Final term Assignment
Probability and Statistics

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I.d = 14712

Submitted to:- Sir Daud.

(Q.1)

(Answer).

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (2,7), (2,8), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (3,7), (3,8), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (4,7), (4,8), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (5,7), (5,8), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), (6,7), (6,8), (7,1), (7,2), (7,3), (7,4), (7,5), (7,6), (7,7), (7,8), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8)\}$

let

$A = \{$ The sum is 7 $\}$

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$B = \{$ The sum is even $\}$

$C = \{$ The sum is greater than 8 $\}$

$D = \{$ The two disc had the same outcomes $\}$.

Now

$A = \{(1,6), (2,5), (3,4), (5,2), (6,1), (4,3)\}$.

$B = \{(1,1), (1,3), (1,5), (1,7), (2,2), (2,4), (2,6), (2,8), (3,1), (3,3), (3,5), (3,7), (4,2), (4,4), (4,6), (4,8), (5,1), (5,3), (5,5), (5,7), (6,2), (6,4), (6,6), (6,8), (7,1), (7,3), (7,5), (7,7), (8,2), (8,4), (8,6), (8,8)\}$.

$C = \{(1,8), (2,7), (2,8), (3,6), (3,7), (3,8), (4,5), (4,6), (4,7), (4,8), (5,4), (5,5), (5,6), (5,7), (5,8), (6,3), (6,4), (6,5), (6,6), (6,7), (6,8), (7,2), (7,3), (7,4), (7,5), (7,6), (7,7), (7,8), (8,1), (8,2), (8,3), (8,4), (8,5), (8,6), (8,7), (8,8)\}$.

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$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8)\}$

$A \cap B = \{\ \}$ OR ϕ

$A \cap C = \{\ \}$

$A \cap D = \{\ \}$

$P(A) = 6/64$, $P(B) = 32/64$

$P(C) = 36/64$, $P(D) = 8/64$.

$P(A \cap B) = 0$, $P(A \cap C) = 0$, $P(A \cap D) = 0$

Hence

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = 0 \times 32/6$$

$$P(A/B) = 0$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = 0 \times 36/64$$

$$P(A/C) = 0$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = 0 \times 8/64$$

$$P(A/D) = 0$$

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(Q.2)
(Answer).

When we are rolling two dice, there are 36 different combinat[ions] Counting these up, there are 15 possibillities less than 7 : (1,1), (1,2), (1,3), (1,4), (1,5) (2,1), (2,2), (2,3), (2,4), (3,1) (3,2), (3,3), (4,1), (4,2), (5,1). The probabilility of getting less than a 7 is

$$\frac{15}{36} = \frac{5}{12}.$$

There are 6 possible combinations of getting a 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), which gives a probability of

$$\frac{6}{36} = \frac{1}{6}.$$

This means that 21 possibilities account for getting less than or equal to 7, so there are 15 remaining possibilities of getting more than 7. This is the same as the probabili[ty]

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of getting less than 7, so the probability must be $\frac{5}{12}$ aswell. In calculating this, we must assume that each combination is equally likely to roll as any other and therefore the dice are fair, orelse the calculations don't work.

(Q.3)
(Answer).

Given that $P = 2/3$ $n = 8$

$$q = 1 - P$$
$$= 1 - 2/3$$
$$q = 1/3$$

Let "X" denotes the number of games won by A, Then.

i) $P(X = 4)$

$$= \binom{8}{4} (2/3)^4 (1/3)^4$$
$$= \frac{1120}{6561}$$
$$= 0.1707.$$

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ii) $P(X \geq 4)$

$= 1 - P(X < 4)$

$= 1 - \sum_{x=0}^{3} \binom{8}{x} (2/3)^x (1/3)^{8-x}$

$= 1 - \Big[(1/3)^8 + 8(2/3)(1/3)^7$

$+ 28(2/3)^2(1/3)^6 +$

$56(2/3)^3(1/3)^5 \Big]$

$= 1 - \frac{1}{6561} [1 + 16 + 112 + 448]$

$= 1 - \frac{577}{6561}$

$= \frac{6561 - 577}{6561}$

$= \frac{5984}{6561}$

$= 0.9121$

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(iii) $P(3 \le X \le 6)$

$$\sum_{x=3}^{6} \binom{8}{x} \left(\tfrac{2}{3}\right)^{x} \left(\tfrac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\tfrac{2}{3}\right)^3\left(\tfrac{1}{3}\right)^5 + \binom{8}{4}\left(\tfrac{2}{3}\right)^4 \left(\tfrac{1}{3}\right)^4 + \binom{8}{5}\left(\tfrac{2}{3}\right)^5\left(\tfrac{1}{3}\right)^3 + \binom{8}{6}\left(\tfrac{2}{3}\right)^6\left(\tfrac{1}{3}\right)^2$$

$$= \frac{8}{(3)^8}\left[56 + 140 + 224 + 224\right]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = 0.7852$$

(Q.4)
(Answer).

Proof:-

Since the C_i's form a partition of the sample space, we can apply the law of total probability for $A \cap B$.

$$P(A \cap B) = \sum_{i=1}^{M} P(A \cap B / C_i) P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{M} P(A / C_i) P(B / C_i) P(C_i)$$

$\because$ (A and B are conditional independent)

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$$P(A \cap B) = \sum_{i=1}^{M} P(A/C_i) P(B) P(C_i)$$

$\because$ (B is independent of all C_i's).

$$P(A \cap B) = P(B) \sum_{i=1}^{M} P(A/C_i) P(C_i)$$

$$P(A \cap B) = P(B) P(A).$$

$\because$ (law of total probability)

Hence A and B are independent.

(Q.5)

(Answer).

· Mean and Variance of Binomial Random Variables:-

The Probability function for a binomial random variable is

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

This is the probability of having x successes in a series of n independent trials when the probability of success in any one of the trials is p.

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If X is a random variable with this probability distribution

$$E(X) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

since the $x=0$ term variables, let $y = x-1$ and $m = n-1$, subbing $x = y+1$ and $n = m+1$ into the last sum (and using the fact that the limits $x = 1$ and $x = n$ correspond to $y = 0$ and $y = n-1 = m$, respectively).

$$E(X) = \sum_{y=0}^{m} \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)p \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

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The binomial theorem says that

$$(a+b)^m = \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} a^y b^{m-y}$$

Setting $a=p$ and $b=1-p$

$$\sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^y (1-p)^{m-y}$$

$$= \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} a^y b^{m-y} =$$

$$(a+b)^m = (p+1-p)^m = 1$$

So that $\boxed{E(X) = np}$

Similarly, but this time using $y = x-2$ and $m = n-2$

$$E(X(X-1)) = \sum_{x=0}^{n} x(x-1) \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

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$$= \sum_{x=2}^{n} \frac{n!}{(x-2)(n-x)} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}.$$

$$= n(n-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= n(n-1)p^2 (p+(1-p))^m$$

$$= n(n-1)p^2.$$

So the variance of x is

$$E(X^2) - E(X)^2 = E(X(X-1)) + E(x) - E(x)^2 = n(n-1)p^2 + np - (np)^2$$

$$= \boxed{np(1-p)}$$

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(Q.6)
(Answer).

· Bi-nomial Distribution:-

A binomial distribution can be though of as simply the probability of a success or Failure out come in an experiment or survey that is repeated multiple times.

$$P(x-x)\ f(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

Binomial Frequency Distribution:-

If the binomial probability distribution is multiplied by N, the number of experiments or sets, the resulting distribution is known as the bio - - - - - -

$$N \binom{n}{x} p^{x} q^{n-x}$$

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(Q.7)
(Answer).

· Coefficient of Variation:-

For Data Set A:-

$$CV = \frac{6}{\bar{x}} \times 100$$

$$CV = 3/45 = 100$$

$$CV = 6.7$$

For Data Set B:-

$$CV = 11/60 \times 100$$

$$CV = 18.3$$

For Data Set C:-

$$CV = 5/50 \times 100$$

$$CV = 10$$

For Data Set D:-

$$CV = 15/25 \times 100$$

$$CV = 60$$