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Sub: ElectroMagnetism

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Assignment

so now

$dw = -(20\times10^{-6})\ [100\,a\rho - 200\,a\phi + 300\,az]\cdot[0.371\,8x - 0.557\,ay$

$+ 0.743\,az]\ (6\times10^{-6})$

$= -(20\times10^{-6})\ [37.1\,(a\rho.2x) - 55.7\,(\partial\rho.\partial y) -$

$74.2\,(a\,\phi.2x) + 111.4\,(2\phi.dy) + 222.9]\,(6\times10^{-1})$

where, at $P$, $(d\rho.dx) = (\partial\phi.d\,y) = \cos(40^\circ) = 0.766$,

$(d\rho.\partial y) = \sin(40^\circ) = 0.643$ and $(d\phi.dx)$

$= -\sin(40^\circ) = -0.643$. substituting These resolution

$dw = -(20\times10^{-6})\ [28.4 - 35.8 + 47.7 +$

$85.3 + 222\,9]$

$(6\times10^{-6}) = -41.8\,nJ$

D) The value of $E$ at $P$ ($\rho = 2$, $\phi = 40^\circ$, $z = 3$) is given as $E = 100\, a_\rho - 200\, a_\phi + 300\, a_z$ V/m. Determine the incremental work required to more a 20 uC charged a distance of 60 um.

Ans) In the direction of $a_\rho$: The incremental work is given by $dw = -qE \cdot dL$, where in this case, $dL = d\rho\, a_\rho = 6\times10^{-6}\, a_\rho$. Thus

* In the direction of $a_\phi$ in this case $dL = 2\, d\phi\, a_\phi = 6 \times 10^{-6}\, a_\phi$, and so

$dw = -(20\times10^{-6})(-200)(6\times10^{-6}) = 2.4\times10^{-8}\,J = \underline{24\,nJ}$

* In the direction of $a_z$: Here $dL = dz\, a_z = 6\times10^{-6}$

$dw = -(20\times10^{-6})(300)(6\times10^{-6}) = -3.6\times10^{-8}\,J = \underline{-36\,nJ}$

* In the direction of $E$: Here, $dL = 6\times10^{-6}\, a_E$, where

$$a_E = \frac{100 a_\rho - 200 a_\phi + 300 a_z}{(100^2 + 200^2 + 300^2)^{\frac{1}{2}}} = 0.267 a_\rho - 0.535 a_\phi + 0.802 a_z$$

Thus

$dw = (20\times10^{-6})\,[100 a_\rho - 200 a_\phi + 300 a_z]\,[0.267 a_\rho - 0.535 a_\phi + 0.802 a_z]$

$(6\times10^{-6}) = \underline{-44.9\,nJ}$

In the direction of $a_G = 2a_x - 3a_y + 4a_z$. In this case, $dL = 6\times10^{-6}\, a_G$, where

$$a_G = \frac{2a_x - 3a_y + 4a_z}{[2^2 + 3^2 + 4^2]^{\frac{1}{2}}} = 0.371 a_x - 0.557 a_y + 0.743 a_z$$

$$\left(\frac{-22x + 3ay - 2z}{\sqrt{14}}\right)(10^{-3})$$

$$= \frac{-(4\times10^{-3})}{\sqrt{14}}(-800-900-500)$$

$$= 2.35\ J$$

$\Rightarrow$ in the direction of

$G = 22x - 3ay + 4az$ ; in this case, $dl = 6\times10^{-6}\ 2G$, where

$$2G = \frac{22x - 3ay + 4az}{(2^2+3^2+4^2)^{1/2}}$$

$$= 0.3712x - 0.557ay + 0.743z$$

So now

$$dw = -(20\times10^{-6})(100\,2p - 200\,2y + 300\,2z) \cdot (0.3718x - 0.557ay + 0.474az)(6\times10^{-6})$$

$$= -(20\times10^{-6})(37.1\,(ap\cdot ax) - 55.7\,(2p\cdot 2y) -$$

$$74.2\,(a\phi\cdot 2x + 111.4\,(2\phi \cdot 2y) +$$

Q2. Let $E = 400a_x - 300a_y + 500a_z$ in the neighborhood of point $P(6, 2, -3)$. Find the incremental work done in moving a 4-C charge a distance of 1mm in the direction specified by:

$\Rightarrow$ $a_x + a_y + a_z$ : we write

$$dw = -qE \cdot dL = -4(400a_x - 300a_y + 500a_z) - \frac{(a_x + a_y + a_z)}{\sqrt{3}}(10^{-3})$$

$$= -\left(\frac{4 \times 10^{-3}}{\sqrt{3}}\right)(400 - 300 + 500 = -1.39\,J$$

$\Rightarrow$ $-2a_x + 3a_y - 3z$ : The computation is similar to that of part a, but we change the direction.

$$dw = -qE \cdot dL = -4(400a_x - 300a_y + 500a_z$$

either point the initial
force will be same.
Thus the answer $du =$
$31\ uJ$ as part a. This
is also found by going
through the same
procedure as part a
but with the direction
(role of P and G)
reversed.

Q3. if $E = 120\ 2\rho$ V/m. Find
the incremental amount
of work done in moving
a 50 uC charge a
distance of 2mm from.

$222.9]\,(6\times10^{-6})$

where, at P, $(2p \cdot 2x) = (2\phi \cdot 2y) = \cos(40^\circ) = 0.766$, $(2p \cdot 2y) = \sin(40^\circ) = 0.643$, and $(2\phi \cdot 2x) = -\sin(40^\circ) = 0.643$. Subsituting these resolution

$$dw = -(2\times10^{-6})\,[28.4 - 35.8 + 47.7 + 85.3 + 222.9]\,](6\times10^{-6}) = -41.8\,nJ.$$

line between P & G would involve moving along a chord of a circle whose radius $\sqrt{5}$. Halfway along this line point of symmetry in field (make a sketch to see this). This means that when starting from

not depend on $\phi$ or $z$.
Note also that P and Q are
at the same radius($\sqrt{5}$)
from the axis but have
different $\phi$ and $z$ coordinates.
We ~~two~~ could just as
well position the two points
at the same $z$ as well
position the two points
at same location and
the problem would not
change. if this were so
then moving along straight.

Q4) Compote the value of G

— — — — — using the path.

(A)
Ans. Straight line of segments
A (1 -1, 2) to B (1,1,2) to
P (2,1,2) in general
we p have

$[G \cdot dL = ] \; 2y \, dx$

The change of $x$ occurs
when moving b/w B and P
during which $y = 1$ Thus

$$\int_A^P G \cdot dL = \int_B^P 2y \, dx = \int 2(1) \, dx = \boxed{2}$$

(B) Straight line segment
A (1, -1, 2), (2, -1, 2) to
P (2, 1, 2) In case the
change in $x$ occur
when moving from A to C

Ans. P (1,2,3) toward Q (2,1,4).
The vector along this
direction will be Q - P = (1,-1,1)
from which $a_{PQ} = (a_x - a_y + a_z)$
$\sqrt{3}$. we now write $dw = -qE\,dL =$
$-(50 \times 10^{-6})\left[120\,2p \cdot \frac{(a_x - a_y + a_z)}{\sqrt{3}}\right](2 \times 10^{-2})$

At P, $\phi = \tan^{-1}(2/1) = 63.4^{\circ}$ This
$(2p \cdot 2x) = \cos(63.4) = 0.447$
and $(2p \cdot 2y) = \sin(63.4) = 0.894$
Sobsituting these, we obtained
$dw = 3.14J$.

=> Q (2,1,4) toward P (1,2,3).
A little thought is in
order ~~side~~ here. Note thate
the field has only a
radial component and does

during

which $y = -1$ thus

$$\int_{A}^{y} G \cdot dl \Big]_{A}^{\omega} \cdot 2y\,dx \Big]_{1}^{2} 2(-1)\,dx \boxed{-2}$$

Qs) For $G = 3xy^3 a_x + 2z a_y$.

Now Thing S - - - - - - -

in that path does matter.

Ans. straight line $y = x-1,\ z = 1$

we obtain

$$\int G \cdot dL \Big]_{2}^{4} 3xy^2\,dx + \Big]_{1}^{3} 2z\,dy \Big]_{2}^{4}$$

$$3x(x-1)^2\,dx + \boxed{96}$$

$$\int_{1}^{3} 2(1)\,dy)$$

b. Parabola $6y = x^2 + 2,\ z = 1$

we obtain

$$\int G \cdot dL = \int_2^4 3xy^2\,dx + \int_2^3 2z\,dy$$

$$\Rightarrow \int_2^4 \frac{1}{12}\,x\,(x^2+2)^2\,dx + \int_1^3 2(1)\,dy = \boxed{82}$$