$$x^3y''' + 2x^2y' + 2y = 10x + \frac{10}{x}.$$

Sol:.
There we need some correct because $2x^2y'$ are not same w should try to solve for $2x^2y''$

$$x^3y''' + 2x^2y'' + 2y = 10x + 10/x$$

$$x^3\frac{d^3y}{dx} + 2x^2\frac{d^2}{dx}y + 2y = 10x + 10x^{-1} \rightarrow$$

$$(x^3D^3 + 2x^2D + 2)y = 10x + 10x^{-1} \rightarrow 1$$

let $x = e^t \Rightarrow t = \ln x$.

$xD = D$

$x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$

$x^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta^2 + 2\Delta$.

subsituting in eq 1

$$(\Delta^3 - 3\Delta^2 + 2\Delta + 2\Delta^2 - 2\Delta)y = 10x + 10^{-}$$

$$(\Delta^3 - \Delta^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{e^t}$$

②

using synthatic devision.

$$\begin{array}{r|rrrr} & 1 & -1 & 0 & 2 \\ -1 & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & \boxed{0} \end{array}$$

Now using quadratic equation

$a = 1, \; b = -2, \; c = 2$

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2a}$$

$$\Delta = \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-4}}{2} \Rightarrow \frac{2 \pm \sqrt{(-2)^2}}{2}$$

$$\Delta = \frac{2 \pm 2i}{2} = \frac{\cancel{2}(1 + i)}{\cancel{2}}$$

$$\frac{\cancel{2}(1 - i)}{\cancel{2}}$$

$$y_c = e^{-x}(c_1 \cos t + 2 \sin t)$$

Now for Particular solution

$$y_P = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot 10e^{-t}$$

$$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-1}}{(1)^3 - (1) + 2}$$

$$= 5e^t + 5e^t$$

General sol:

$$y = y_c + y_p$$

$$y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5e^t + 5$$

Put $e^t = x$ & $t = \ln x$.

$$y = e^{-x}(c_1 \ln x + c_2 \sin x) + 5e^x + 5e^-$$

Ans

Q2: $x^3y''' + 4x^2y'' - 5xy' - 15y = x^4$

$$x^3\frac{d^3y}{dx^2} + 4x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} - 15y = x^4$$

$$\left(x^3\frac{d^3}{dx^2} + 4x^2\frac{d^2}{dx^2} - 5x\frac{d}{dx} - 15\right)y = x^4$$

$$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4 \rightarrow (1)$$

let $x = e^t \Rightarrow t = \ln x$

$xD = \Delta$

$x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$

$\Delta^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta^2 + 2\Delta$

substituting into eq (1)

$$\Delta^3 - 3\Delta^2 + 2\Delta + 4\Delta^2 - 4\Delta - 5\Delta - 15 = x^4$$

$$\Rightarrow \Delta^3 + \Delta^2 - 7\Delta - 15 = e^{4t}$$

Complementry set: y_c:

solve by synthatic eq.

	1	1	-7	-15
3		3	12	15
	1	4	+5	6

$\Delta^2 + 4\Delta + 5 = 0$

$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$

$= \frac{-4 \pm \sqrt{16 - 20}}{2} \Rightarrow \frac{-4 \pm 2i}{2}$

$\Delta = \frac{\cancel{2}(-2 \pm i)}{\cancel{2}} \Rightarrow$

$y_c = e^{4t}(c_1 \cos t + c_2 \sin t)$

Particular integral :-

$yp = \frac{1}{F(\Delta)} e^{4t}$

$= \frac{1}{\Delta^3 + \Delta^2 - 7\Delta - 15} e^{4t}$

$= \frac{1}{4^3 + 4^2 - 7(4) - 15} e^{4t}$

$yp = \frac{1}{37} e^{4t}$

The general sol is :

$y = y_c + yp$

$y = e^{4t}(c_1 \cos t + c_2 \cos t) + \frac{1}{37} e^{4t}$

Again Put $t = \ln n$ & $n = \ln n$.

$y = e^{n'}(c_1 \cos \ln n + c_2 \sin \log n) + \frac{1}{} e^{4n}$

Q3:-

$x^2 y'' + 2xy' - 6y = 10x^2$

Sol

$y(1) = 1$ & $y'(1) = -6$.

$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 6y = 10x^2$

$\left(x^2 \frac{d^2}{dx^2} + 2x \frac{d}{dx} - 6\right) y = 10x^2$

Put $xD = \Delta$

$x^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$.

$x = e^t$ & $\log x = t$

$(\Delta^2 - \Delta + 2\Delta - 6) y = 10 e^{2t}$

$(\Delta^2 + \Delta - 6) y = 10 e^{2t}$

The characteristic equation

$\Delta^2 + \Delta - 6 = 0$

$\Delta^2 + 3\Delta - 2\Delta - 6 = 0$

$\Delta(\Delta + 3) - 2(\Delta + 3) = 0$

$(\Delta + 3)(\Delta - 2)$

$\Delta = -3$,

$\Delta = 2$.

Since Root are ~~general~~ Real & distinct

For $y_c = ?$

$$y_c = c_1 e^{-3t} + c_2 e^{2t}.$$

For $y_p = ?$

$$y_p = \frac{1}{D^2 - D - 6} 10 e^{2t}$$

$$= \frac{10 e^{2t}}{D^2 - D - 6}.$$

$$= 10 \cdot \frac{1}{0} e^{2t} \rightarrow \text{fail.}$$

now.

$$10 \frac{1}{\frac{d}{dD}(D^2 + D - 6)} e^{2t}.$$

$$y_p = 2te^{2t}.$$

General solution.

$$y = y_c + y_p.$$

$$= c_1 e^{-3t} + c_2 e^{2t} + 2te^{2t}$$

$$y = c_1 x^{-3} + c_2 x^2 + 2(\log x) x^2 \rightarrow B.$$

Put $y(1) = 1$ i.e. $x = 1, y = 1$ in eq B

$$1 = c_1 + c_2 \rightarrow C$$

now differentiate eq B w.r.t x.

$y' = -3c_1 x^{-4} + 2c_2 x + \frac{2}{x}(x^2 + 4x\log x.$

$-6 = -3c_1 + 2c_2 + 2 + 0.$

$-8 = -3c_1 + 2c_2. \rightarrow$

xing eq (1) with (2) & -ing from D.

$2c_1 + 2c_2 = 2.$

$\mp 8c_1 \pm 2c_2 = \mp 8$

$5c_1 = 10.$

$\boxed{c_1 = 2}$

$-8 = -3(2) + 2c_2.$

$\boxed{c_2 = -1}$

now Put the value of c_1 & c_2 in eq (B)

$y = 2x^{-3} - x^2 + 2(\ln x \cdot x(x^2)$

$y = \frac{2}{x^3} - x^2 + 2x^2\log x$ Ans

(4)

Q: $x^2y'' + 7xy' + 5y = x^5$

Sol:

$$x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5$$

$$\left(x^2\frac{d^2}{dx^2} + 7x\frac{d}{dx} + 5\right)y = x^5$$

$$(x^2D^2 + 7xD + 5)y = x^5 \rightarrow (1)$$

let $x = e^t \Rightarrow t = \ln x$

$x\Delta = \Delta$

$x^2\Delta^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$

Subsitoting into eq (1)

$\Delta^2 - \Delta + 7\Delta + 5 = 0$

$\Rightarrow (\Delta^2 + 6\Delta + 5)y = e^{5t}$

Complementry sol: y_c:

$\Delta^2 + \Delta + 5\Delta + 5 = 0$

$\Delta(\Delta+1) + 5(\Delta+1)$

$(\Delta+1)(\Delta+5)$

$\Delta = -1, \Delta = -5$

$$y_c = C_1e^{\Delta_1 x} + C_2e^{\Delta_2 x}$$

$e^{-x} \quad + C_2 e^{-5x}$

Particular integral :

$$y_p = \frac{1}{D^2+6D+5} e^{5t}$$

$$y_p = \frac{1}{25+30+5} e^{5t}$$

$$= \frac{1}{60} e^{5t}$$

General sol

$$y = c_1 e^{-t} + c_2 e^{-5t} + \frac{1}{60} e^{5t}$$

Replace $x = e^t \Rightarrow t = \ln x$.

$$y = c_1 e^{\ln x} + c_2 e^{-5\ln x} + \frac{1}{60} e^{5\ln x} \rightarrow (B)$$

$x = 0$ Put in this equ
no in eq (B) $e^0 = 1$

Put $y(0) = 2$ i.e $y = 2$ & $x = 2$

$$2 = c_1(2)^{-5} + c_2(2)^{-1} + \frac{1}{60}(2)$$

$$2 = -32c_1 - 2c_2 + \frac{1}{60}(38)$$

$$2 = -32c_1 - 2c_2 + 8/15$$

$$\frac{22}{15} = -32c_1 - 2c_2 \rightarrow (C)$$

now differentiate eq(B) w.r.t (x)

$y' = -5c_1 x^{-6} - c_2 x^{-2} + \frac{1}{12} x^4 \rightarrow$

Put $y'(2) = 2$ i.e $y' = 2$ & $x = 2$ in above equ

$2 = -5c_1 c_2^{-6} - c_2(2)^{-2} + \frac{1}{12}(2)^4$.

$2 = -5c_1(-64) - c_2(4) + \frac{1}{12}(16)$

$2 = 320c_1 + 4c_2 + 4/3$

$\frac{2}{3} = 320c + 4c_2 \rightarrow$ (D)

xing eq (C) with 2 and then in eq(C)

from (D))

$-\frac{44}{15} = 64c_1 + 41c_2$.

$-\frac{44}{15} = 64c + 4c_1$.

$\frac{34}{15} = -256c_1$

$c_1 = \frac{34}{15} \times 256$.

$\boxed{c_1 = 580}$

Put the value of c_1 in eq(C)

$\frac{22}{15} = -32(580) - 2c_2$.

$\frac{22}{15} + 18560 = -2c_2$.

$$\boxed{c_2 = -9280}$$

Now Put the value of c_1 & c_2.

in eq (B).

$$y = 580x^{-5} - 9280x^{-1} + \frac{1}{60}x^5$$ Ans

Q 5: $(x+1)^2 y_2 - 3(x+1)y' + 4y = x^2$

let: $= (x+1) = z$,

$= (x+1)^2 = z^2$

$x = e^t \Rightarrow \ln x = t$

$\Rightarrow z^2 y_2 - 3zy + 4y = x^2$

$zy = \Delta$

$zy = \Delta(\Delta - 1) = \Delta^2 - \Delta$

~~$\Delta^2 - \Delta + 3\Delta$~~

$(\Delta^2 - \Delta - 3\Delta + 4)y = x^2$

$(\Delta^2 - 4\Delta + 4)y = e^{2t}$

complementry set y_c:

$\Delta^2 - 2\Delta - 2\Delta + 4 = 0$

$\Delta(\Delta - 2) - 2(\Delta - 2)$

$(\Delta - 2)(\Delta - 2) = 0$

$\Delta = 2, 2$

$y_c = (C_1 + C_2 x + \cdots)\, e^{mx}$

$= (C_1 + C_2 x + \cdots)\, e^{2x}$

Particular Integral:

$$y_P = \frac{1}{F(D)} e^{2t}$$

$$= \frac{1}{D^2 - 4D + 4} e^{2t}$$

$$= \frac{1}{(2)^2 - 4(2) + 4} e^{2t}$$

$$\frac{1}{0} e^{2t}$$

F(D) should not equal to zer

$\Rightarrow D^2 - 4D + 4.$

$2D - 4 \rightarrow (2)$

If we put 2 in eq (2)

= again give zero

take 2nd time derivative

$= \frac{d}{dD}(2D - 4)$

$= 2 \Rightarrow y_P = \frac{1}{2} e^{2t}$

$$y_p = \frac{1}{2} e^{2t}$$

The general sol is

$$y = y_c + y_p$$

$$y = (c_1 + c_2 x + \cdots) e^{2x} + \frac{1}{2} e^{2t}$$

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Assignment #1

ubmitted to

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