Date:___________

Name :- Muhammad Daniyal

Id NO:- 17011

Class timming:- wednesday (8:am to 11:-00 am)

Subject:- Discrete mathatics

Program:- BS(CS)

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"Question no 1"

what is Recurrence Relation? And explain Repeated substitution method with help of example:-

Answer:-

Recurrence Relation:-

A recurrence relation is an equation that recursively defines a sequence where the next Term is a function of the previous terms (Expressing F_n as Some Combination of F_i with $i<n$). Example – Fibonacci Series – $F_n = F_{n-1} + F_{n-2}$

Repeated Substitution method:-

Example:-

$$T(n) = \begin{cases} 1, & \text{if } n = 1 \\ 4T(n/2) + n, & \text{if } n \geq 2 \end{cases} \qquad n = 2^k$$

(a) $T(n) = n + 1\,(n/2)$

Lets write $T(n)$ as a function of $(n/2)$

$K = 1;$

$$T\left(\frac{n}{2}\right) = \frac{n}{2} + 4T\left(\frac{n/2}{2}\right) = \frac{n}{2} + 4T\left(\frac{n}{4}\right)$$

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$T(n) = n + 4\left[\frac{n}{2} + 4T\left(\frac{n}{4}\right)\right] = n + \frac{4n}{2} + (4^2)T\left(\frac{n}{4}\right)$

$T(n) = n + 2n + 16T\underbrace{\left(\frac{n}{4}\right)}$

Lets write $T(n)$ as a function of $\left(\frac{n}{4}\right)$

$\underline{K=2;}$

$T\left(\frac{n}{4}\right) = \frac{n}{4} + 4T\left(\frac{n/2}{4}\right) = \frac{n}{4} + 4T\left(\frac{n}{8}\right)$

$T(n) = n + 2n + 16\left[\frac{n}{4} + 4T\left(\frac{n}{8}\right)\right].$

$= n + 2n + \frac{16n}{4} + (16)(4)T\left(\frac{n}{8}\right)$

$T(n) = n + 2n + 4n + 64T\left(\frac{n}{8}\right)$

Lets write $T(n)$ as a function of $(n/8)$

$\underline{K=3;}$

$T\left(\frac{n}{4}\right) = \frac{n}{4} + 4T\left(\frac{n/2}{4}\right) = \frac{n}{4} + 4T\left(\frac{n}{8}\right)$

$T(n) = n + 2n + 4n + 64T\underbrace{\left(\frac{n}{8}\right)}$

Lets write $T(n)$ as a function of $\left(\frac{n}{8}\right)$

$T\left(\frac{n}{8}\right) = \frac{n}{8} + 4T\left(\frac{n/2}{8}\right) = \frac{n}{8} + 4T\left(\frac{n}{16}\right)$

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$T(n) = n + 2n + 4n + 64\left[\frac{n}{8} + 4T(n/16)\right]$

$= n + 2n + 4n + \frac{64n}{8} + (64)(4)T(n/16)$

$T(n) = n + 2n + 4n + 8n + 256T(n/16)$

At this point we can see a patteren so we stop "repeating substitution" And write a general expression that describes the behaviour.

(a) $T(n) = n + 2n + 4n + 8n + 256T(n/16)$

$T(n) = n(1 + 2 + 4 + 8) + 256T(n/16)$

$T(n) = n(2^0 + 2^1 + 2^2 + 2^3) + (4)^4 T(n/2^4)$

we can now generalize as follows

$T(n) = n(2^0 + 2^1 + 2^2 + 2^3 + \text{-----} + (2^{k-1}) + 4^k T(n/2^k)$

$T(n) = n \sum_{i=0}^{n-1} 2^i + 4^k T(n/2^k)$

The problem says that $n = 2^k$ so,

$T(n) = n \sum_{i=0}^{n-1} 2^i + 4^k T(2^k/2^k)$

$T(n) = n \sum_{i=0}^{k-1} 2^i + 4^k T(1)$

The problem says that $T(n) = 1$ if $n = 1$

$T(n) = n \sum_{i=0}^{n-1} 2^i + 4^k (1)$

$T(n) = n \sum_{i=0}^{k-1} 2^i + 4^k$

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"Question no 2"

Premises:

If there was a ball game, then traveling was difficult. If they arrived on time, then traveling was not difficult. They arrived on time

Conclusion:-

There was no ball game.

Determine symbollicaly using rule of Inference.

P: There was a ball game.

q: Traveling was difficult.

r: They arrived on time.

Solution:-

1 →	$P \to q$	Premises
2 →	$r \to \sim q$	Premises
3 →	r	Premises
4 →	$\sim q$	2, 3 modus ponens
5 →	$\sim q \to \sim P$	1, Contrapositive
	$\sim P$	4, 5 Modus ponens

Conclusion:-

$\sim P$ = There was no ball game.

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"Question no 3"

Determine logicaly Symbolically using rule of infeunce.

Solution:-

P: Claghorn has wide Support.
q: claghorn is asked to run for the Senate
r: claghorn yells "Eureka"

$P \to q$	Premises
$r \to \sim q$	Premises
r	Premises
$\sim q$	2,3 Modus ponus
$\sim q \to \sim P$	1, Contrapositive
$\sim P$	5,6 modus ponus

Conclusion:-

$\sim P$ = Claghorn does not have wide Support.

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"Question no 4"

Answer:-

Pigeon hole principle:-

~~Th~~ The pigeon hole principle states, that if n items states that are put into m containers, with n>m, then at least one container must contain more than one item. If you have more "objects" than you have "holes", at least one hole must have multiple objects init. A real-life example could be, "If you have three gloves, then you have at least two right hand gloves, or at least two left hand gloves, because you have three objects, but only two categories to put them into (right to left). The seemingly obvious statement, a type of counting agrement, canbe used to demonstrate possibly unexpected results. For example if you know that the ~~type~~ population of London is greater by (at least two people) than the maximum number of hairs than Canbe present on a human head, then the pegion hole principle reqjuires that there must be (at least) two people in london who have the Same number of hairs on their heads.

The principle have severl generalizations and Can be Stated in various ways. In a more quantified version. for natural number K and m, if $n = Km + 1$ objects are distributed among m sets. then the pigeonhole principle asserts that at least one of the sets will Contains at least $K+1$ objects. For arbitrary n and m generalizes to $K+1 = [(n-1)/m] + 1 = [n/m]$, ~~when~~

Example for pigeonhole:-

1) Finding maximum and minimum value in array. Let the minimum and maximum values be "min" and "max" respectively.

2) Also find range as "max-min+1"

3) Visit each element of the array then put each element in its pigeonhole.

4) Start the loop all over the pigeonhole.

Input data:

8	3	2	7	4	6	8

0	
1	
2	
3	
4	
5	
6	

Range: $8-2+1=7$

$arr[i]-min$

$8-2=6$

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8	3	2	7	4	6	8

0	2
1	3
2	4
3	
4	6
5	7
6	8 8

Range: 8 − 2 + 1 = 7

arr [i] − min

3 − 2 = 1

Now

2 − 2 = 0

Now:-

Subtract third array

arr [i] − min

7 − 2 = 5

Now:-

arr [i] − min

4 − 2 = 2

Now:-

arr [i] − min

6 − 2 = 4

Now:-

8 − 2 = 6

Now this is the example of pegion hole the in one hole there are two pegions init.

SMNotes

"Question no 6"

Answer:-

Let P, Q and R be the three Control panels.

Let S be the System.

Let 0 represent that the Switch in the Control panel is in the off ~~possib~~ position.

Let 1 represents that the System is disabled and S = 1 represents that the System is enabled.

If two or three of the Control panels have the Switches in the on positions then the System is enabled.

$P = 0$ and $Q = 1$ and $R = 1$

$P = 1$ and $Q = 0$ and $R = 1$, then $S = 1$.

If one or ~~more~~ none of the Control panels have the Switches in the on ~~Switches~~ position, then the System is disabled.

P=1 and Q=0 and R=0, then S=0
P=0 and Q=1 and R=0, then S=0
P=0 and Q=0 and R=1, then S=0
P=0 and Q=0 and R=0, then S=0

Combining this information in an input/output table, we then obtain:

Input			output
P	Q	R	S
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

Boolean expression:-

Negation $\sim p$: not p

Disjunction $p \vee q$: p or q

Conjunction $p \wedge q$: p and q

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If two or the three Control panels have the Switches inthe on positions $(S=1)$ then their $P \wedge Q$, $P \wedge R$ or $Q \wedge R$ is equal to 1 (or all three are true at the same time when all three Control panels have the Switches inthe on position), The boolean expression is then the disjunction of the these three Statments

$$(P \wedge Q) \vee (Q \wedge R) \vee (P \wedge R)$$

Circuit:-

A "not gate" outputs the negation $\sim a$ of the input a.

An "OR Gate" outputs the disjunction $a \vee b$ of the inputs a and b.

An AND Gate outputs the Conjunction $a \wedge b$ of the inputs a and b.

$$(P \wedge \sim Q) \vee (\sim P \wedge Q).$$

The negation will occur before the Conjunctions and disjunction, thus we first negate P by using a NOT gate with input P and we negate

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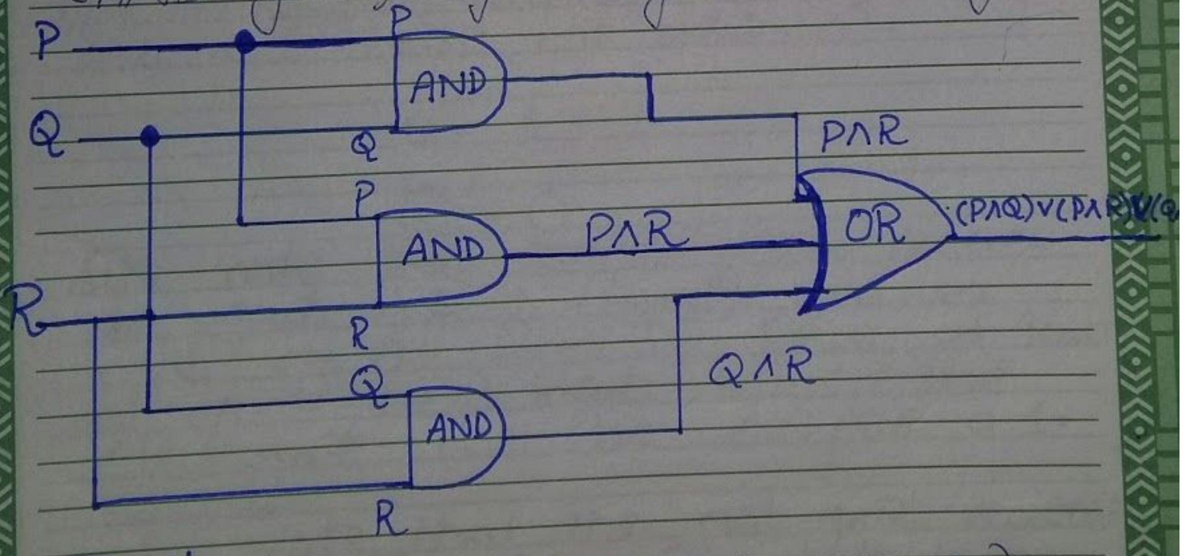
Q by using a NOT gate with input Q.

Next the conjunction will occur before the disjunction in $(P \wedge \sim Q) \vee (\sim P \wedge Q)$ due to the two pairs of baskets. We then need to use an AND gate with input P and $\sim Q$ to form $P \wedge \sim Q$ while we use an AND gate with input $\sim P$ and Q to form $\sim P \wedge Q$.

Finally, we need to take the disjunction of $P \wedge \sim Q$ and $\sim P \wedge Q$ (output AND gates) by using an OR gate.



Result:- $(P \wedge Q) \vee (P \wedge R) \vee (Q \wedge R)$

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"Question no 7"

Give the output Signals for the Circuit.

1) Answer:-

A Not gate Changes the input from a 0 to a 1 or form a 1 to a 0.
An OR gate outputs a 1 if at least one of the inputs is a 1 (else it outputs 0).

An AND gate outputs a 1 if both inputs are a 1 (else it outputs 0).

P = 1

Q = 0

We note that the OR gate recieves an input of 1 (from P) and an input of 0 (from Q). Since at least one of the inputs is a 1. the OR gate will be output a 1.

we Note that the "not gate" recieves an input of 0 (from Q), thus the NOT gate will output a 1.

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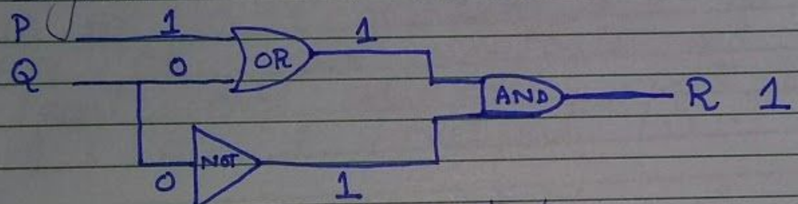
Finally, we then note that the AND gate recieves an input of 1 from the OR gate and an input 1 from the NOT gate. As both inputs are a 1, the AND gate will output a 1 as well thus the output signals is a R = 1.



Inputs		Outputs
P	Q	R
1	0	0
1	1	1
0	0	0
0	1	0

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Part II:-

input Signals P=1 , Q=0 , R=0

Answer:-

A "Not gate" Changes the input from a 0 to a 1 or from a 1 to a 0. An "OR gate" outputs a 1 if at least one of the input is a 1 (else it output 0).

A "AND gate output a 1 if both inputs are a 1 (else it outputs 0)

P = 1 , Q = 0, R = 0

we note that NOT gate recieves an input 0 (from Q). Thus the not gate is output a 1.

we note the AND gate recieves an input of 1 (from P) and an input of 1 (from Q). Since both inputs are a 1, the AND gate will output a 1.

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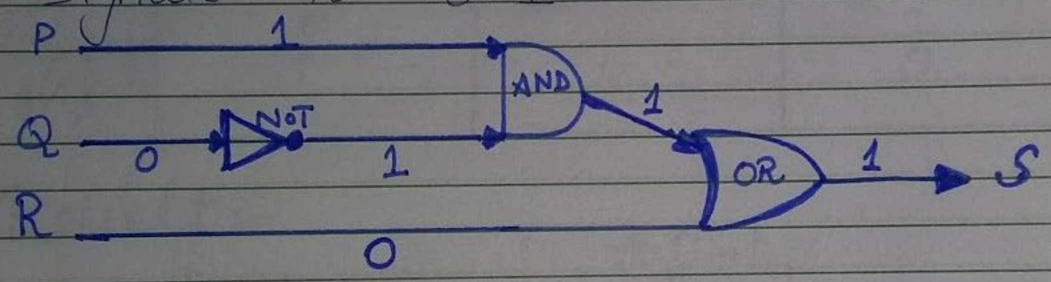
. Finally, we note that the OR gate recieves an input of 1 from the AND gate and an input of 0 form R. As at least one of the inputs is a 1, the OR gate will output a 1 as well thus the output signals is $S=1$.



Similarly, we can derive the output of each gate for every possible input P, Q and R.

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P	Q	R	output NOT Gate	output AND Gate	output OR Gate
1	1	1	0	0	1
1	1	0	0	0	0
1	0	1	1	1	1
1	0	0	1	1	1
0	1	1	0	0	1
0	1	0	0	0	0
0	0	1	1	0	1
0	0	0	1	0	0

Result:-

P	Q	R	output S
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

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"Part 3"

input Signals P=0 , Q=0, R=0

Answer:-

A "NOT gate" Changes the input from a 0 to a 1 or from a 1 to 0.

An "OR gate" outputs a 1 if at least one of the input is a 1 (else it outputs 0).

An "AND Gate" outputs a 1 if both inputs are a 1 (else it outputs 0.

we note that the leftmost OR gate recievese an input of 0 (from P) and an input of 0 (from Q). Since at least one of the inputs is a 0, the OR gate will output a 0.

we note that the AND gate recieves an input of 0 (from Q) and an input of 0 (from R). Since both inputs are a 0. the AND gate will output a 0.

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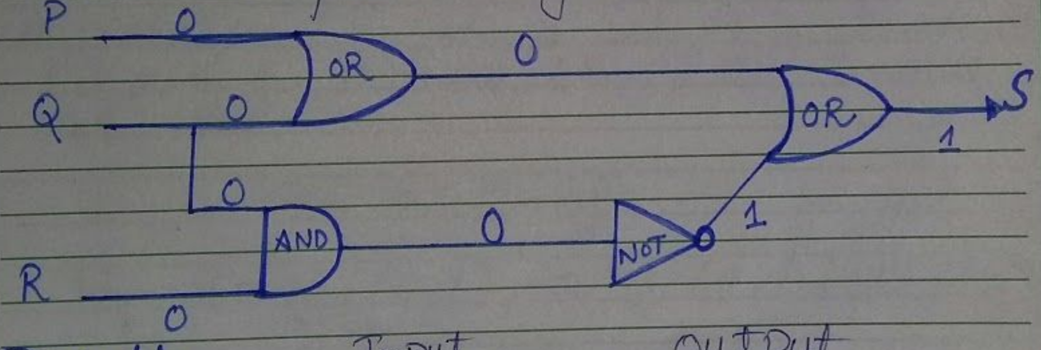
we then note that the NOT gate recieves an input of 0 (from the AND gate), thus the NOT gate will output a 1.

Finally, we then note that the right most OR gate recieves an input of 0 from the left most OR gate an input of 1 from the NOT gate. As at least one of the input is a 1, thus the OR gate will output a 1 as well and thus the output signals is $S = 1$.



Result:-

Input			Output
P	Q	R	S
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

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"Question no 8"

Answer:-

$A = \{0, 1, 2, 3\}$

A relation R on a Set A is reflexive if $(a,a) \in R$ for every element $\forall a \in R$

A relation R on a Set A is Symmeteric if $(b,a) \in R$ whenever $(a,b) \forall \in R$

A relation R on a Set A is Transitive if $(a,b) \in R$ and $(b,c) \in R$ implies $\forall (a,c) \in R$.

$A = \{0, 1, 2, 3\}$

$R_1 = \{(0,0), (0,1), (0,3), (1,1), (1,0), (2,3), (3,3)\}$

(a) Directed Graph:-

we note that A Contains 4 elements and thus we will draw 4 points.

we label these points 0, 1, 2, 3 (which are the element of A.).

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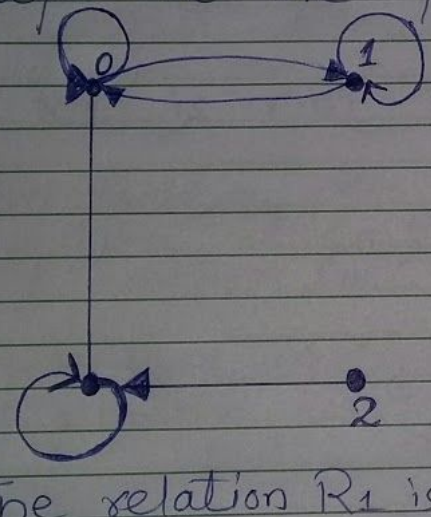
For every element $(x,y) \in R$ with $x \neq y$, we draw an errow from x to y.

For every element $(x,x) \in R$, we draw a loop at the point x.



B) The relation R_1 is relflecsive if $(a,a) \in R_1$ for every element $a \in A$.

Since $A = \{0, 1, 2, 3\}$, R_1 is relfecsive if it Contains $(0,0)$, $(1,1)$, $(2,2)$, $(3,3)$

we note that R_1 does not Contain $(2,2)$ and thus R_1 is not reflexive.

"R_1 is not reflexsive"

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C) The relation R_1 on a set A is Symmetric if $(b,a) \in R_1$ whenever $(a,b) \in R_1$

we not that $(0,3) \in R_1$, while $(3,0) \notin R_1$ thus R_1 is not Symmetric.

"R_1 is not Reflexsive"

D) The relation R_1 on a set is transitive if $(a,b) \in R_1$ and $(b,c) \in R_1$ implies $(a,c) \in R_1$

we note that $(1,0) \in R_1$ and $(0,3) \in R_1$, while $(1,3) \in R_1$ and thus R_1 is not transitive.

"R_1 is not transitive"

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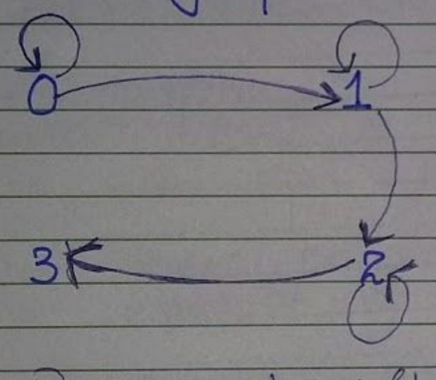
$R_2 = \{(0,0)(0,1)(1,1)(1,2)(2,2)(2,3)\}$

$A = \{0, 1, 2, 3\}$

$R_2 = \{(0,0)(0,1)(1,1)(1,2)(2,2)(2,3)\}$

a) Directed graph:-



b) No, R_2 is not reflexsive because there is no Loop at 3.

C) No, R_2 is not Symmetric because between two numbers where there exists an arrow, the arrow goes only one direction.

d) NO, R_2 is not transitive though there is an arrows between 0 and 1 and also an arrow between 1 and 2, but there is no arrow between 0 and 2

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"Part 3"

$R_3 = \{(2,3), (3,2)\}$

Answer:-

$A = \{0,1,2,3,4\}$

$R_3 = \{(2,3), (3,4)\}$

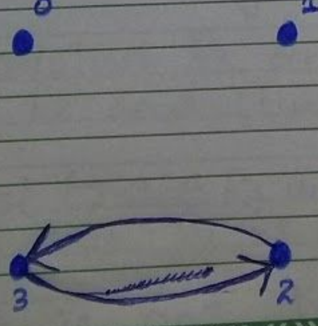
(a) Directed graph:-

we note that A Contains 4 elements and thus we will draw 4 points

we label these points 0, 1, 2, 3 (which are the elements of A.)

For every element $(x,y) \in R$ with $x \neq y$ we draw an arrow from x to y

For every element $(x,x) \in R$, we draw a Loop at the point x.



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b) The relation R_3 is reflexive if $(a,a) \in R_3$ for every element $a \in A$.

Since $A = \{0, 1, 2, 3\}$, R_3 is reflexsive if it Contains $(0,0), (1,1), (2,2) (3,3)$

we note that R_3 does not Contain $(0,0)$ and thus R_3 is not reflexive

"R_3 is not reflexive"

C) The relation R_3 on a Set A is Symmetric if $(b,a) \in R_3$ whenever $(a,b) \in R_3$

we note that $(2,3) \in R_3$ while $(3,2) \in R_3$ and there are no other ordered pairs in R_3, thus R_3 is Symmetric.

"R_3 is Symmetric"

d) The relation R_3 on a Set A is Transitive if $(a,b) \in R_3$ and $(b,c) \in R_3$ implies $(a,c) \in R_3$

we know that $(2,3) \in R_3$ and $(3,2) \in R_3$ while $(2,2) \notin R_3$ and thus R_3 is not Transitive

"R_3 is not transitive"

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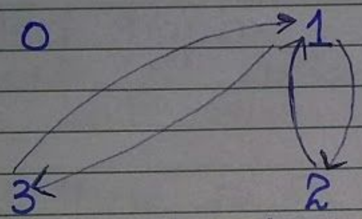
$R_4 = \{(1,2)(2,1)(1,3)(3,1)\}$

$A = \{0, 1, 2, 3\}$

$R_4 = \{(1,2)(2,1)(1,3)(3,1)\}$

a)



b) No, R_4 is not reflexive because there is no loop at 1.

"R_4 is not reflexsive"

c) Yes, R_4 is Symmetric because between two numbers where there exist an arrow, the arrows go both ways

"R_4 is Symmetric"

d) No, R_4 is not transitive though there are arrows between 2 and 1 and also arrows between 1 and 3 but there are no arrows between 2 and 3

"R_4 is not transitive"

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$R_5 = \{(0,0)(0,1)(0,2)(1,2)\}$

$A = \{0, 1, 2, 3\}$

$R_5 = \{(0,0)(0,1)(0,2)(1,2)\}$

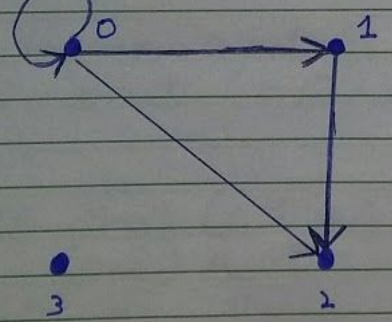
a) Directed graph:-

we note A contains 4 elements and thus we will draw 4 points.

we label these points 0, 1, 2, 3 (which are the elements of A.)

For every element $(x,y) \in R$ with $x \neq y$, we draw an arrow from x to y

For every element $(x,x) \in R$, we draw a loop at the point x.



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b) The Relation R_5 is reflexive if $(a,a) \in R_5$ for every element $a \in A$.

Since $A = \{0,1,2,3\}$, R_5 is reflexive if it Contains $(0,0)$ $(1,1)$ $(2,2)$ $(3,3)$

we note R_5 does not Contain $(1,1)$ and thus R_5 is not reflexive.

"R_5 is not reflexive"

C) The R_5 on a Set A is Symmetric if $(b,a) \in R_5$ whenever $(a,b) \in R_5$

we note that $(0,1) \in R_5$ while $(1,0) \in R_5$ and thus R_5 is not Symmetric

"R_5 is not Symmetric"

d) The R_5 on a set A is Transitive if $(a,b) \in R_5$ and $(b,c) \in R_5$ implies $(a,c) \in R_5$

we note that $(0,1) \in R_5$ and $(1,2) \in R_5$ while $(0,2) \in R_5$ and thus R_5 is transitive as there are no other pairs for which transitivity Canbe Checked

"R_5 is transitive"

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$R_6 = \{(0,1), (0,2)\}$

a) Directed graph:-

$A = \{0, 1, 2, 3\}$

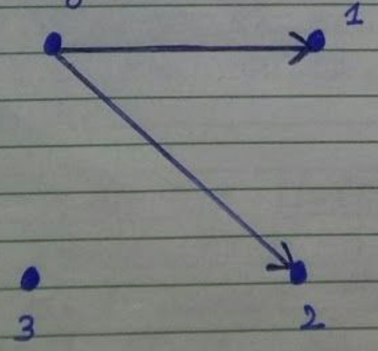
$R_6 = \{(0,1)\ (0,2)\}$

we note that A Contains 4 elements and thus we will draw 4 points

we label these points 0, 1, 2, 3 (which are the elements of A)

For every element $(x,y) \in R$, with $x \neq y$, we draw an arrow from x to y.

For every element $(x,x) \in R$, we draw a loop at the point x.



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b) The relation R_6 is reflexive if $(a,a) \in R_6$ for every element $a \in A$.

Since $A = \{0, 1, 2, 3\}$, R_6 is reflexive if it Contains $(0,0), (1,1), (2,2), (3,3)$

we note that R_6 does not Contain $(0,0)$ and thus R_6 is not reflexive.

"R_6 is not reflexive"

C) The relation R_6 on a Set A is Symmetric if $(b,a) \in R_6$ whenever $(a,b) \in R_6$

we note that $(0,1) \in R_6$ while $(1,0) \notin R_6$ and thus R_6 is not Symmetric.

"R_6 is not Symmetric"

d) The relation R_6 on a Set A is transitive if $(a,b) \in R_6$ and $(b,c) \in R_6$ implies $(a,c) \in R_6$

we note that the if-Statment $(a,b) \in R_6$ and $(b,c) \in R_6$ is never true (as there exists no Such pairs in R_6). when the if-Statement is false and thus R_6 is transitive.

"R_6 is transitive"

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$R_7 = \{(0,3),(2,3)\}$

$A = \{0, 1, 2, 3\}$

$R_7 = \{(0,3),(2,3)\}$

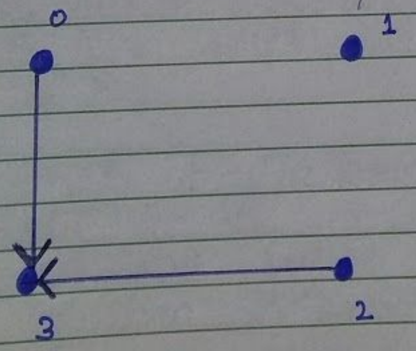
a) Directed graph:-

we note that A Contains 4 elements and thus we will draw 4 points

we label these points 0, 1, 2, 3 (which are the elements of A).

For every element $(x,y) \in R$ $x \neq y$, we draw an arrow from x to y.

For every element $(x,x) \in R$, we draw a loop at the point x.



SMH Notes

Date

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b) The relation R_7 is reflexive if $(a,a) \in R_7$ for every element $a \in A$.

Since $A = \{0,1,2,3\}$, R_7 is reflexive if it contains $(0,0)(1,1)(2,2)(3,3)$

we note that R_7 dose not contain $(0,0)$ and thus R_7 is not reflexive.

"R_7 is not reflexive"

c) The R_7 on a set A symmetric if $(b,a) \in R_7$ whenever $(a,b) \in R_7$

we note that $(0,3) \in R_7$ while $(3,0) \notin R_7$ and thus R_7 is not symmetric.

"R_7 is not symmetric"

d) The R_7 on a set A is transitive if $(a,b) \in R_7$ and $(b,c) \in R_7$ implies $(a,c) \in R_7$

We note that if-statement $(a,b) \in R_7$ and $(b,c) \in R_7$ is never true (as there exist no such pairs in R_7. How ever an if-then statement is always true

~~when the~~ "R_7 is transitive"

SMHNotes

$R_8 = \{(0,0)\ (1,1)\}$

$A = \{0,1,2,3\}$

$R_8 = \{(0,0),(1,1)\}$

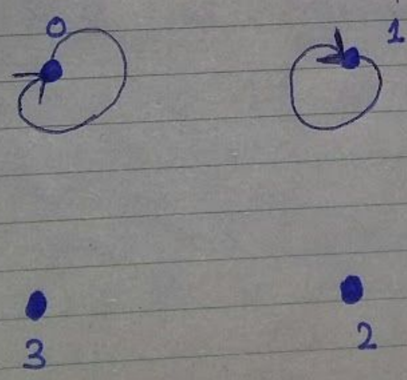
a) Directed graph:-

we note that A contains 4 elements and thus we will draw 4 points.

we label these points 0, 1, 2, 3 (which are the elements of A).

For every element $(x,y) \in R$ with $x \neq y$, we draw an arrow from x to y

For every element $(x,x) \in R$, we draw a loop at the point x.



MASTER NOTES

Date: ____________

b) The relation R_8 is reflexive if $(a,a) \in R_8$ for every element $a \in A$.

Since $A = \{0, 1, 2, 3\}$ R_8 is reflexive if it Contains $(0,0)$ $(1,1)$ $(2,2)$ $(3,3)$

we note that R_8 does not contains $(2,2)$ and thus R_8 is not reflexive.

"R_8 is not reflexive"

c) The relation R_8 on a Set A is Symmetric if $(b,c) \in R_8$ whenever $(a,b) \in R_8$

we note that $(0,0) \in R_8$ while $(0,0) \in R_8$ and $(1,1) \in R_8$ while $(1,1) \in R_8$. There are no other elements in R_8. So the R_8 is Symmetric.

"R_8 is Symmetric"

d) The R_8 on a Set A is ~~Symmetric~~ Transitive if $(a,b) \in R_8$ and $(b,c) \in R_8$ implies $(a,c) \in R_8$

we note that the Statement if "$(a,b) \in R_8$ and $(b,c) \in R_8$" is never True for two distinct pairs (as there no other such pairs in R_8). when if statement is false So R_8 is transitive.

"R_8 is Transitive"

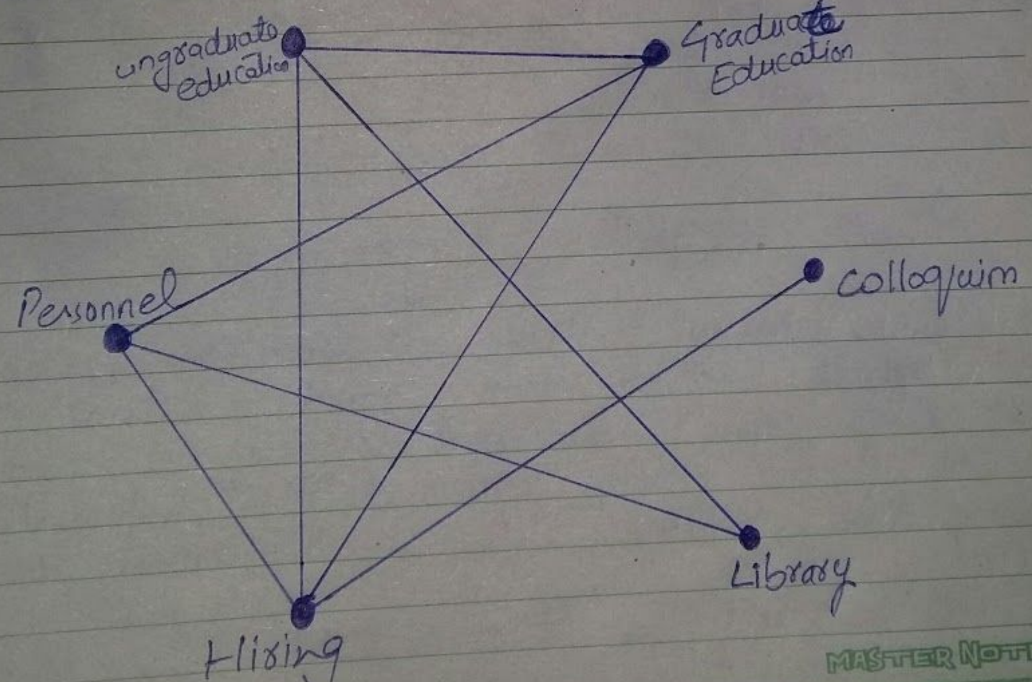
Date: ________

"Question no 10"

Answer:-

we have been given 6 Committes: undergraduate Education, Graduate education Colloquim, Library, Hiring and personnel. Let us then represent the scheduling problem by a graph with 6 vertices and each vertex represents one of the Commitees.

Next, we draw an edge between two vertices, if the two Commitees have a Common member.



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Next, we will assign a color to each other vertex such that no two adjacent vertices have the same color (thus no two vertices that are connected by an edge will have the same color.

we note that the vertex "Hiring" has the most edges connecting the vertex to another vertex. Thus let us color this hiring vertex first the color Red.

"Hiring" = Red

Next, the vertex "Library" is the only vertex that is not connected to the vertex "Hiring" by an edge and thus we can color "Library" the same as "Hiring". Let us colour the vertex "Library" with the color ~~purple~~

"Library" = ~~purple~~ "Red"

Next, we note the vertex "personnel" is connet to vertex "Hiring" and vertex ~~library~~ "library" by an edge. Thus we cannot color the vertex "Personnel" with Red, let us then color this vertex with purple.

"Personnel" = purple.

Date: ____________

Next, we note vertex "Graduate Education is Connected to vertex "Hiring" and vertex "personnel" by an edge. Thus we Cannot Color the vertex "personnel with Red nor with purple, let us then Color this vertex with blue.

"Graduate Education = blue

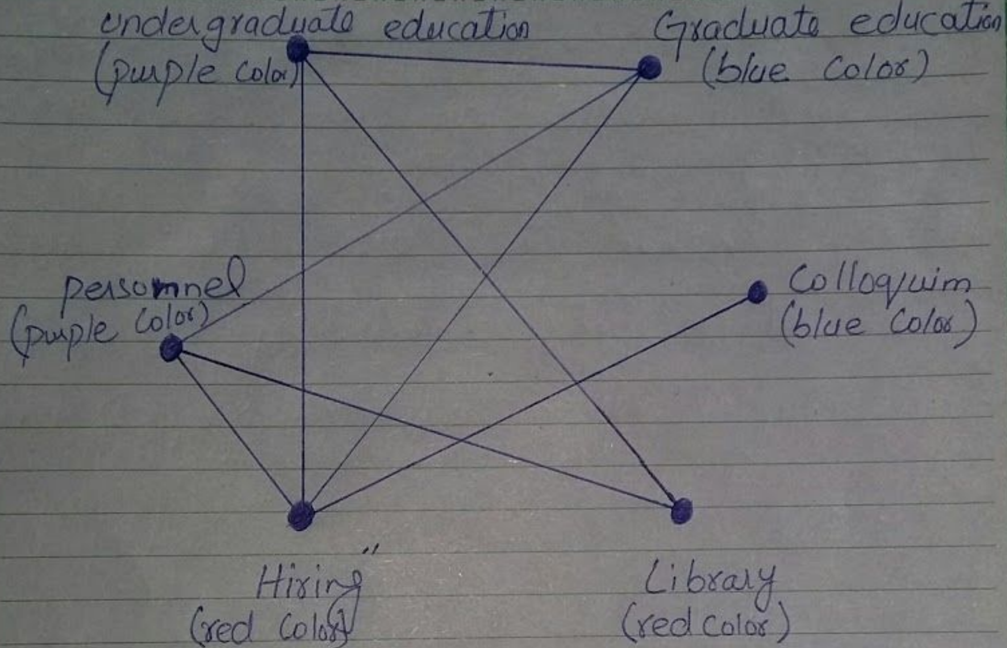
Next, we not that vertex "ungraduate education" is Connected to vertex "hiring" and vertex "Graduate education" by an edge. Thus we Cannot Color the vertex "personnel" with Red nor with blue, let us then Color this vertex with purple

"undergraduate education" = purple.

And then Connect the Colloqium with the color blue

"Colloquium" = blue

we then note that we Colored all vertices such that no adjecent vertices have the same Color and we used exactly 3 Colors.



we can then schedule the meeting by assigning each time slot to a color.

Thus let us asign the first time slot to the red colors, the second time slot to the blue colors and the third time slot to the purple color.

First time slot: Hiring and Library
Second time slot: Colloquim and Graduate education
Third time slot: undergraduate education and personnel.