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Question No 1

Consider the given below matrix as the augmented matrix of a linear system. Explain in your words the next elementary row operation that should be performed in order to solve this system.

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\,Last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

Solution:

my ID is 16054

so

$ID3 = 0$

$-ID\,Last = -4$

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

This is augmented matrix.

The linear equation of this augmented matrix is

$$x_1 \quad 3x_3 = 5 \quad \text{— eq (1)}$$
$$x_2 \quad -4x_3 = 7 \quad \text{— eq (2)}$$
$$x_3 = -6 \quad \text{— eq (3)}$$
$$x_4 = 0 \quad \text{— eq (4)}$$

$$\begin{matrix} x_1 & & 3x_3 & = 5 \\ & x_2 & -4x_3 & = 7 \\ & & x_3 & = -6 \\ & & & x_4 = 0 \end{matrix} \quad \left|\left| \begin{bmatrix} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}\right.\right.$$

So First we multiplying $-3R_3$ and then add to R_1.

$$-3R_3 + R_1$$

$$\begin{matrix} x_1 & & 0 & = 23 \\ & x_2 & -4x_3 & = 7 \\ & & x_3 & = -6 \\ & & & x_4 = 0 \end{matrix} \quad \left|\left| \begin{bmatrix} 1 & 0 & 3-3 & 0 & 5+18 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}\right.\right.$$

$-3R_3 + R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 23 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

Now multiplying row 3 R_3 by 4 and add to R_2.

$$4R_3 + R_2$$

$x_1 = 23$

$x_2 = -17$

$x_3 = -6$

$x_4 = 0$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 23 \\ 0 & 1 & -4+4 & 0 & 7-24 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{4R_3+R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 23 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

So this is the last reduced echelon form or system of linear equation.

So

$x_1 = 23$

$x_2 = -17$

$x_3 = -6$

$x_4 = 0$

Verification:

put value of x_1 and x_3 in eq - (1)

$23 + 3(-6) = 5$

$23 - 18 = 5$

$5 = 5$ — true

put value of x_2 and x_3 in eq (2).

$-17 - 4(-6) = 7$

$-17 + 24 = 7$

$7 = 7$ — true

So the last system of equation are.

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$$\begin{bmatrix} 1 & 0 & 0 & 0 & 23 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Question No 2

Find the elementory row operation that transform the first matrix into

(a) Second and reverse row operation that transforms the second matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

First matrix into Second matrix:

The following operation can be performed to transform the first matrix into second matrix.

$R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0-0 & 2-2 & -5-(-8) & -1-4 \end{bmatrix} R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

So this ~~row~~ elementry row operation transform first matrix into second.

Second matrix into first matrix:

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

while to transform the second matrix into first the following operation can be performed.

$R_3 + 2R_2$ which is reversed operation.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0+0 & 0+2 & 3+(-8) & 4+(-5) \end{bmatrix} R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

so that the reverse row operation that transforms the second matrix into first.

Question No 2 (b)

(b) Below given are the some matrices. Find which one is the row echelon form and which is reduced row echelon form. Explain in your own words for each of the selection in detail.

(a) $$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

is in echelon form.

Answer:

it is ~~ech~~ in echelon form because it satisfies all the following conditions.

(1) All the entries in a column below a leading entry are zero.

(2) Each leading entry of a row is in a column to the right of the leading entry of the above row.

(3) To satisfy the 3rd condition there is no zero-row which should be below all non-zero rows.

$$\text{(b)} \quad \begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Answer :

it is in reduced echelon form because it is already in echelon form and satisfy the further two conditions.

(1) All the leading entries in non-zero rows are 1.

(2) Each leading 1 is the only non zero entry in its column.

(C)
$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

<u>Answer:</u>

it is in echelon form because it satisfy all the following conditions.

(1) All the entries in a column below a leading entry are zero.

(2) Each leading entry of a

row is in column to the right of the leading entry of the above row.

(3) To satisfy the 3rd condition there is no zero-row which should be below the all non-zero rows.

d) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

Answer:

it is neither in echelon form nor in reduced echelon form because it doesn't satisfy the following condition.

(3) All the zero rows are below the non zero rows.

(Question No 3)

part a

Difference between echelon form and Reduced row echelon form.

The following differences of Row echelon form and reduced row echelon form.

Echelon form:

A rectangular matrix is in echelon (or row echelon form) if it has the following three properties:

(1) All nonzero rows are above any rows of all zeros.

(2) Each leading entry of a row is in a

column to the right of the leading entry of the row above it.

(3) All entries in a column below a leading entry are zeros.

Reduced Echelon form :

if a matrix in echelon form satisfies the following additional conditions, then it is in reduced echelon form (or reduced row echelon form).

(1) The leading entry in each non-zero row is 1.

(2) Each leading 1 is the only non-zero entry in its column.

Examples:

example of echelon form

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 5/2 \end{bmatrix} \quad \begin{bmatrix} \bullet & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$*$ → can be some number

~~the~~ example of reduced row echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$*$ → Can be some number

Practical Use of Reduced echelon form:

The practical use of Reduced Row echelon form is in Balancing chemical equations.

For example:

The equation is

$$Fe + O_2 \longrightarrow Fe_2O_3$$

Balance the equation.
In balancing the equation, let p, q and r be the unknown variables such that

$$pFe + qO_2 \longrightarrow rFe_2O_3$$

we compare the number of Iron (Fe) and oxygen (O) atoms of the reactants with the number of atoms of the product. We obtain the following set of equations:

$$Fe : p = 2r$$
$$O : 2q = 3r$$

The homogenous system of equations becomes

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

where

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & -3 \end{bmatrix}$$

is an echelon form.
The following operation used for soloving the question. Now this echelon for convert to reduced echelon form.

$R_2 \longleftrightarrow \frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & \frac{2}{2} & \frac{-3}{2} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3/2 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

Thus, $Ru = 0$ becomes

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3/2 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

upon expanding we have

$$p - 2r = 0$$

$$\boxed{p = 2r}$$

$$p - \frac{3}{2}r = 0$$

$$\boxed{p = \frac{3}{2}r}$$

so the nullspace solution.

$$x = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \begin{bmatrix} 2r \\ \frac{3}{2}r \\ 1r \end{bmatrix}$$

So There are three pivot table p, q, and one free variable r, if we choose

$r = 1$.

then $p = 2$, $q = \frac{3}{2}$,

Ignore the fraction.

So $q = 3$.

so the equation is balanced.

$$2Fe + 3O_2 \rightarrow 2F_2O_3$$

other practical is Reduced echelon form is to solve the linear equation and to solve augmented matrix.

Question No 3

Part B

Find an echelon form for the below Matrix using row operations.

$$\begin{bmatrix} 1 & ID2 & 8 \\ 2 & 8 & -1 \\ -ID3 & 0 & 0 \\ 1 & -4 & ID\text{-}First\text{-}last \end{bmatrix}$$

Solution:

My ID is 16054

So

ID2 = 6

-ID3 = -0 = 0

ID - first - Last = 14

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 0 & 0 & 0 \\ 1 & -4 & 14 \end{bmatrix}$$

First we interchange, R_3 into R_4. so the operation is

$R_3 \longleftrightarrow R_4$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 1 & -4 & 14 \\ 0 & 0 & 0 \end{bmatrix} R_3 \longleftrightarrow R_4$$

Now, multiply 3 times to Row 3 and then ~~mult~~ multiply 2 times to Row one and the adding Row 1 to Row three. so the operation is

$3R_3 + 2R_1$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 3+2 & -12+12 & 42+16 \\ 0 & 0 & 0 \end{bmatrix} \quad 3R_3 + 2R_1$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 5 & 0 & 58 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, multiply four times to Row one and then ~~Negative~~ three times of R_2, and then subtract R_2 from R_1. So the operation are

$4R_1 - 3R_2$

$$\begin{bmatrix} 4-6 & 24-24 & 32+3 \\ 2 & 8 & -1 \\ 5 & 0 & 58 \\ 0 & 0 & 0 \end{bmatrix} \quad 4R_1 - 3R_2$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2 & 8 & -1 \\ 5 & 0 & 58 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, multiply Row three by two and R_1 by five, then add R_1 to R_3. So the operation is.

$$2R_3 + 5R_1$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2 & 8 & -1 \\ 10-10 & 0 & 175+116 \\ 0 & 0 & 0 \end{bmatrix} \quad 2R_3 + 5R_1$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2 & 8 & -1 \\ 0 & 0 & 291 \\ 0 & 0 & 0 \end{bmatrix}$$

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Now, Adding Row one to Row two, so the operation are:

$R_2 + R_1$

$$\begin{bmatrix} -2 & 0 & 35 \\ 2-2 & 8+0 & -1+35 \\ 0 & 0 & 291 \\ 0 & 0 & 0 \end{bmatrix} R_2 + R_1$$

$$\begin{bmatrix} -2 & 0 & 35 \\ 0 & 8 & 34 \\ 0 & 0 & 291 \\ 0 & 0 & 0 \end{bmatrix}$$

So this is the final echelon form.

The END