Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	Sir pir mehar ali	**Total Marks:**	50

Student Details

Name:	**RAFI UD DIN**	**Student ID:**	**12401**

Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $( ) - 4\ (-1) + 4\ (-2) = ( ) - \ (-1)$ To the input $( ) = (-1)\ ( )$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks 7**	**CLO 2**
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $( ) - 0.7\ (-1) + 0.1\ (-2) = 2\ ( ) - \ (-2)$	**Marks 7**	**CLO 2**
Q2.	(a)	Determine the causal signal x(n) having the z-transform $( ) = \frac{1}{(1-2\ ^{-1})(1-\ ^{-1})^2}$ (Hint: Take inverse z-transform using partial fraction method)	**Marks 6**	**CLO 2**
	(b)	Evaluate the inverse z- transform using the complex inversion integral $( ) = \frac{1}{1-\ ^{-1}}$ $\|\|>\|\|$	**Marks 6**	**CLO 2**
Q.3	(a)	A two- pole low pass filter has the system response $( ) = \frac{}{(1-\ ^{-1})^2}$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition H(0) = 1 and $\|(\frac{}{4})\|^2 = \frac{1}{2}$.	**Marks 6**	**CLO 3**

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	**Marks** **6** **CLO** **3**
Q 4	(a)	A finite duration sequence of Length L is given as 1, 0≤≤−1 ( )={0, h Determine the N- point DFT of this sequence for $N \geq L$	**Marks** **6** **CLO** **2**
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $_1(\ ) = \{^2,1,2,1\}$ ↑ $_2(\ ) = \{^1,2,3,4\}$ ↑	**Marks** **6** **CLO** **2**

NAME RAFI UD DIN
ID 12401

Q No 1
(a)

Sol: The characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2$$

$$y_h(n) = c_1 2^n + c_2 n 2^n$$

The particular solution is

$$y_p(n) = k(-1)^n u(n)$$

$$k(-1)^n u(n) - 4k(-1)^{n-1} y(n-1) + 4k(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$ $k(1+4+4) \Rightarrow 2 \Rightarrow k$

$= k = 2/9$ total solution is

$$y(n) = \left[c_1 2^n + c_2 n^{2n} + \frac{2}{9}(-1)^n\right] u(n)$$

For initial condation we have $y(0)$

$y(0) = 1,\ y = 2$ then

$$c_1 + 2/9 = 1$$

$$\Rightarrow c_1 = 7/9$$

$$2c_1 + 2c_2 - 2/9 = 2$$

$\Rightarrow C_2 = 1/3$

Q No 1
part (b)

Sol :- The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = \frac{1}{2}, \frac{1}{5}$$

$$y_h(n) = C_1 \frac{1}{2}^n + C_2 \frac{1}{5}^n$$

with $x(n) = \delta(n)$ we have

$$y(0) = 2$$

$$y(1) = 0.7y(0) = 0 \Rightarrow (1) = 1.4$$

Hence $C_1 + C_2 = 2$ and

$$\frac{1}{2} C_1 + \frac{1}{5} C_2 = 1.4 = 7/5$$

$$\Rightarrow C_1 + 2/5 \, C_2 = 14/5$$

the evaluation yield

$$C_1 = 10/3, \quad C_2 = -4/3$$

$$h(n) = 10/3 \, (1/2)^n - 4/3 \, (1/5)^n \,] \, u(n)$$

The set response is

$$\theta(n) = \sum_{k=0}^{n} h(n-k)$$

$$\frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k} - \frac{4}{3} \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} 5^{k}$$

$$= \frac{10}{3} \left(\frac{1}{2}^{n} \left(2^{n+1} - 1 \right) u(n) - \frac{1}{3} \left(\frac{1}{5} \left(5^{n+1} - 1 \right) \right. \right.$$

$$u(n).$$

QNo2
(a):-
Sol:-

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$X(z) = \frac{1}{(1+2z^{-1})(1-z^{-1})^2}$$

$$X(z) = \frac{1}{4(1+2z^{-1})} + \frac{3}{4}\frac{1}{1-2z^{-1}}$$

$$+ \frac{1}{2}\frac{z^{-1}}{(1-z^{-1})^2}$$

By applying inverse transform.

$$X(n) = 1/8\,(-1)^{x}\,u(x) - 3/8\;u(x) + 1/2\,n\,u(x)$$

$$= \left[1/8\,(-1)^{n} + 3/8 + n/2\right] u(x)$$

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QNo2

(b):-

Sol:-

we have

$$x(n) = \frac{1}{2\pi j} \oint_C \frac{z^{n-1}}{1-az^{-1}} dz$$

$$= \frac{1}{2\pi j} \oint_C \frac{z^n dz}{z-a}$$

Where C is the circule at radius greater than |a| we shall evaluate this integral using with $f(z) = z^n$ we distenguish two cases.

1) If $n \geq 0$, $f(z)$ has only zeros and hense no poles inside C the only poles inside C is $z=a$ Hence

$$x(n) = f(z_0) = a^n \quad n \geq 0$$

2) if $n<0$ $f(z) = z^n$ has an n^{th} order pole at $z=0$ which is also inside C thus There are contrubutions from both poles for $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j} \oint_C \frac{1}{z(z-a)} dz$$

$$= \left.\frac{1}{z-a}\right|_{z=0} + \left.\frac{1}{z}\right|_{z=a} = 0$$

if $n=-2$ we have

$$x(-2) = \frac{1}{2\pi j}\oint_C \frac{1}{z^2(z-a)}\,dz$$

$$= \frac{d}{dz}\left(\frac{1}{z-a}\right)\Bigg|_{z=0} + \frac{1}{z^2}\Bigg|_{z=a} = 0$$

By countinuing the same way we can show that $x(n)=0$ for $n<0$ Thus

$$x(n) = a^n u(n)$$

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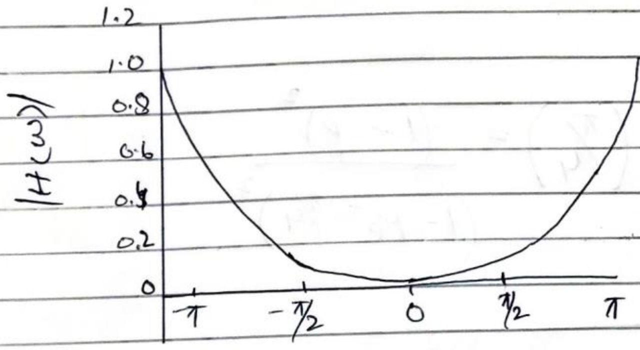
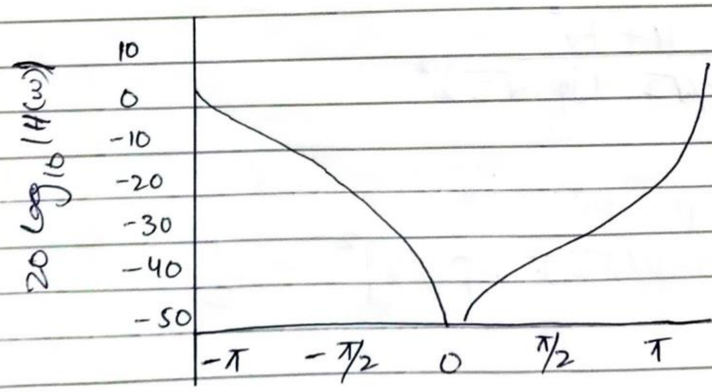
QNo3 (a):-

Sol:- At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence $b_0 = (1-p)^2$





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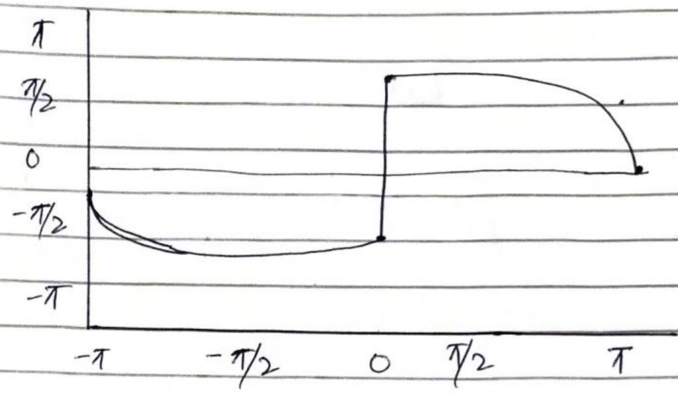


At $\omega = \pi/4$

$$H\left(\pi/4\right) = \frac{(1-p)^2}{\left(1-pe^{-j\pi/4}\right)^2}$$

$$= \frac{(1-p)^2}{1-p(\cos(\pi/4) + jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{1-p/\sqrt{2} + jp/\sqrt{2})^2}$$

$$= \frac{(1-p)^4}{\left[(1-p/\sqrt{2})^2 + p^2/2\right]^2} = \frac{1}{2}$$

QNo3

Part(b):-

Sol: By the filter requirements:-

Poles $P_{1,2} = re^{\pm j\pi/2}$ pass band center

Zeroes $Z_{1,2} = \pm 1$ Stopband Center.

$$\Rightarrow H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)} = G\frac{z^2-1}{z^2+r^2}$$

By the filter requirement

$$H(\pi/2) = G\frac{-2}{-1+r^2} = 1$$

$$= G\frac{1-r^2}{2}$$

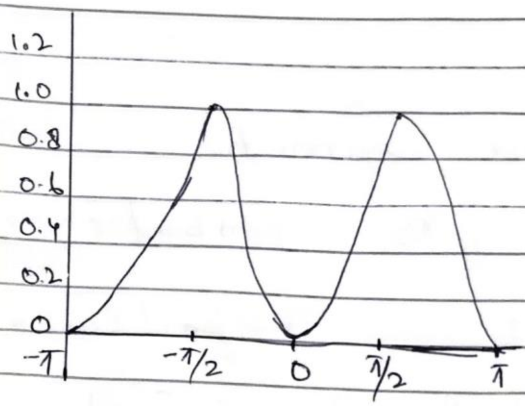
To set r use $H(4\pi/9) = 1/\sqrt{2}$

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2$$

Evaluating gives $r^2 = 0.7$ therefore

$$H(z) = 0.15\frac{1-z^{-2}}{1+0.7z^{-2}}$$


1.2
1.0
0.8
0.6
0.4
0.2
0
-π
-π/2
0
π/2
π


π
π/2
0
-π/2
- π
-π/2
0
π/2
π

QNo 4

(a)

Sol:- The fourier transform of this sequence is

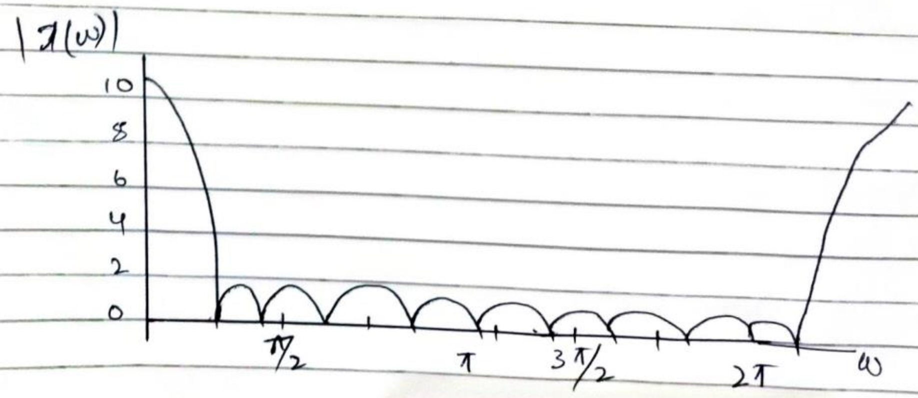
$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

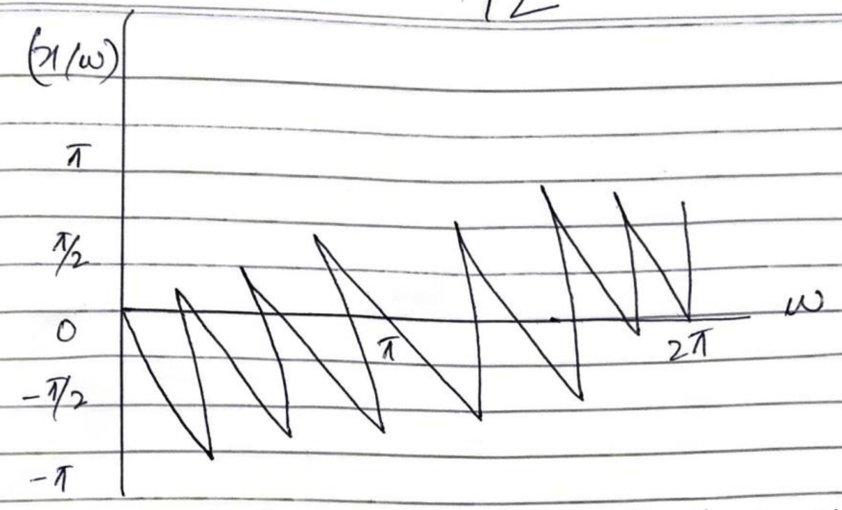
$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega L}} =$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1-e^{-j2\pi K/N}} \qquad K = 0, 1 \dots N-1$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





If N is selected such that $N = L$ then the DFT becomes

$$X(k) = \begin{cases} L & k = 0 \\ 0 & k = 1, 2 \text{ ---- } L-1 \end{cases}$$

thus there is only one nonzero value in DFT.

QNo 4(b)

Sol:- Circular convolution using circular convolution:

$x_1(n) = \{1, 2, 3, 4\}$

and $x_2(n) = \{1, 2, 1, 2\}$

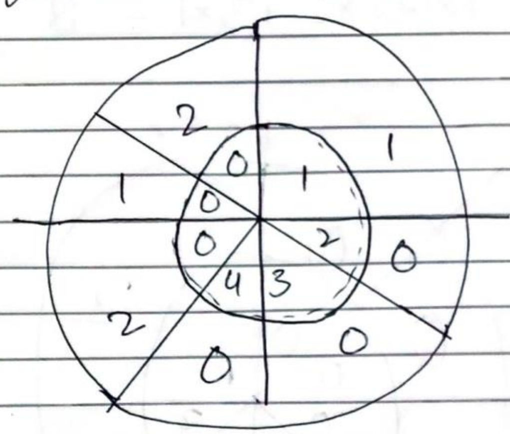
$L = 4$, $M = 4$

Length of $y(n) = L + M - 1 = 4 + 4 - 1 = 7$

$x_1(n) = \{1, 2, 3, 4, 0, 0, 0\}$

and $x_2(n) = \{1, 2, 1, 2, 0, 0, 0\}$

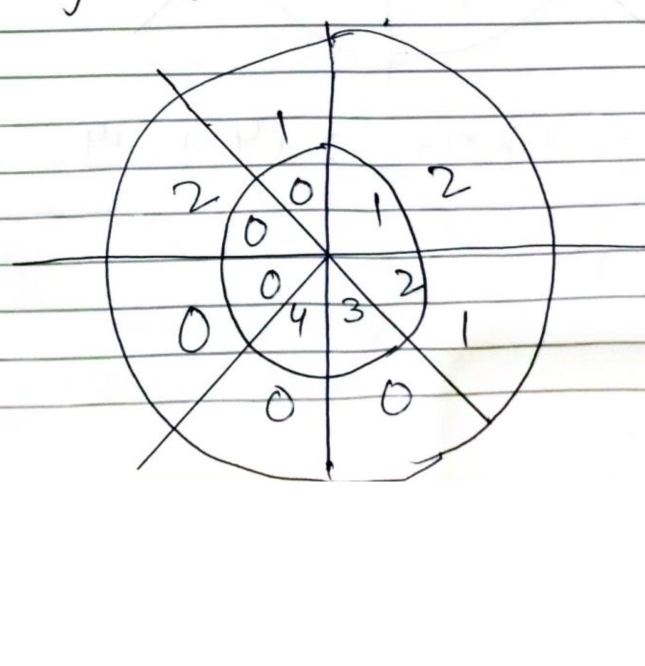
for $y(0)$



$n = 0$

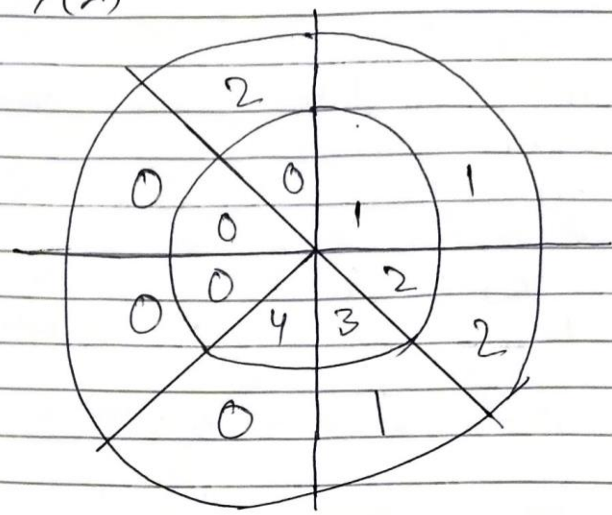
$y(0) = 1 \times 1 = 1$

for $y(1)$



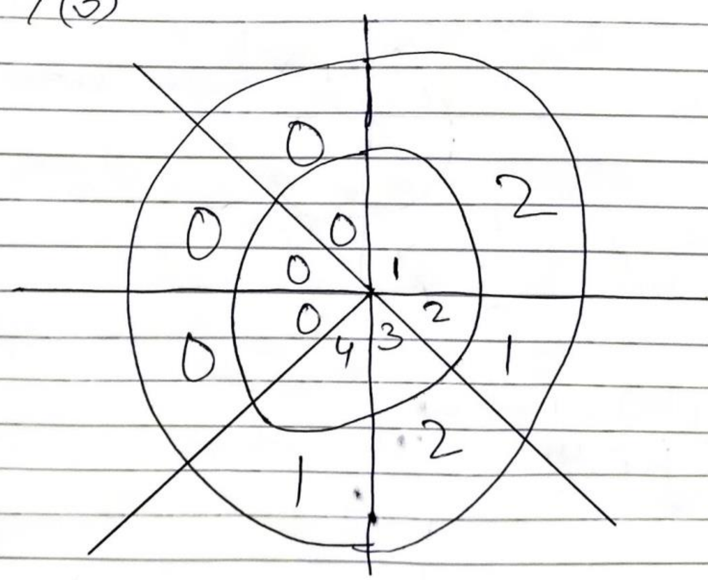
$y(1) = 2 \times 1 + 1 \times 2$
$= 4$

For $Y(2)$



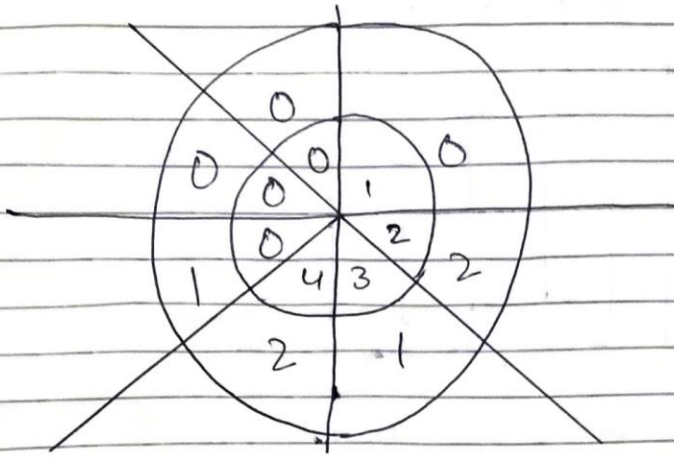
$$Y(2) = 1\times1 + 2\times2 + 3\times1 = 8$$

For $Y(3)$



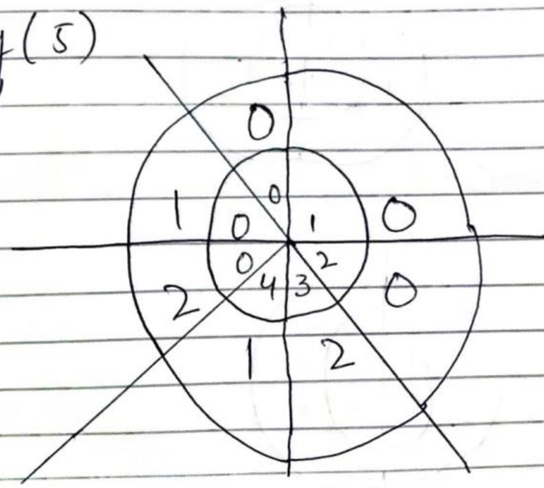
$$Y(3) = 1\times2 + 2\times1 + 3\times2 + 4\times1 = 14$$

For $Y(4)$



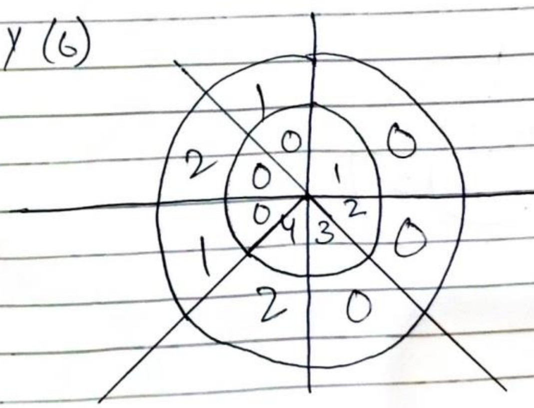
$$Y(4) = 4\times 2 + 3\times 1 + 2\times 2 = 15$$

For $Y(5)$



$$Y(5) = 4\times 1 + 3\times 2 = 10$$

For $Y(6)$



$$Y(6) = 4\times 2 = 8$$

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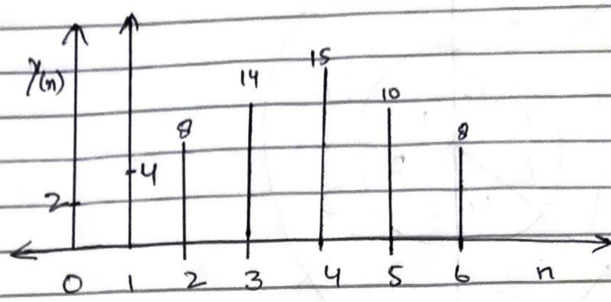
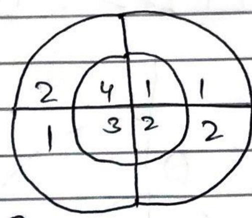
$Y(n) = \{1, 4, 8, 14, 15, 10, 8\}$



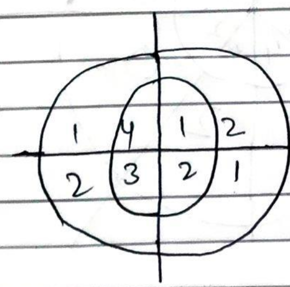
linear circular convolution

For $y(0)$



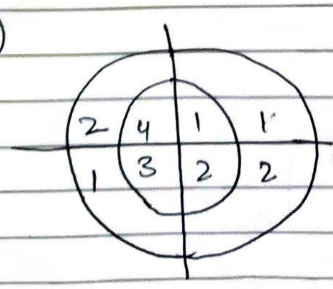
$Y_0(0) = 1+4+3+8 = 16$

For $y(1) =$

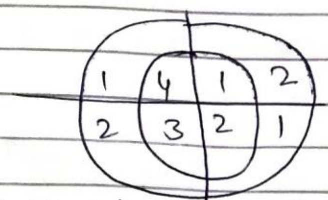


$Y(1) = 2+2+6+4 = 14$

For $Y(2)$

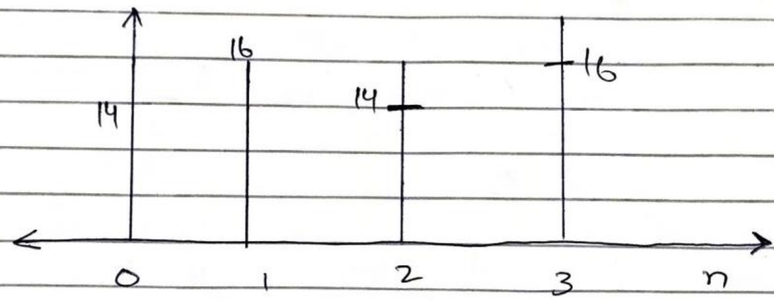


For $y(3)$



$Y(3) = 2+2+6+4 = 14$

$Y_n = \{16, 14, 16, 14\}$



Result: $Y(n) = \{14, 16, 14, 16\}$