

PAGE 1

Page # 1

Name Samiullah

Instructor Mansoor Qadir

Student ID 16054

Department Computer - Science

Semester 2nd

Subject Linear Algebra

Question No 1

Consider the given below matrix as the augmented matrix of a linear system. Explain in your words the next elementary row operation that should be performed in order to solve this system.

# PAGE 2

$$\begin{bmatrix} 1 & 103 & 3 & 0 & 5 \\ 0 & 1 & -10\text{Last} & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 103 \end{bmatrix}$$

Solution:

my 10 is 18054

so

$$103 = 0$$

$$-10\text{Last} = -4$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

This is augmented matrix.

The linear equation of this augmented matrix is

$$x_1 \quad 3x_3 = 5 \quad \text{--- eq ①}$$

$$x_2 \quad -4x_3 = 7 \quad \text{--- eq ②}$$

$$x_3 = -6 \quad \text{--- eq ③}$$

$$x_4 = 0 \quad \text{--- eq ④}$$

$$\begin{array}{l} x_1 \quad 3x_3 = 5 \\ x_2 \quad -4x_3 = 7 \\ x_3 = -6 \\ x_4 = 0 \end{array} \quad \left| \begin{array}{cccc|c} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right.$$

So First we multiplying  
-3R<sub>3</sub> and then add  
to R<sub>1</sub>.

$$-3R_3 + R_1$$

$$\begin{array}{l} x_1 \quad 0 = 23 \\ x_2 \quad -4x_3 = 7 \\ x_3 = -6 \\ x_4 = 0 \end{array} \quad \left| \begin{array}{cccc|c} 1 & 0 & 3-3 & 0 & 5+18 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right.$$

↓  
-3R<sub>3</sub>+R<sub>1</sub>

PAGE 3



$$\begin{bmatrix} 1 & 0 & 0 & 0 & 23 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Now multiplying row 3  
 $R_3$  by 4 and add  
 to  $R_2$ .

$$4R_3 + R_2$$

$$\begin{array}{l} x_1 = 23 \\ x_2 = -17 \\ x_3 = -6 \\ x_4 = 0 \end{array} \quad \left| \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 23 \\ 0 & 1 & -4+4 & 0 & 7-24 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \right.$$

↓  
 $4R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 23 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

So this is the last  
 reduced echelon form or  
 system of linear equation.



So

$$x_1 = 23$$

$$x_2 = -17$$

$$x_3 = -6$$

$$x_4 = 0$$

Verification:

put value of  $x_1$   
and  $x_3$  in eq (1)

$$23 + 3(-6) = 5$$

$$23 - 18 = 5$$

$$5 = 5 \quad \text{— true}$$

put value of  $x_2$   
and  $x_3$  in eq (2).

$$-17 - 4(-6) = 7$$

$$-17 + 24 = 7$$

$$7 = 7 \quad \text{— true}$$

So the last ~~eq~~ system  
of equation are.

# PAGE 6

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 23 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Q-

Question No 2

Find the elementary row operation that transform the first matrix into

(a) second and reverse row operation that transforms the second matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

First matrix into second matrix:

The following operation can be performed to transform the first matrix into second matrix.

$$R_3 - 2R_2$$



$$\begin{array}{c}
 \left[ \begin{array}{cccc}
 1 & 3 & -1 & 5 \\
 0 & 1 & -4 & 2 \\
 0 & 0 & 3 & -5
 \end{array} \right] \\
 \begin{array}{l}
 \\
 R_3 - 2R_2 \\
 0-0 \quad 2-2 \quad -5-(-8) \quad -1-4
 \end{array}
 \end{array}$$

$$\left[ \begin{array}{cccc}
 1 & 3 & -1 & 5 \\
 0 & 1 & -4 & 2 \\
 0 & 0 & 3 & -5
 \end{array} \right]$$

So this ~~is~~ elementary  
 row operation transform  
 first matrix into  
 second.

Second matrix into first matrix:

$$\left[ \begin{array}{cccc}
 1 & 3 & -1 & 5 \\
 0 & 1 & -4 & 2 \\
 0 & 0 & 3 & -5
 \end{array} \right]$$

while to transform  
 the second matrix  
 into first the  
 following ~~is~~ operation

can be performed.

$R_3 + 2R_2$  which is reversed operation.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0+0 & 0+2 & 3+(-8) & 4+(-5) \end{bmatrix} \quad R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

so that the reverse row operation that transforms the second matrix into first.

## Question No 2 (b)

(b) Below given are the some matrices. Find which one is the row echelon form and which is reduced row echelon form. Explain in your own words for each of the selection in detail.

$$(a) \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

is in echelon form.

Answer:

it is ~~an~~ in echelon form because it satisfies all the following conditions.



- (1) All the entries in a column below a leading entry are zero.
- (2) Each leading entry of a row is in a column to the right of the leading entry of the above row.
- (3) To satisfy the 3rd condition there is no zero-row which should be below the all non-zero rows.

$$(b) \begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Answer :

it is in reduced echelon form because it is already

in echelon form and satisfy the further two conditions.

(1) All the leading entries in non-zero rows are 1.

(2) Each leading 1 is the only non-zero entry in its column.

$$(1) \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Answer:

it is in echelon form because it satisfy all the following conditions.

(1) All the entries in a column below a leading entry are zero.

(2) Each leading entry of a



row is in column to the right of the leading entry of the above row.

(3) To satisfy the 3rd condition there is no zero-row which should be below the all non-zero rows.

$$d) \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

Answer:

it is neither in echelon form nor in reduced echelon form because it doesn't satisfy the following condition.

(3) All the zero rows are below the non zero rows.



## Question No 3

### part a

Difference between Echelon form and Reduced row echelon form.

The following differences of Row echelon form and reduced row echelon form.

#### Echelon form :

A rectangular matrix is in echelon (or row echelon form) if it has the following three properties:

- (1) All non zero rows are above any rows of all zeros.
- (2) Each leading entry of a row is in a

column to the right  
of the leading entry  
of the row above  
it.

(3) All entries in a column  
below a leading entry  
are zeros.

## Reduced Echelon form :

if a matrix in  
echelon form satisfies  
the following additional  
conditions, then it is  
in reduced echelon  
form (or reduced row  
echelon form).

(1) The leading entry in  
each non-zero row  
is 1.

(2) Each leading 1 is  
the only non-zero  
entry in its column.

Examples :

example of echelon form

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 5/2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

\* → can be some number

~~the~~ example of reduced row echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

\* → can be some number

## Practical Use of Reduced

### Echelon form :

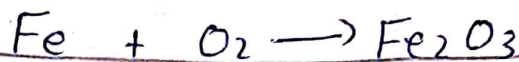
The practical use of Reduced Row echelon form is in Balancing



chemical equations.

For example:

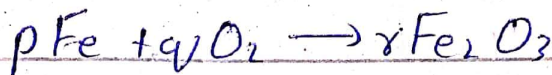
The equation  
is



Balance the equation.

In balancing the equation,

let  $p$ ,  $q$  and  $x$  be  
the unknown variables  
such that



we compare the number  
of Iron (Fe) and oxygen  
(O) atoms of the

reactants with the number  
of atoms of the product.

We obtain the following  
set of equations:

$$\text{Fe} : p = 2x$$

$$\text{O} : 2q = 3x$$

The homogenous system of  
equations becomes

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

where

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & -3 \end{bmatrix}$$

is an echelon form.

The following operation used for solving the question. Now this echelon form convert to reduced echelon form.

$$R_2 \leftrightarrow \frac{1}{2} R_2$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & \frac{2}{2} & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

Thus,  $Rx = 0$  becomes

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -\frac{3}{2} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$



upon expanding we have

$$p - 2x = 0$$

$$\boxed{p = 2x}$$

$$p - \frac{3}{2}x = 0$$

$$\boxed{p = \frac{3}{2}x}$$

so the nullspace solution.

$$x = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 2x \\ \frac{3}{2}x \\ 1x \end{bmatrix}$$

so There are three pivot  
table  $p$ ,  $q$ , and one free  
variable  $x$ , if we choose

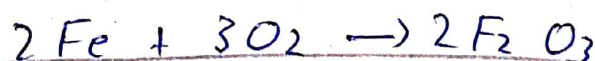
$$x = 1$$

then  $p = 2$ ,  $q = \frac{3}{2}$ .

Ignore the fraction.

$$\text{so } q = 3.$$

so the equation is  
balanced





page 20

other practical is  
Reduced echelon form  
is to solve the  
linear equation and  
to solve augmented  
matrix.

Question No 3

Part B

Find an echelon form for the below matrix using row operations.

$$\begin{bmatrix} 1 & 102 & 8 \\ 2 & 8 & -1 \\ -103 & 0 & 0 \\ 1 & -4 & 10 - \text{First-Last} \end{bmatrix}$$

Solution:

My ID is 16054

So

$$102 = 6$$

$$-103 = -0 = 0$$

$$10 - \text{First-Last} = 14$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 0 & 0 & 0 \\ 1 & -4 & 14 \end{bmatrix}$$

First we interchange  $R_3$  into  $R_4$ . so the operation is

$$R_3 \leftrightarrow R_4$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 1 & -4 & 14 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \leftrightarrow R_4$$

Now, multiply 3 times to Row 3 and then ~~multiply~~ multiply 2 times to Row one and the adding Row 1 to Row three. so the operation is

$$3R_3 + 2R_1$$



$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 3+2 & -12+12 & 42+16 \\ 0 & 0 & 0 \end{bmatrix} \quad 3R_3 + 2R_1$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 5 & 0 & 58 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, multiply four times to Row one and then ~~negative~~ three times of  $R_2$ , and then subtract  $R_2$  from  $R_1$ .

So the operation are

$$4R_1 - 3R_2$$

$$\begin{bmatrix} 4-24 & 24-24 & 32+3 \\ 2 & 8 & -1 \\ 5 & 0 & 58 \\ 0 & 0 & 0 \end{bmatrix} \quad 4R_1 - 3R_2$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2 & 8 & -1 \\ 5 & 0 & 58 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, multiply ~~Row~~ Row three  
by two and  $R_1$   
by five, then  
add  $R_1$  to  $R_3$ .  
So the operation  
is

$$2R_3 + 5R_1$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2 & 8 & -1 \\ 10-10 & 0 & 175+116 \\ 0 & 0 & 0 \end{bmatrix} \quad 2R_3 + 5R_1$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2 & 8 & -1 \\ 0 & 0 & 291 \\ 0 & 0 & 0 \end{bmatrix}$$



# last page

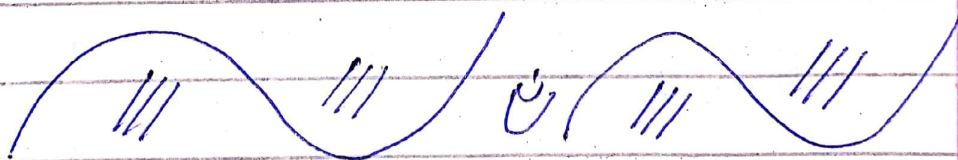
Now, Adding Row one  
to Row two, so  
the operation are:

$$R_2 + R_1$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2-2 & 8+0 & -1+35 \\ 0 & 0 & 291 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 + R_1$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 0 & 8 & 34 \\ 0 & 0 & 291 \\ 0 & 0 & 0 \end{bmatrix}$$

So this is the  
final echelon form.



The End