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Section = B -

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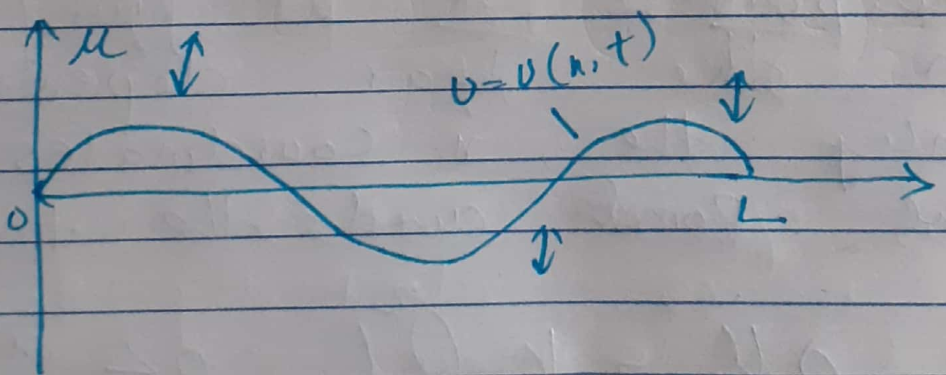
Application of Partial Differential equation

(1) Wave equation:-

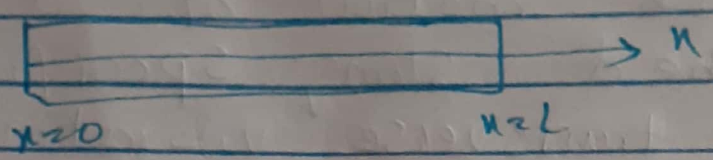
The simplest situation to give rise to the one-dimensional wave equation is the motion of a stretched string - specifically the transverse vibrations of a string such as the string of a musical instrument. Assume that a string is placed alone the x-axis is streched and then fixed at end $x=0$ and $x=L$.



$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

(ii) Heat conduction equation:-

Consider a long thin bar or wire of constant cross-section and of homogenous meterial oriented along the x-axis

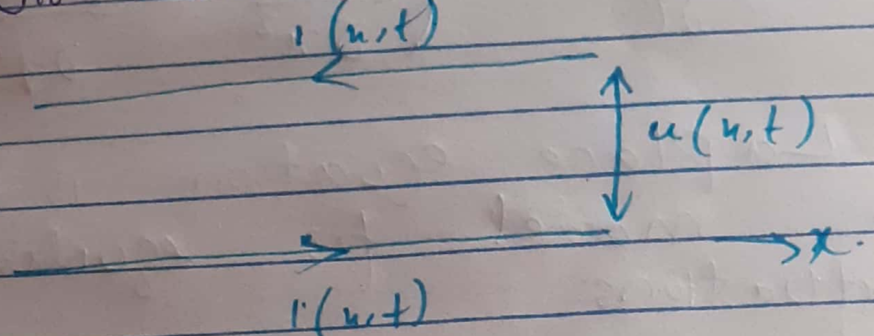


Imagine that the bas is thermally insulated laterally and is sufficiently thin that heat flows (by conduction) only in the x-axis direction then the temprature u at any point in the bar depends only the x-coordinates of the point and the time t

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \qquad \frac{0 \leq x \leq L}{t \geq 0}$$

(iii) Transmission Line Equations:-

In a long electrical cable or a telephone wire both the current and voltage depend upon position along the wire as well as the time.



Using basic law of electrical circuit theory the the electrical ~~cotton~~ current i(x,t) satisfies

$$\frac{\partial^2 i}{\partial x^2} = LC\frac{\partial^2 i}{\partial t^2} + (RC+GL)\frac{\partial i}{\partial t} + RGi$$

(iv) Laplace's Equation:-

if you look back the two dimensional heat conduction equation:

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Laplace equation is always a model for equilibrium situations.

Other important PDEs in Science & Engineering:-

(i) Poisson equation:-

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$$

This equation arises in electronics, elesticity theory and else where.

(ii) Helmholtz equation:-

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k^2 u = 0.$$

(iii) Schrodinger Equation:

$$\frac{-h^2}{8\pi^2 m}\left(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}+\frac{\partial^2 \psi}{\partial z^2}\right)=E\psi^2$$

(iv) Transverse vibrations equation:-

$$a^2\frac{\partial^4 u}{\partial x^4}+\frac{\partial^2 u}{\partial t^2}=0.$$