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Q NO# 01 :-

$$\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2) \quad y(0)=0$$

So $x = 0$, $y = 0$.

$$dy = e^{y} \cdot e^{-t} \sec(y)(1+t^2)\, dt.$$

$$\frac{1}{e^{y} \cdot \sec(y)} dt = (1+t^2) e^{-t} dt.$$

As $\cos(y) = \dfrac{1}{\sec(y)}$

$$\int e^{-y} \cos y \, dy = \int (1+t^2) e^{-t} dt$$

using integration by parts.

$$e^{-y} \int \cos y \, dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y} \right) =$$

$$(1+t^2) \int e^{-t} - \int \left(\int e^{-t} \cdot \frac{d}{dt} (1+t^2) \right)$$

eqn ①

L.H.S

$$e^{-y} \int \cos y \, dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y} \right)$$

$$e^{-y} \sin y - \int (\sin y - e^{-y} (-1))$$

$\Rightarrow e^{-y}\sin y + \int(\sin y \cdot e^{-y})$

$\Rightarrow e^{-y}\sin y + \int(\sin y \cdot e^{-y})$

$\Rightarrow e^{-y}\sin y + \int(e^{-y}\sin y)$

Again using integration by parts.

$\Rightarrow e^{-y}\sin y + e^{-y}(-\cos y) - \int(\int \sin y \frac{d}{dy} e^{-y})$

$\Rightarrow e^{-y}\sin y + e^{-y}\cos - \int(-\cos y \frac{e^{-y}}{-1})$

$\Rightarrow e^{-y}\sin y - e^{-y}\cos y - \int(\cos y \, e^{-y})$

$\Rightarrow$ Since $\int(\cos y \, e^{-y}) = L.H.S$

Since it is again same to the first one so L.H.S will become.

$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S$

$2\,LHS = e^{-y}(\sin y - \cos y)$

$$\boxed{L.H.S = \frac{e^{-y}(\sin y - \cos y)}{2}}$$

Now taking R.H.S.

$\int (1+t^2)\, e^{-t}\, dt \}.$

$\Rightarrow (1+t^2) \int e^{-t} - \int (\int e^{-t} \frac{d}{dt} (1+t^2))$

$\Rightarrow (1+t^2)\, e^{-t} + \int -e^{-t} (2t))$

$\Rightarrow (1+t^2)\, e^{-t} + \int ((2t)\, e^{-t})$

Again using integration by parts.

$\Rightarrow (1+t^2)\, e^{-t} + (2t \int e^{-t} - \int (e^{-t} \frac{d}{dt} 2t))$

$\Rightarrow -(1+t^2)\, e^{-t} + (-2t e^{-t} - \int (-e^{-t} 2)$

$\Rightarrow -(1+t^2)\, e^{-t} + (-2t e^{-t} + \int (2e^{-t}))$

$\Rightarrow -(1+t^2)\, e^{-t} + (-2t e^{-t} - 2e^{-t}) + c$

$\Rightarrow -(1+t^2)\, e^{-t} - 2te^{-t} - 2e^{-t} + c$

$\Rightarrow -e^{-t} - e^{-t}t^2 - 2te^{-t} - 2e^{-t} + c$

$\Rightarrow -(t^2+2t+3)\, e^{-t} + c = R.H.S$

Now take L.H.S = R.H.S

P.T.O

$$\Rightarrow \frac{e^{-y}(\sin y-\cos y)}{2} = -(t^2+2t+3)e^{-t}+c$$

we know that

$$\Rightarrow t=0,\ y=0.$$

put it above.

$$\Rightarrow t(0-1) = -3+c$$

$$c = 5/2$$

put value of c

$$\Rightarrow \boxed{\frac{e^{-y}}{2}(\sin y-\cos y) = -(x^2+2t+3)e^{-t} + 5/2}$$

Ans.

QNO#02:-

$$(\sqrt{x+y}+\sqrt{x-y})\,dx-(\sqrt{x+y}-\sqrt{x-y})\,dy=0$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}} \longrightarrow (1)$$

this is Homogeneous Differential eqn in x & y to solve this put

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}.$$

thus eqn (1) Becomes-

$$\Rightarrow v + x\frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+v+1-v+2\sqrt{1-v^2})}{2v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{2(1+\sqrt{1-v^2}}{v}$$

P.T.O

$$\Rightarrow \quad x \frac{dv}{dx} = \frac{1 + \sqrt{1-v^2} - v^2}{v}$$

$$\Rightarrow \quad x \frac{dv}{dx} = \frac{\sqrt{1-v^2}\left(1 + \sqrt{1-v^2}\right)}{v}$$

$$\Rightarrow \quad \frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \frac{dx}{x}$$

Taking integral on b/s.

$$\Rightarrow \int \frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1+v^2}\right)} = \int \frac{dx}{x}$$

$$\Rightarrow \quad \text{put } 1+\sqrt{1-v^2} = t$$

$$\Rightarrow \frac{1}{2}(1-2)^{-1/2}(-2v)\,dv = dt.$$

$$\Rightarrow \quad \frac{v\,dv}{\sqrt{1-v^2}} = -dt.$$

$$\Rightarrow \quad \int -\frac{dt}{t} = \int \frac{dx}{x}$$

$\Rightarrow \quad -\ln t = \ln x + \ln c$

$\Rightarrow \quad -\ln\left(1+\sqrt{1-v^2}\right) = -\ln cx$

~~$\ln\left|1+\sqrt{1-v^2}\right| = \frac{1}{cx}$~~

$\Rightarrow \quad \cancel{\ln}\left(1+\sqrt{1-v^2}\right) = \cancel{\ln}(cx)^{-1}$

$\Rightarrow \quad 1+\sqrt{1-v^2} = 1/cx$

$\Rightarrow \quad 1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}$

$\Rightarrow \quad 1+\frac{\sqrt{x^2-y^2}}{x^2} = \frac{1}{cx}$

$\Rightarrow \quad x+\sqrt{x^2-y^2} = \frac{1}{c}$

$\Rightarrow \quad x+\sqrt{x^2-y^2} = c_1 \quad \because \frac{1}{c} = c_1$

Which is a required solution.

Q NO # 03:- $(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x.$

Sol:- $(D^4 + D^2)\,y = 3x^2 + 4\sin x - 2\cos x.$

$\Rightarrow f(D)\,y = f(x)$

As it is non-homogenous linear equation. So solution will be.

$$y = y_c + y_p \quad \text{—} \quad (i)$$

Complementry solution y_c:

$D^4 + D^2 = 0 \Rightarrow D^2(D^2+1) = 0$

Either $D^2 = 0 \Rightarrow D = 0.$

$D^2 + 1 = 0 \Rightarrow D^2 = -1.$

$D = \sqrt{-1} \Rightarrow \boxed{D = i}$ or $\boxed{D = 0 + i}$

Roots are real & complex.

$y_c = c_1 e^{0x} + e^{0x}(c_2 \cos x + c_3 \sin x)$

$y_c = c_1 + c_2 \cos x + c_2 \sin x.$

$y_p = \frac{1}{f(D)} F(x).$

P.T.O

$$y_p = \frac{1}{D^4+D^2}(3x^2+4\sin x-2\cos x)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}.$$

$$f(D) = D^4+D^2$$

at $D=0 \Rightarrow f(0)=0$

so $f'(D) = 4D^3+2D$

Now also for $D=0 \Rightarrow f'(D)=0$

again differentiating.

$$f''(D) = 12D+2$$

so D is $= 0$.

$$f''(0) = 12(0)+2 = 2$$

so replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_p = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2}\cdot 4\sin x - \frac{x^2}{12D+2}\,2\cos x.$$

P.T.O

So putting $D = 0$ in all.

$$y_p = \frac{x^2 \cdot 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$y_p = \frac{3x^2}{2} + \frac{4x^2\sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x.$$

So putting in equation (i)

$$y = C_1 + C_2\cos x + C_3\cos x + 3/_2\, x^4 + 2x^2\sin x - x^2\cos x.$$

$$y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + 3/_2\, x^4.$$

Ans

The End of paper.