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SECTION : "B"

SUBJECT : INTRODUCTION TO STRUCTURAL DYNAMICS & EARTHQUACK ENGINEERING

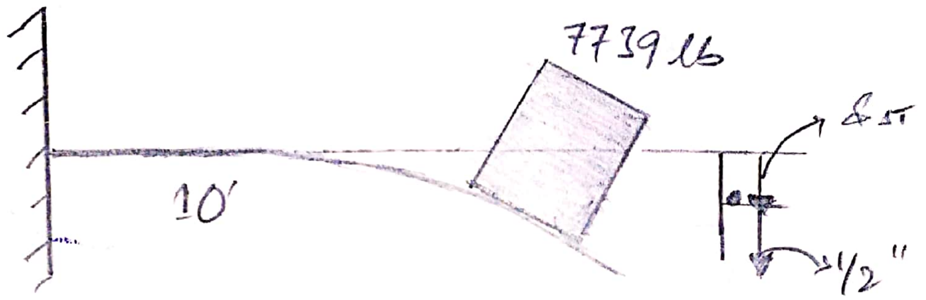
Qno # 01

Given Data:

$E = 29000$ ksi , $I = 150$ in^4

δ_{st} = deflection due to 7739 lb

Static load Beam is pulled 1/2" downwards.



Required Data:

→ Natural time period of system.

→ Develop and solve the equation of motion.

→ Draw graph to show the variation of Equilibrant Static forces with time.

Solution: General equation of motion for SDOF System is ;

$$ku + c\dot{u} + m\ddot{u} = P(t)$$

Since System is undamped $(c=0)$ under going for vibration $P(t) = 0$

Hence gernal EOM become

$$ku + m\ddot{u} = 0 \rightarrow (A)$$

$$k = \frac{3EI}{L^3} = \frac{3 \times 29000 \frac{k}{in^2} \times 150}{(10 \times 12 \text{ in})^3}$$

$$k = 7.55 \text{ k/in.}$$

→ In order to eliminate chances of mistake during calculations, it is more appropriate to use fundamental Units like lb, ft, sec or kg, in, sec.

$$k = 7.55 \text{ k/in}$$

$$k = 90625 \text{ lb/ft}$$

$$m = \frac{w}{g} = \frac{7739}{32.2} = 240.34 \text{ slugs}$$

$$\omega_n = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{90625}{240.34}}$$

$\omega_n = 19.41$ rad/sec

$$T_n = \frac{2\pi}{\omega_n} \Rightarrow \frac{2\pi}{19.41} = 0.323 \text{ sec}$$

put the values of m and k in (A).

$90625u + 240.34\ddot{u} = 0$

where K is in lb/ft and m is lb.

lb sec/ft^2 = slugs.

→ General solution to EOM for undamped free vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$u_n(0) = 1/2'' = \frac{1}{24}$ ft and $\dot{u}(0) = 0$

$$u(t) = \frac{1}{24} \times \cos(19.41t) + 0$$

$$= \frac{1}{24} \times \cos(19.41t).$$

Equivalent static force at anytime "t" is

$$f_s(t) = k \cdot u(t)$$

$$= \frac{90625 \times \cos(19.41t)}{24}$$

$3776 \cos(19.41 t)$

Amplitude of dynamics displacement, u_0 for undamped free vibration is

$$u_0 = \sqrt{\left[(u(0))^2 + \left(\frac{v(0)}{\omega_n}\right)^2\right]}$$

$$= \sqrt{[(1/24)^2 + 0)}$$

$$= \sqrt{(1/24)^2}$$

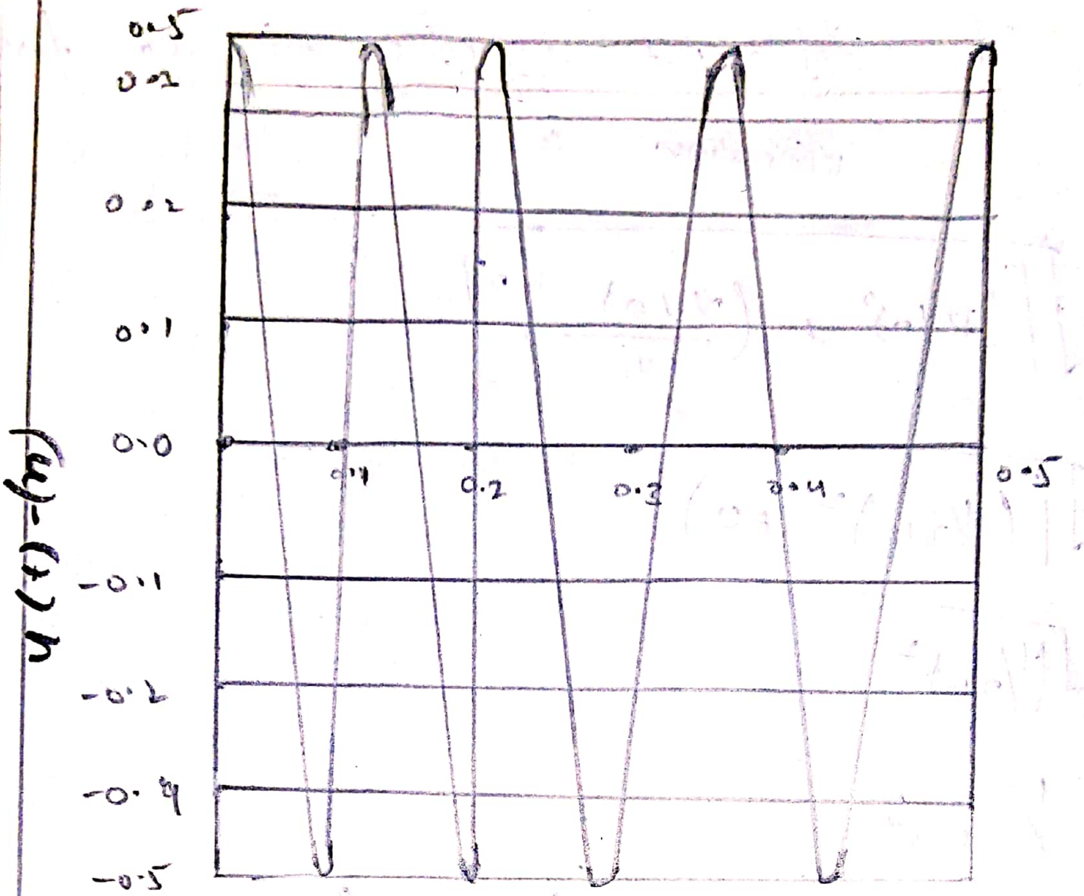
$$= 1/24 \text{ ft}$$

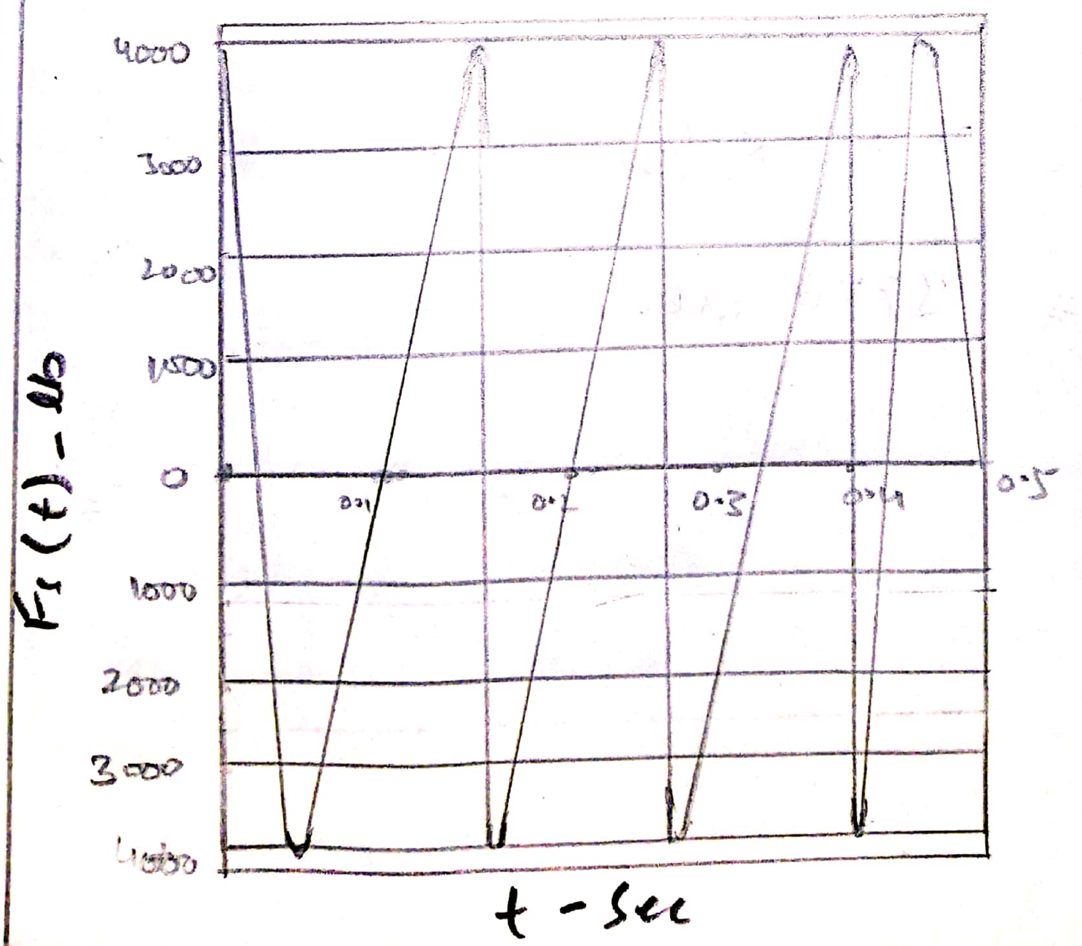
Amplitude of equivalent static force.

F_{s0} .

$$k u_0 = 90625 \times 1/24$$

$$k u_0 = 3776 \text{ lb.}$$


u(t)-(m)
0.5
0.4
0.2
0.1
0.0
-0.1
-0.2
-0.4
-0.5
0.1
0.2
0.3
0.4
0.5


Fs(t) - lb
4000
3000
2000
1500
0
1000
2000
3000
4000
0.1
0.2
0.3
0.4
0.5
t - sec

Qno # 02

Answer:

Given Data:

ξ (damping ratio) of reinforced concrete with consideration cracking

= 3 - 5 %

= 3 %

Data of Beam from Question # 01

Required

→ Develop and solve the equation showing variation in equivalent static force with time.

→ Draw graph to show the variation of displacement with time and the variation of equivalent static force with time.

Sol: EOM for damped free vibration is

$ku + cu + mu'' = 0 \rightarrow$ (A)

As from Question 1

$K = 90625$ lb/ft , $M = 240.34$ lb sec²/ft

$w_n = 19.41$ rad/sec

$c = \xi \times 2 m w_n$

$c = 3\% \times 2(240.34)(19.41)$

$c = 0.03 \times 2(240.34)(19.41)$

$c = 279.89$ lb sec/ft

→ Put the value in eq (A)

$90625 + 279.89 + 240.34 = 0$

Solution to the EOM for damped free vibration is

$$u(t) = e^{-\xi w_n t}\left[u(0)\cos(w_D t) + \frac{1}{w_D}\left[u'(0) + u(0)\xi w_n\right]\sin(w_D t)\right)$$

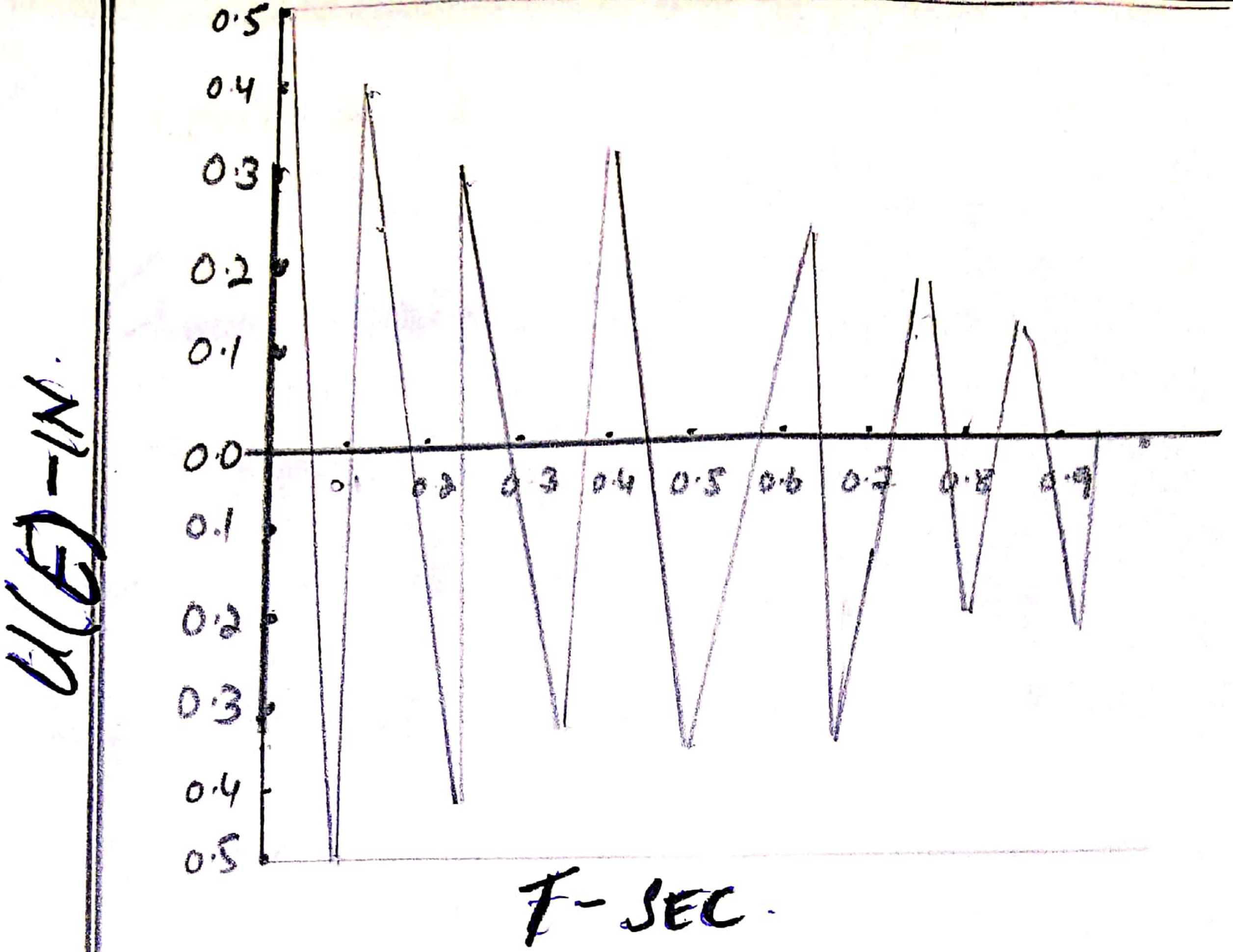
$w_D = 19.41$ rad/sec

$$u(t) = e^{-0.582t}\left[0.041 \times \cos(19.41t) + 0.00125 \times \sin(19.41t)\right)$$

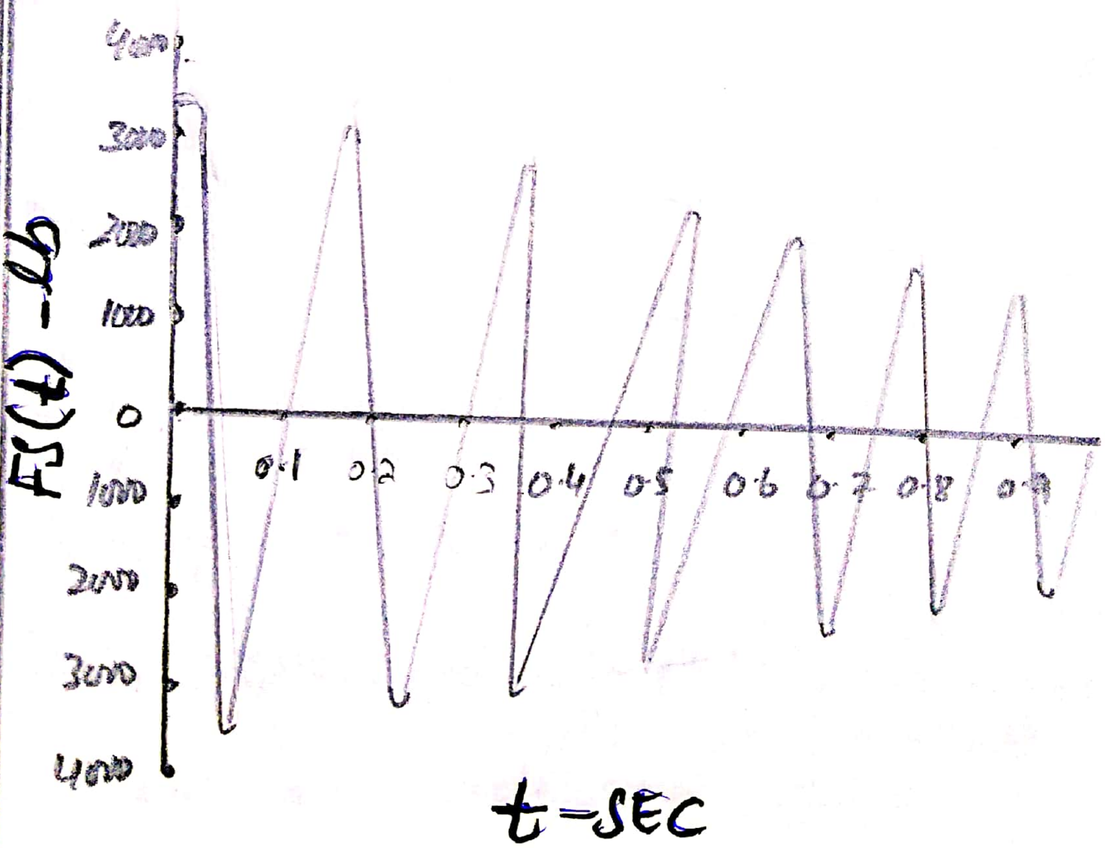
$$F_s(t) = k \cdot u(t) = 90625 \times u(t)$$

$$F_s(t) = e^{-0.582t}\left[(90625 \times 0.041) \cos(19.41t)\ 90625 \times 0.00125 \sin(19.41)\right)$$

$$F_s(t) = e^{-0.582t}\left(3715.62 \cdot \cos(19.41t) + 113.28 \times \sin(19.41)\right)$$



U(t) - IN
0.5
0.4
0.3
0.2
0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
t - SEC



FS(t) - lb
4000
3000
2000
1000
0
1000
2000
3000
4000
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
t - SEC

Qno # 3

Given data:

Force = 60 kips

Displacement of tank = $\frac{ID}{1000}$ = $\frac{7739}{1000}$ = 7.73"

Time taken to complete 7 cycles = 3.57 sec

amplitude of ~~amplitude~~ displacement = 2.286 cm.
= 0.9"

Required:

a) Damping Ratio

b) Natural period of Un-damped Vibration.

c) Stiffness of Structure

d) Weight of tank.

e) damping co-efficient

f) The displacement amplitude to 0.5"
Number of cycle to reduced.

Sol:- Displacement of tank, $u_1 = 7.73$

After, 7 cycles

$J = 7$, $u_{J+1} = u_8 = 0.9"$

a) **Damping Ratio :**

$$J = \frac{1}{2\pi\xi} \ln\left[\frac{u_1}{u_{J+1}}\right]$$

$$7 = \frac{1}{2\pi\xi} \ln\left(\frac{7.73}{0.9}\right)$$

$$\xi = 0.0488$$

$$\xi = 4.88\%$$

b) **Natural Period of Undamped Vibration. T_n = ?**

As, the 7 cycles of vibration are completed in 3.57 sec.

→ Time req to complete one cycle,

$$T_D = \frac{3.57}{7} = 0.51 \text{ sec}$$

$$T_D = 0.51 \text{ sec.}$$

Now,

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n \sqrt{1-\zeta^2})}$$

$$T_D = \frac{T_n}{(1-\zeta)^2}$$

$$T_n = T_D \times \sqrt{1-\zeta^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0488)^2}$$

$$T_n = 0.5094 = 0.51 \text{ sec}$$

$$T_n = 0.51 \text{ sec}$$

c) STIFFNESS OF STRUCTURE, K = ?

$$K = \frac{60 \times \cos 60^\circ}{7.73} = 3.880 \text{ k/in}$$

$$K = 3.9 \text{ k/in}$$

$$K = 46920 \text{ lb/ft}$$

d) Weight of tank, w = ?

$$\omega_n = \sqrt{k/M} = k/_{w/g} \qquad \because w = mg$$

$$\Rightarrow \sqrt{\frac{k \times g}{W}}$$

$$\omega_n = \sqrt{\frac{k \times g}{W}}$$ taking square.

$$\omega_n^2 = \frac{k \times g}{W}$$

$$W = \frac{k \times g}{\omega_n^2}$$

also; $$\omega_n = \frac{2\pi}{T_n}$$

$$W = \frac{k \times g}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$= k \times g \frac{T_n^2}{4\pi^2}$$

$$= \left[\frac{46920}{ft} \times \frac{32.2\ ft}{sec^2}\right] \times \left[\frac{(0.57\ sec)^2}{4\pi}\right]$$

$$= 9953.93\ lb$$

$$= 9.95\ k.$$

e) Damping Co-efficient, c = ?

$$\zeta = \frac{c}{2m\omega_n}$$

$$C = \zeta \times 2m\omega_n$$

$$= \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$= \frac{0.0488 \times 4 \times \pi \times \frac{9953.9}{32.2}}{0.51}$$

$$C = 371.71 \text{ lb-sec/ft}$$

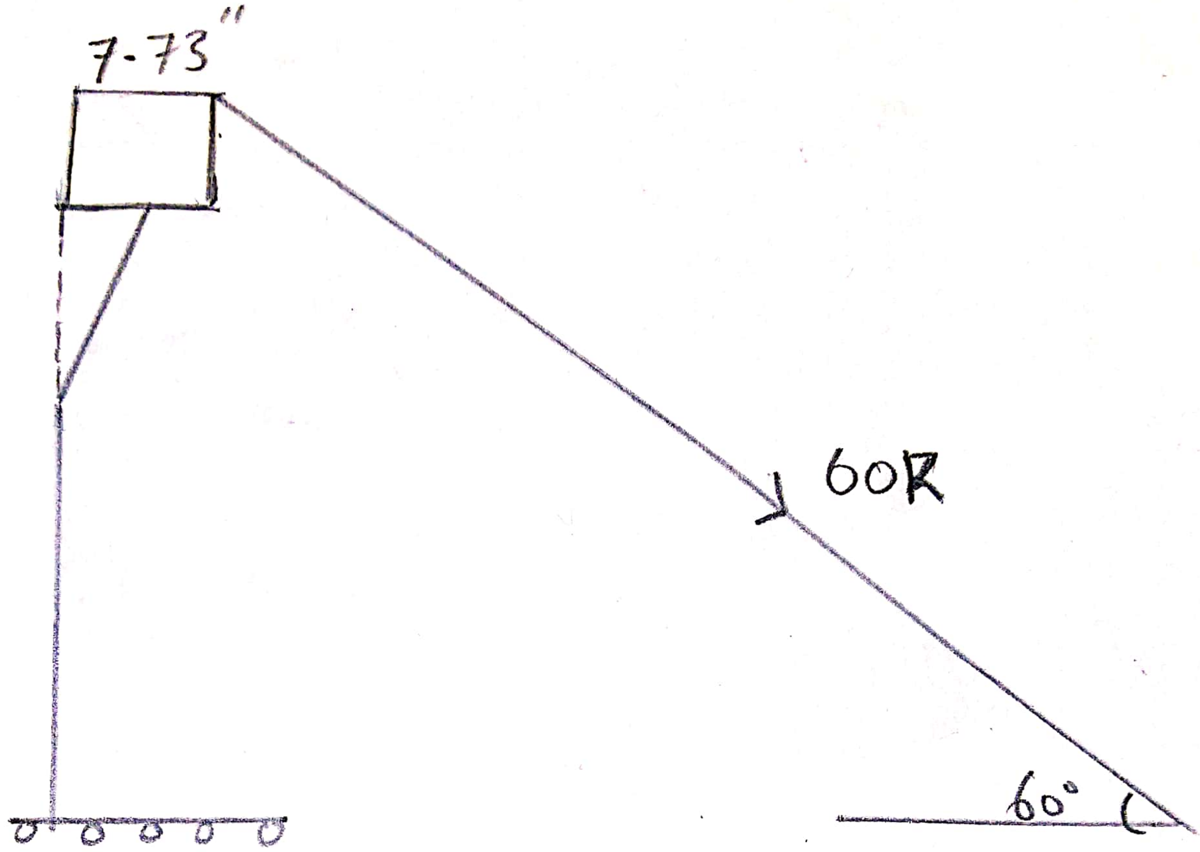
F) Number of cycles to reduce the displacement amplitude to 0.5", J=?

$$J = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{J+1}}\right)$$

$$J = \frac{1}{2\pi \times 0.0488} \ln\left|\frac{7.73}{0.5}\right|$$

$$J = 8.93$$

$$\simeq 9 \text{ cycles.}$$


7·73"
60R
60°