R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right]$$ $1/5 \times R_1$

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$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right| \quad R_2 \times 5$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right| \quad R_3 - R_2$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right| \quad \begin{array}{l} R_3 \longleftrightarrow R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right| \quad R_2 \times -5$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right|$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right|$$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad 5/4 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = (3/4, 31/21, -11/21, 1/3)$$

$$x = 3/4, \quad y = 31/21$$

$$z = -11/21, \quad m = 1/3$$

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Q No 4:-

verify That:

$u(x,t) = \sin(x+2t)$

is a solution of The one dimensional equation.

Sol:- Given That:

$u(x,t) = \sin(x+2t)$

Differentiat w.r.t x Partialy

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\sin(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)(1+0)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\cos(x+2t)$$

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$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)(1+0)$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)}$$

and $u(x,t) = \sin(x+2t)$

Differentiate w.r.t "t".

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t)(0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2)-\sin(x+2t)(0+2)$$

$$\boxed{\frac{\partial^2 u}{dt^2} = -4\sin(x+2t)}$$

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As we Know That one-diamensional wave equation is:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$-4\sin(x+2t) = c^2[-\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2 \sin(x+2t)$$

$$-4\sin(x+2t) + c^2 \sin(x+2t) = 0$$

For The orbitrary constant

$$c = \pm 2$$

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for The orbitrary Constant.

$$c = 2$$

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