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Section = B

Subject = Differential Equation

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Question #01

(i) The order of Matrix A is $m \times p$ and the order of Matrix B is $p \times n$ Then the order of Matrix AB is?

Ans The order of Matrix $AB = m \times n$

(ii) The number of non-zero rows in Echelon form?

Ans/Sol non zero rows in Echelon form is Rank of Matrix.

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Ans $|B| = 0$

$|B| = 1 \times a - 4 \times 2 = 0$

$\Rightarrow a - 8 = 0$

$a = 8$

If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= (2i)(-i) - (i)(i)$$

$$= 2i^2 - i^2$$

As $i^2 = -1$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$|A| = 3$$

(v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is $= ?$ ~~Scalar Matrix~~

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is Scalar Matrix

(vi) Solution of $\frac{dy}{dx} + 2xy = y$?

Solution

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

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$$\frac{dy}{dx} = y(1-2x)$$

$$\int \frac{1}{y}\, dy = \int (1-2x)\, dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + C$$

$$y = e^{x-x^2} + C$$

vii) The order and Degree of Differetial Equation $\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is

order $= 1$

Degree $= 6$

(viii) The order and Degree of $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{dy}{dx^2}\right)$ is =?

Order $= 2$

Degree = undefine

Pages | The Differential Equation $2\frac{dy}{dx}+x^2y=2x+3$, $y(0)=5$

$$2\frac{dy}{dx}+x^2y=2x+3$$

$$\int 2\,dy=\int((2x+3)-x^2y)\,dx$$

$$2y=\frac{2x^2}{2}+3x-y\frac{x^3}{3}+C$$

$$2y=\frac{2x^2}{2}+3x-y\frac{x^3}{3}+c$$

$$y=\frac{x^2}{2}+\frac{3x}{2}-\frac{x^3y}{6}+c$$

Put $x=0$, $y=5$

$$5=0+0-0+c$$

$$5=c$$ [So it is Homogeneous Equation]

So

$$y=\frac{x^2}{2}+\frac{3x}{2}-\frac{x^3y}{6}+5$$

X $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

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Solution

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\mathcal{R} \begin{vmatrix} 1 & a & a^2 \\ 1-1 & b-a & b^2-a^2 \\ 1-1 & c-a & c^2-a^2 \end{vmatrix} \quad \begin{matrix} \cancel{R_2-R_1} \\ R_2-R_1 \\ R_3-R_1 \end{matrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

Expand By C_1

$$= 1 \begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix} - 0 + 0$$

$$= \{(b-a)(c^2-a^2)\} - \{(b^2-a^2)(c-a)\}$$

$$= (b-a)(c-a)(c-\cancel{a}-b+\cancel{a})$$

$$= (b-a)(c-a)(c-b)$$

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Question # 2

i) Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

As the product of factors which are liner in abc?

Sol

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1,

$$= a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

Taking common (abc)

$$= abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$= abc(bc(c-d) - ac(c-a) + ab^2(b-a)$$

Answer

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Part(ii)

ii Find the Eign value

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic eq $|A - \lambda I| = 0$ (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinant

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 3-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$2-\lambda\begin{vmatrix}3-\lambda & -1 & -1\\ -1 & 3-\lambda & -1\\ -1 & -1 & 2-\lambda\end{vmatrix}-(-1)\begin{vmatrix}-1 & -1 & -1\\ -1 & 3-\lambda & -1\\ 0 & -1 & 2-\lambda\end{vmatrix}-1$$

$$\begin{vmatrix}-1 & 3-\lambda & -1\\ -1 & -1 & -1\\ 0 & -1 & 2-\lambda\end{vmatrix}=0 \quad -(1)$$

Again,

$$\begin{vmatrix}3-\lambda & -1 & -1\\ -1 & 3-\lambda & -1\\ -1 & -1 & 2-\lambda\end{vmatrix} \quad \text{Expand by } R_1$$

$$3-\lambda\begin{vmatrix}3-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix}-(-1)\begin{vmatrix}-1 & -1\\ -1 & 2-\lambda\end{vmatrix}-1\begin{vmatrix}-1 & 3-\lambda\\ -1 & -1\end{vmatrix}$$

$$=(3-\lambda)\left[(3-\lambda)(2-\lambda)-(-1)(1)\right]+1\left[(-1)(2-\lambda)-(-1)\right.$$

$$\left.(2-\lambda)-(-1)(-1)\right]-1\left[(-1)(-1)-(-1)(3-\lambda)\right]$$

$$=(3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)-1(1+3-\lambda)$$

$$=(3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

$$=3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$=-\lambda^3+8\lambda^2-18\lambda+8 \quad -(a)$$

Now

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} \text{ Expand by } C_1$$

$$-1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\cancel{\lambda}+\lambda^2-1)+(-2+\lambda-1)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda^2+6\lambda-8 \quad — (b)$$

$$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} \text{ Expand by } C_1$$

$$\Rightarrow -1\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$= -\left[-(2+\lambda-1) + (6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda^2+6\lambda-8 \quad — (c)$$

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Now putting ⓐ, ⓑ, ⓒ in eq ①

$\Rightarrow (2-\lambda)(-\lambda+8\lambda^2-18\lambda+8)-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8=0$

$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda$

$-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8=0$

$=\lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+18\lambda^2-\lambda^2-\lambda^2-36\lambda$

$-8\lambda+6\lambda+6\lambda+16\lambda+\cancel{16}-\cancel{8}-\cancel{8}=0$

$\lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

Now

Solve upper equation By synthetic Equation.

	1	-10	32	-32
2			16	32
	1	-8	16	0

So we get

$(\lambda-2)(\lambda^3-8\lambda+16\lambda)=0$

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

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$\lambda = 0$ | $\lambda - 2 = 0$ | $\lambda^2 - 8\lambda + 16 = 0$

$\lambda = 2$ | $\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$

$(\lambda - 4) = 0$ | $(\lambda - 4) = 0$

$\lambda = 4$ | $\lambda = 4$

So

$\lambda_1 = 0$

$\lambda_2 = 2$

$\lambda_3 = 4$

$\lambda = 4$ Ans

Question #3

The rate of change in the form of differential equation is given by $(x^2+3y^2)\,dx-2xy\,dy=0$
Find the general Solution at $x=2$, $y=6$

Solution

$$(x^2+3y^2)dx-2xy\,dy=0$$

$$x=2 \text{ and } y=6$$

Now

$$(x^2+3y^2)\,dx=2xy\,dy$$

Dividing b.H.S by $2xy\,dx$

$$\frac{dy}{dx}=\frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx}=\frac{\cancel{x^2}}{2\cancel{x}y}+\frac{3\cancel{y^2}}{2x\cancel{y}}$$

$$\frac{dy}{dx}=\frac{1}{2}\left[\frac{x}{y}+\frac{3y}{x}\right]-①$$

Let $y=vx$

Diff

$$dy=v\,dx+x\,dv$$

Dividing b dx

$$\frac{dy}{dx}=\frac{v+x\,dv}{dx}\quad — Ⓐ$$

Put (A) in (i)

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{v\cancel{x}} + \frac{3v\cancel{x}}{\cancel{x}}\right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying B.H.S By 2

$$2v + \frac{2x\,dv}{dx} = \frac{1}{v} + 3v$$

$$\frac{2x\,dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$\frac{2x\,dv}{dx} = \frac{1}{v} + v$$

$$\frac{2x\,dv}{dx} = \frac{1+v^2}{v}$$

Multiplying B.H.S By $\frac{dx}{dv}$

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

Multi B.H.S By $\frac{v}{x(1+v^2)}$

we get

$$\frac{2v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Now Taking "$\int$" on B.S

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Taking "e" on B·S

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

Now put $v = y/x$

$$1 + (y/x)^2 = xc$$

$$\frac{x^2+y}{x^2} = xc$$

Now $x^2 + y^2 = x^3 c$ — (i)

Put $x = 2$ $y = 6$

$$(2)^2 + (6)^2 = (2)^2 c$$

$$4 + 36 = 8c$$

$$\underline{40} = 8c$$

$$\frac{40}{8} = c$$

$$c = 5$$

Now Put $c = 5$ in (ii)

$x^2 + y^2 = 5x^3$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x-1)$

Now

Taking "$\sqrt{}$" on Both Sides

$\sqrt{y^2} = +\sqrt{x^2} \times \sqrt{5x-1}$

$\cancel{\sqrt{y^2}} = -\cancel{\sqrt{x^2}} \times \sqrt{5x-1}$

$y = \pm x \sqrt{5x-1}$ Ans