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SECTION - "B"

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SEMESTER - 2nd

PAPER - Linear Algebra

QUESTION No: (1)

Consider the given below matrix as the augmented matrix of a linear system. Explain in your words the next Elementary row operation that should be performed in order to solve this system. Where ID<sub>3</sub> is the 3rd digit in your ID and ID<sub>last</sub> is the last digit of your ID in inverse e.g. if your ID is 12345 then ID<sub>last</sub> is 5.

(ANSWER)



$$\left[ \begin{array}{cccc|c} 1 & 3 & 3 & 0 & 5 \\ 0 & 1 & -8 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right] \rightarrow -3R_2 + R_1$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 27 & 0 & -16 \\ 0 & 1 & -8 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right] \rightarrow 8R_3 + R_2$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 27 & 0 & -16 \\ 0 & 1 & -8 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right] \rightarrow 8R_3 + R_2$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 27 & 0 & -16 \\ 0 & 1 & 0 & 0 & -41 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right] \rightarrow -27R_3 + R_1$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 146 \\ 0 & 1 & 0 & 0 & -41 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right] \text{Ans} \leftarrow$$



## QUESTION NO:- (2)

Part (a):-

Find the elementary row operation that transforms the first matrix into Second and reverse row operation that transforms the Second matrix into first.

(ANSWER)

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \rightarrow R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$\leftarrow \longrightarrow$



$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} \rightarrow R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$



Part (b):

Given below are some matrices. Find whether these are in the forms written in front of them or not. Explain in your own words for each of the selection in detail:

(a)  $\begin{bmatrix} p & 0 & 0 & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & -r & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$  is in echelon form

Yes, it is an echelon form because in echelon form the number of zero increase row by row.



⊙  $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  is in echelon form

It is not in echelon form  
it is in reduced echelon form

⊙  $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$  is in reduced echelon form.

It is an echelon form  
Not reduce echelon form.

⊙

⊙  $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$  is in reduced echelon form.

It is also echelon Form

QUESTION No. (3)

Part (a)

The row echelon form is used to solve the system of linear equations. What is the difference between the row echelon and reduced row echelon form? What is the practical



Use of reduced row echelon form? Give one example.

Q (ANSWER)

Difference between the row echelon and reduced row echelon form:

The difference are given below:

The row echelon form of a matrix is not unique, which means there are infinite answer possible when you perform row reduction.

Reduced row echelon form is at the other end of the spectrum; it is unique, which means row-reduction on a matrix will produce the same answer no matter how you perform the same row operations.

Practical use of Reduced Row Echelon Form:-

Reduced row echelon form is a type of matrix used to solve system of linear equations. Reduced Row echelon form has four requirements.

- The first non-zero number in the first row (the leading entry) is the number 1.

The second row also starts with the number 1, which is further to the right than the leading entry in the first row. For every subsequent row, the number 1 must be further to the right.

- The leading entry in each row must be the only non-zero number in its column.
- Any non-zero rows are placed at the bottom of the matrix.

Example:

$$\begin{bmatrix} 1 & 0 & a_1 & 0 & b_1 \\ 0 & 1 & a_2 & 0 & b_2 \\ 0 & 0 & 0 & 1 & b_3 \end{bmatrix}$$

• Part (b):

Find an echelon form for the below matrix using row operations. Where ID<sub>2</sub> is 2nd digit in your ID e.g. if your ID is 12345, ID<sub>2</sub> = 2, ID<sub>3</sub> = 3, ID-first-last is the first and last digit of your ID i.e. 15

$$\begin{bmatrix} 1 & \text{ID}_2 & 8 \\ 2 & 8 & -1 \\ -\text{ID}_3 & 0 & 0 \\ 1 & -4 & \text{ID-First-Last} \end{bmatrix}$$



(Answer)

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -3 & 0 & 0 \\ 1 & -4 & 18 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -3 & 0 & 0 \\ 1 & -4 & 18 \end{bmatrix} \quad -2R_4 + R_2$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 16 & -37 \\ -3 & 0 & 0 \\ 1 & -4 & 18 \end{bmatrix} \quad 3R_1 + R_3$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 16 & -37 \\ 0 & 18 & 24 \\ 1 & -4 & 18 \end{bmatrix} \quad \begin{array}{l} R_1 - R_4 \\ \bullet R_1 + R_4 \end{array}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 16 & -37 \\ 0 & 18 & 24 \\ 0 & 10 & -10 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 16 & -37 \\ 0 & 18 & 24 \\ 0 & 10 & -10 \end{bmatrix} \quad \begin{array}{l} \frac{1}{16} R_2 \\ \end{array}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 1 & -\frac{37}{16} \\ 0 & 18 & 24 \\ 0 & 10 & -10 \end{bmatrix} \quad -18R_2 + R_3$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 1 & -\frac{37}{16} \\ 0 & 0 & \frac{525}{8} \\ 0 & 10 & -10 \end{bmatrix} \quad \begin{array}{l} 10R_2 - R_4 \\ \end{array}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 1 & -\frac{37}{16} \\ 0 & 0 & \frac{525}{8} \\ 0 & 0 & -\frac{105}{8} \end{bmatrix}$$

THE END.