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SUBJECT: MATHEMATICS.2.

PROGRAM: B.tech electrical

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Q1. Estimate the general solution of
 $y' = (x+2)y^2$.

$$\Rightarrow \frac{dy}{dx} = (x+2)y^2$$

$$\Rightarrow \frac{dy}{y^2} = (x+2) dx$$

$$\Rightarrow y^{-2} dy = (x+2) dx$$

$$\Rightarrow \int y^{-2} dy = \int (x+2) dx$$

$$\Rightarrow \frac{y^{-2+1}}{-2+1} = \frac{x^2 + 2x + C}{2}$$

$$\Rightarrow -y^{-1} = \frac{x^2 + 2x + C}{2}$$

$$\Rightarrow -y^{-1} = \frac{x^2 + 4x + 2C}{2}$$

$$\Rightarrow \frac{1}{-(x^2 + 4x + 2C)} = \frac{1}{y^2}$$

$$\Rightarrow y = \frac{-2}{x^2 + 4x + C}$$

$$\Rightarrow y = -2(x^2 + 4x + C)^{-1}$$

Hence $-2(x^2 + 4x + C)^{-1}$ is general solution of y.

Page 2.
Q2. $x^3 dx + y^3 dy = 0$

⇒ Ist method

$$u = \int Mx dx + \int Ny dy \longrightarrow *$$

$$\Rightarrow \int Mx dx = \int x^3 dx = \frac{x^4}{4} + C$$

$$\Rightarrow \int Ny dy = \int y^3 dy = \frac{y^4}{4} + C$$

Put in *

$$u = \frac{x^4}{4} + C + \frac{y^4}{4} + C$$

$$u = \frac{x^4}{4} + \frac{y^4}{4} + 2C = 0$$

$$u = \frac{x^4}{4} + \frac{y^4}{4} = -2C$$

$$\Rightarrow \frac{x^4}{4} + \frac{y^4}{4} = C \quad \text{Ans}$$

⇒ 2nd method

Differential homogeneous method

$$x^3 dx = -y^3 dy$$

$$-\frac{x^3}{y^3} = \frac{dy}{dx}$$

÷ by x^3 .

$$-\frac{1}{\frac{y^3}{x^3}} = \frac{dy}{dx} \longrightarrow \textcircled{1}$$

Page 13.

& new independent variable "v"
by making substitution.

$$\Rightarrow \frac{y}{x} = v$$

$$\Rightarrow y = xv$$

Product
rule.

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Put in eq (1).

$$-\frac{1}{v^3} = v + x \frac{dv}{dx}$$

$$-\frac{1}{v^3} - v = x \frac{dv}{dx}$$

$$\frac{(1 - v^4)}{v^3} = x \frac{dv}{dx}$$

$$\frac{dx}{x} = \frac{dv}{\frac{(1 - v^4)}{v^3}}$$

$$\frac{dx}{x} = \frac{v^3}{1 - v^4} dv$$

$$\frac{dx}{x} = \frac{v^3}{v^4 - 1} dv$$

integrating

$$\int \frac{dx}{x} = \int \frac{v^3}{v^4 - 1} dv$$

$$\int \frac{dx}{x} = \frac{1}{3} \int \frac{3v^3 dv}{v^4 - 1}$$

$$\log x + \log c = \frac{1}{3} \log (v^4 - 1)$$

$$3 \log x + \log c = \log (v^4 - 1)$$

$$\log (xc)^3 = \log (v^4 - 1)$$

$$xc = v^4 - 1$$

$$v^4 = \frac{y^4}{x^4}$$

$$x^3 c^3 = \frac{y^4}{x^4} - 1$$

$$x^3 c^3 = \frac{y^4 - x^4}{x^4} \quad \therefore c^3 = c$$

∴

$$c = \frac{y^4 - x^4}{x^4} \quad \text{Ans.}$$

Q3

$$yy' + 4x = 0$$

$$y \frac{dy}{dx} + 4x = 0$$

$$y dy = -4x dx$$

Taking integration

$$\int y dy = \int -4x dx$$

$$\frac{y^2}{2} = -4 \frac{x^2}{2} + C$$

$$\frac{y^2}{2} = -2x^2 + C$$

$$y^2 = -4x^2 + 2C$$

$$y^2 = -4x^2 + C \quad \therefore 2C = C$$

Taking $\sqrt{\quad}$

$$\sqrt{y^2} = \sqrt{-4x^2 + C}$$

$$y = \sqrt{C - 4x^2}$$