Name : Rizwan Jalil
ID : 7879
Section : A
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Program : B.S Civil Engineering
Assignment : Plain and Reinforced concrete Design
Submitted : Engr Fawad Khan

QUESTION: 1

Explain in detail types of stirrups with figures and also explain ACI codes for shear design.

Ans: **STIRRUP:**

Stirrups are closed-loop bars tied at regular intervals in beam reinforcement to hold the bars in position.

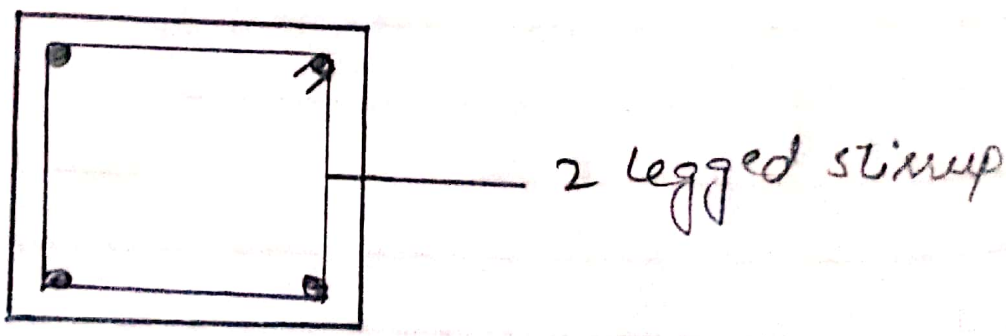
TYPES OF STIRRUPS:

1. **SINGLE LEGGED STIRRUP:**

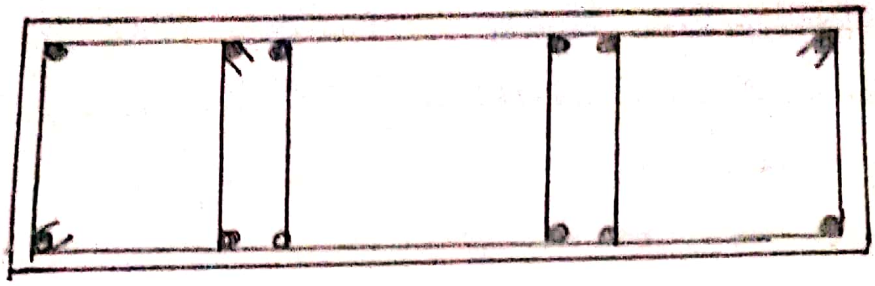
The single-leg stirrups have rarely been used because they are most used when binding only two rods.

2. **TWO LEGGED STIRRUP:**

It is most commonly and widely used stirrup. Minimum 4 bars are required for providing this stirrup.



4. Six Legged Stirrup:



ACI Code For Shear Design Of A Beam:

According to ACI-318, following are the formulas used for the shear design of a Beam.

1. **Critical Section:** Critical section occurs at 45° and is at distance (d)" from the face of support which is equal to effective depth.

2. **Shear Strength Capacity Of Concrete Is:**

$$V_c = 2 \times \sqrt{f'c} \times b_w \times d$$

3. **Minimum Web Reinforcement:**

If $V_u \leq \phi V_c$, then theoritically no web reinforcement is required. However ACI code require provision of atleast a minimum area of web reinforcement equal to,

$\phi = 0.75 \longrightarrow$ for shear design

($\because V_u$ = Total factored shear applied at a given section)

⇒ For Minimum Reinforcement area:

$$A_{v\,min} = \frac{0.75 \times \sqrt{f'c} \times b_w \times s}{f_y} \text{ or } \frac{50 \times b_w \times s}{f_y} \rightarrow \left[\begin{array}{l}\text{higher} \\ \text{value is} \\ \text{selected}\end{array}\right]$$

By interchanging the above formulae, we can obtain the formula for maximum spacing:

$$S_{max} = \frac{Av \times fy}{0.75 \times \sqrt{f'c} \times bw} \text{ or } \frac{Av \times fy}{50 \times bw} \rightarrow \left[\text{lesser value is selected}\right]$$

4. No web-reinforcement is required if $Vu < \frac{1}{2} \phi Vc$

⇒ Between critical section "Vu" and "ϕVc", spacing b/w web reinforcement can be find by,

$$s = \frac{\phi \times Av \times fy \times d}{Vu - \phi Vc}$$

5. If $Vs \leq 4 \times \sqrt{f'c} \times bw \times d$, then max spacing for stirrups will be the smallest of the following:

1. 24"
2. $d/2$
3. $S_{max} = \frac{Av \times fy}{0.75 \times \sqrt{f'c} \times bw}$
4. $S_{max} = \frac{Av \times fy}{50 \times bw}$

∴ (Vs = Shear force carried by web reinforcement)

⇒ If $Vs > 4 \times \sqrt{f'c} \times bw \times d$
↓
Max. spacing will be halved

⇒ If $Vs > 8 \times \sqrt{f'c} \times bw \times d$
↓
Then either increase cross-sectional dimensions or increase $f'c$.

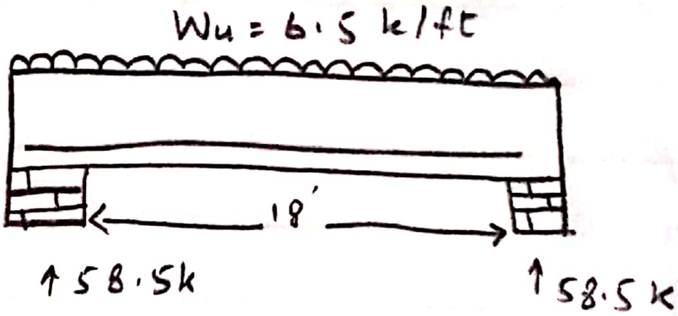
QUESTION: 2

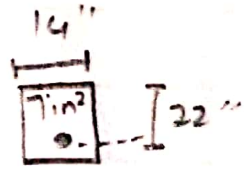
A simply supported rectangular beam 14" wide having an effective depth of 22" to carry a lateral load of 6.5k/ft on a 18' simple span. It is reinforced with 7in² of tensile steel area, if $f'_c = 4$ ksi and $f_y = 60$ ksi, then design the beam for shear.

GIVEN:

Breadth of web of beam (b_w) = 14"
Effective depth (d) = 22"
Given load = 6.5 k/ft
Steel Area = 7in²
f'_c = 4 ksi

SOLUTION:





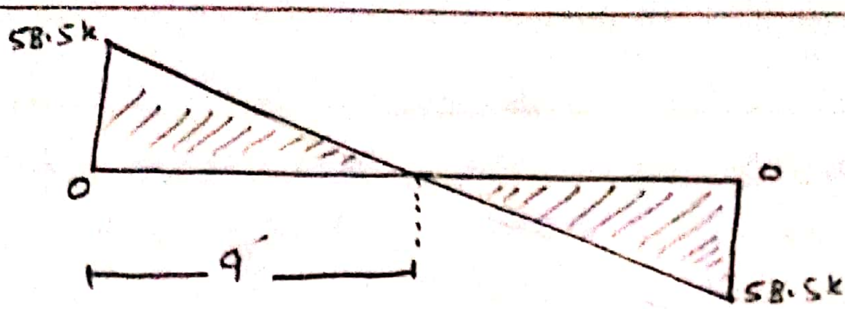
STEP: 1: (Reactions on Supports)

Finding the reactions due to applied load

$$\text{Total Load} = \frac{6.5 \times 18}{2} = 58.5 \text{ kips}$$

STEP: 2: (Shear Force Diagram)

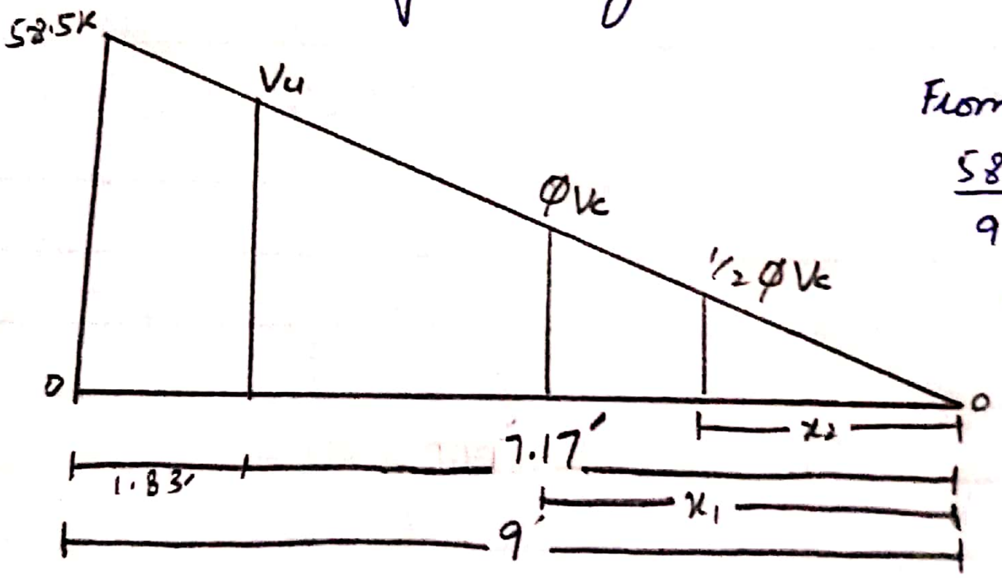
The required shear diagram will be



STEP: 3:

Finding the value of critical shear "Vu" and its location. As, we know that critical shear is located at distance "d" from face of support (d) = 22" = 1.83'

⇒ We will find the values of critical shear at distance "d" by use of similar triangles.



From Similar Triangles

$$\frac{58.5}{9} = \frac{Vu}{8.17}$$

$$Vu = \frac{58.5 \times 7.17}{9}$$

$$Vu = 46.61 \text{kips}$$

STEP: 4:

Finding the value of "ØVc" and "1/2 ØVc" and also the distances from zero shear to right side.

By formula,

$$\phi Vc = \phi \times 2 \times \sqrt{f'c} \times bw \times d$$

$$= 0.75 \times 2 \times \sqrt{4000} \times 14 \times 22 = 29219 \text{ lbs}$$

$$= 29.21 \text{ kips}$$

⇒ Location of ϕV_c by similar triangles,

$$\frac{58.5}{9} = \frac{\phi V_c}{x_1} \Rightarrow \frac{58.5}{9} = \frac{29.21}{x_1}$$

$$\Rightarrow x_1 = 4.49'$$

⇒ Similarly, $\frac{1}{2}\phi V_c = \phi V_c/2 \Rightarrow 29.21/2 = 14.60$ kips

⇒ Location of $\frac{1}{2}\phi V_c$ will be,

$$\frac{58.5}{9} = \frac{14.60}{x^2} \Rightarrow x_2 = 2.24'$$

STEP: 5

⇒ Finding the value of ϕV_s

By formula, $V_u = \phi V_s + \phi V_c$

$$\rightarrow \phi V_s = V_u - \phi V_c$$
$$= 46.61 - 29.21$$
$$\phi V_s = 17.4 \text{ kips}$$

STEP: 6

Check on section adequacy,

By formula,

$$= \phi \times 8 \times \sqrt{f'c} \times bw \times d$$
$$= 0.75 \times 8 \times \sqrt{4000} \times 14 \times 22 = 116877 \text{ lbs}$$
$$= 116.87 \text{ kips}$$

As $\phi \times 8 \times \sqrt{f'c} \times bw \times d > \phi V_s$

So section is Adequate!

STEP: 7

Check on Maximum spacing for stirrups.

By formula,

$$= \phi \times 4 \times \sqrt{f'c} \times bw \times d$$
$$= 0.75 \times 4 \times \sqrt{4000} \times 14 \times 22 \quad = 58438 \text{ lbs}$$
$$= 58.43 \text{ kips}$$

As $\phi \times 4 \times \sqrt{f'c} \times bw \times d > \phi Vs$

So maximum will be selected from the following 4 conditions.

[Here we are using #3 stirrup
dia = (3/8)" = 0.375"
So Area = $\frac{\pi}{4}(0.375)^2 = 0.11 in^2$
For 2-legged stirrup
→ Area × 2 = 0.11 × 2 = 0.22 in^2]

1. $S_{max} = 24"$
2. $d/2 = 22/2 = 11"$
3. $S_{max} = \dfrac{Av \times fy}{0.75 \times \sqrt{f'c} \times bw}$

$$S_{max} = \frac{0.22 \times 6000}{0.75 \times \sqrt{4000} \times 14} = 19.87"$$

4. $S_{max} = \dfrac{Av \times fy}{50 \times bw} = \dfrac{0.22 \times 6000}{50 \times 14} = 18.85"$

From above 4 conditions, least value of spacing for #3, 2 legged stirrup will be selected as $S_{max} = 11"$

STEP : 8

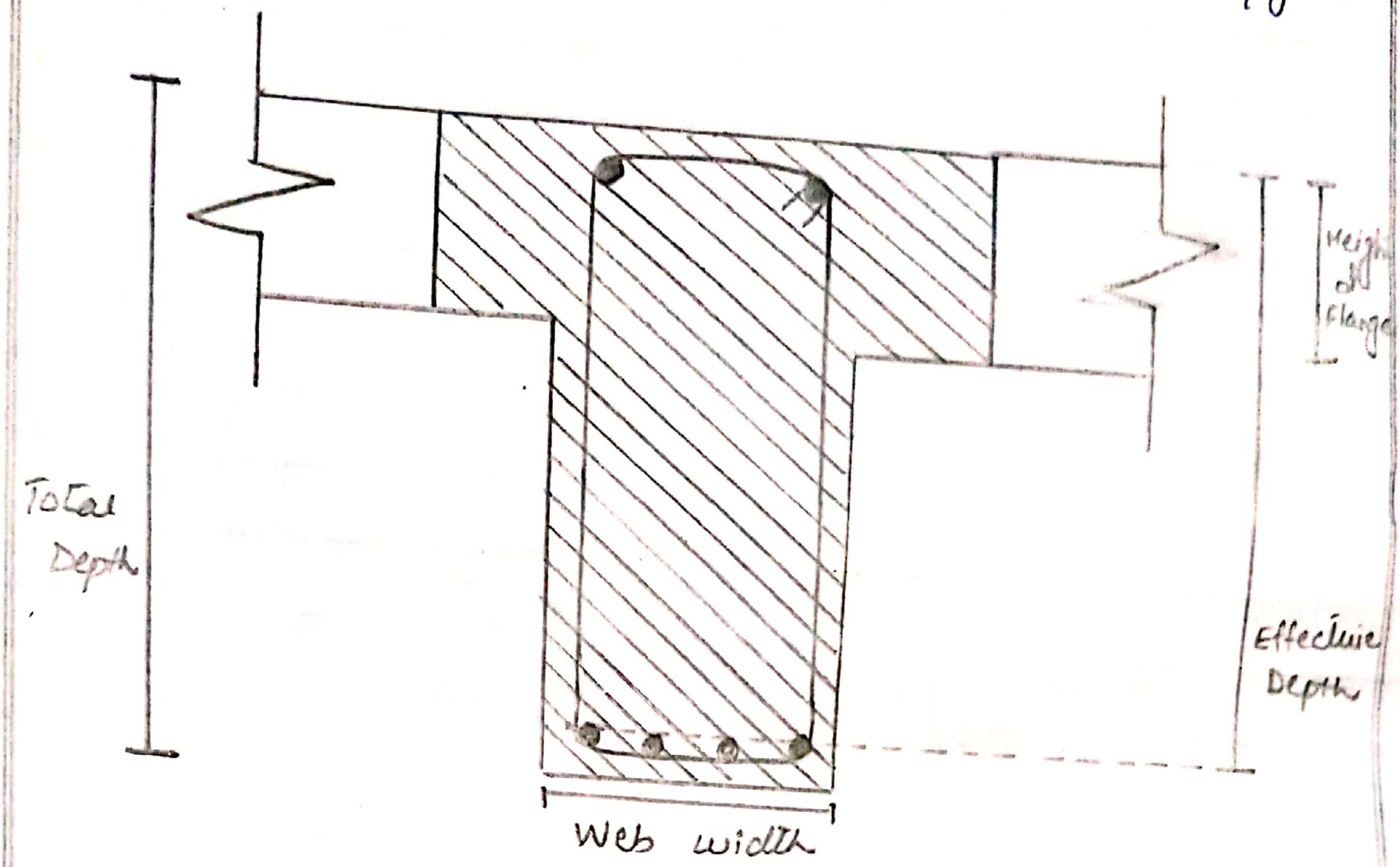
Stirrups spacing from/at critical section will be, By formula,

$$S = \frac{\phi \times Av \times fy \times d}{Vu - \phi Vc} = \frac{0.75 \times 0.22 \times 60 \times 22}{46.61 - 29.21}$$

$$S = 12.5" \approx 12"$$

So 12" c/c

are called T-Beams



=> Because of their T-shape, these beams are called T-Beams.

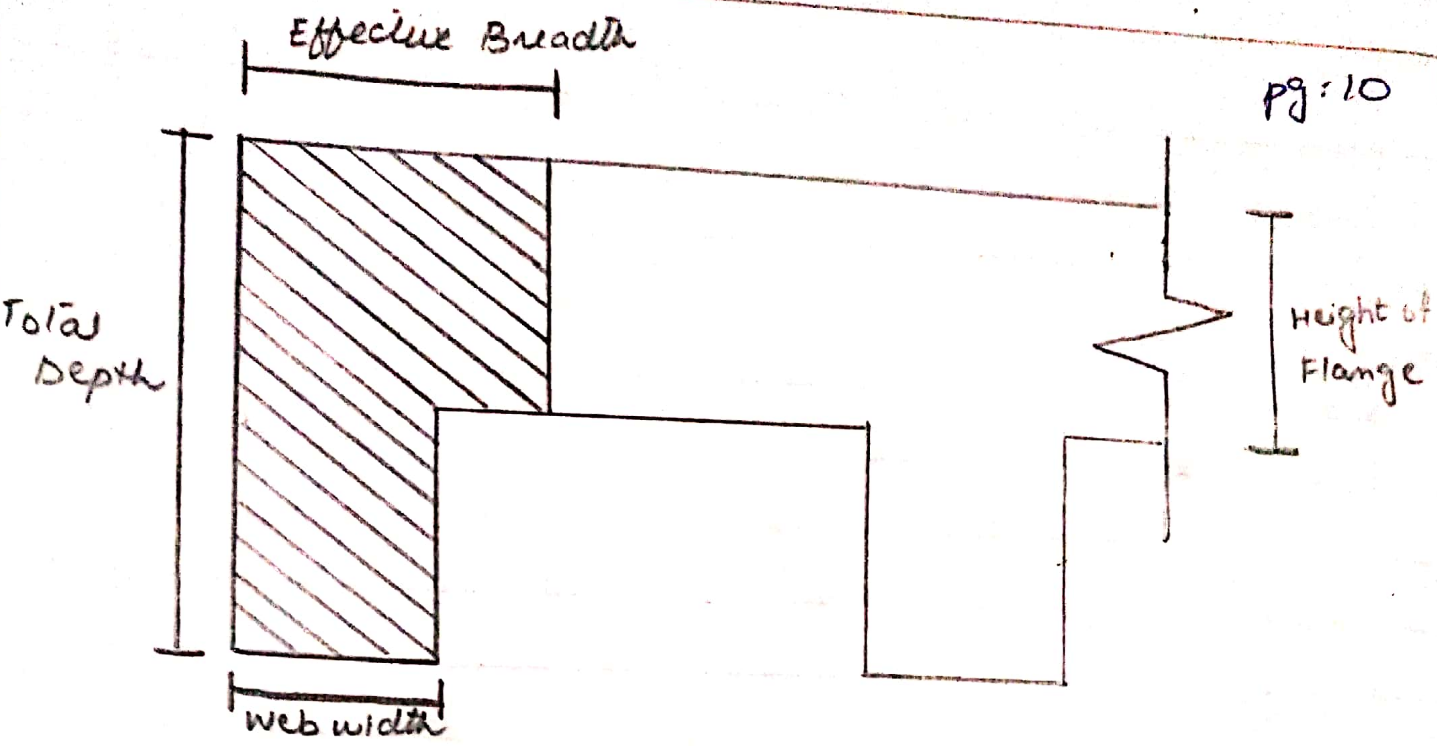
=> It is provided at the center of the slab to resists the loads.

=> The upper most area of the beam attached to the slab is called Flange.

=> The bottom rectangular portion of the beam is called web of the beam.

L-BEAM:

=> L-Beam is L-shaped structure that is in contact with the slab and present at the corner of the floor.



=> L-Beams are also called Edge Beams.
=> It is always provided at the corners of the slab.
=> L-Beams are Typical floor beams because of their reduced overall structural depth, the beams are in prestressed or reinforced concrete.

FLEXURAL ANALYSIS OF T-BEAM:

Flexural Analysis of T-Beam consists of the following steps.

1- For finding the ultimate Factored moment, we use the following Formula,

$$MU = \frac{Wu \times L^2}{8}$$

∵ (Wu = Total Factored load
L = Total span of the beam)

2. Effective width (bc) for T-Beam is calculated as:

1. $16(h_f) + b_w$
2. c/c distance
3. Span/4
4. $\frac{CTS}{2} + b_w$

$\because$ (h_f = height of flange
CTS = clear transverse span)

⇒ We have to select the least value from above formulas

⇒ If c/c distance is given, then there is no need of "$\frac{CTS}{2} + b_w$"

3. Checking whether Rectangular or T-Beam Analysis is required:

i. If $a > h_f$ → Special Analysis is required

ii. If $a < h_f$ → Rectangular beam Analysis is required

Where

(a = Depth of compression block
h_f = Height of flange)

4. For Finding Area of Steel, we have to use

$$A_{st} = \frac{M_u}{\phi \times f_y \times (d - a/2)}$$

Where

$$a = \frac{A_{st} \times f_y}{0.85 \times f'_c \times b_w}$$

$\because$ ϕ = Strength reduction
d = Effective depth
a = Compression block depth
b_w = web width of beam

5. For checking the range of Reinforcement Ratio,

$$\rho_{max} = 0.85 \times \beta \times \frac{f'_c}{f_y} \times \left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right)$$

$$S_{max} = \frac{200}{fy}$$

$$S = \frac{Ast}{b \times d}$$

6. Formula for finding No. of bars required is,

$$\text{No. of bars} = \frac{\text{Area of steel}}{\text{Area of single bar}}$$

7. For checking Minimum width for bars accomodation

b_{min} = 2 (clear cover) + 2 (dia of Stirrup) + No. of Bars (dia of bar) + (spacing b/w bar bar (dia of bar))

8. Design Moment is given by,

$Md = \phi \times fy \times Ast \times (d - a/2) \rightarrow$ if $a < hf$

$Md = \phi \times [As \times fy \times (d - hf/2) + (As - Ast) \times fy \times (d - a/2)] \rightarrow$ if $a > hf$

QUESTION: 4

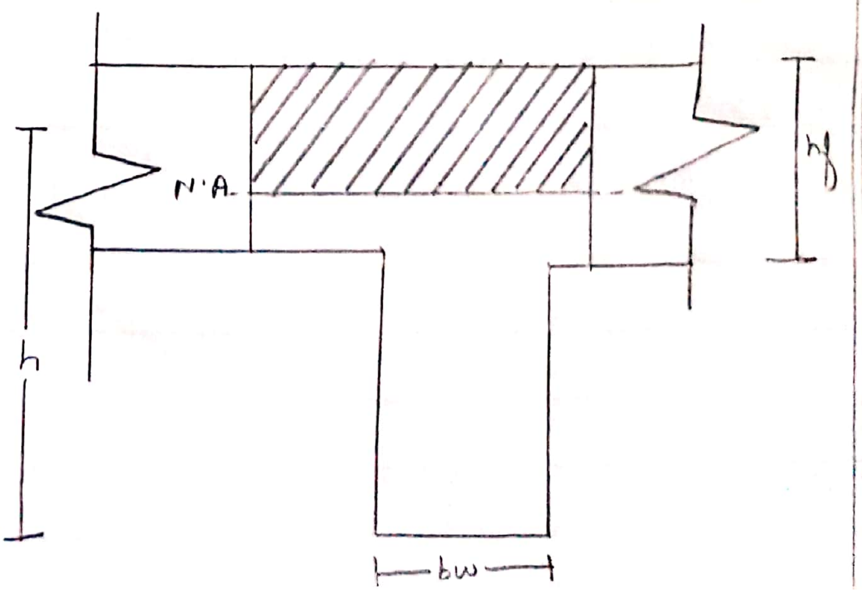
What is the difference b/w CASE-1 AND CASE-2 IN THE Design of T-Beam?

CASE-I:

From the figure

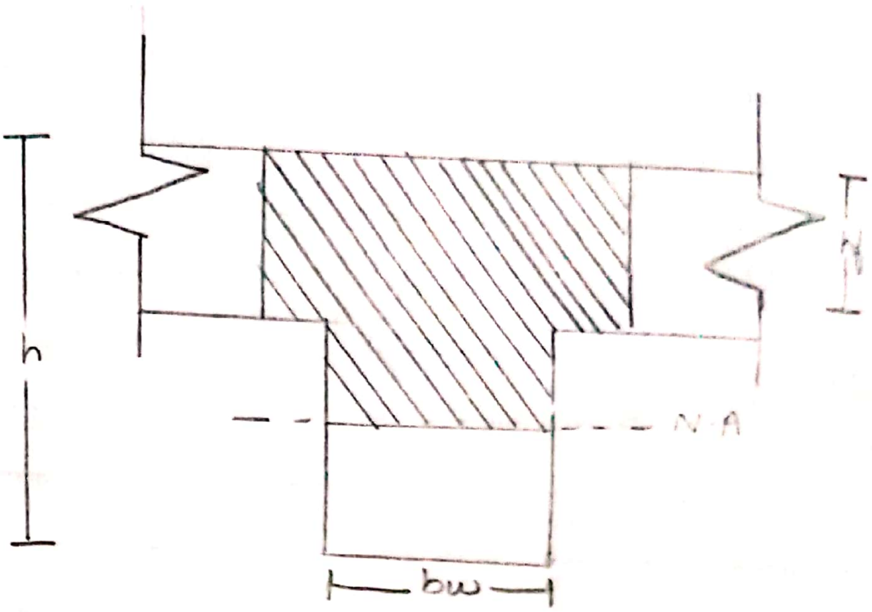
$a < hf$

So in this case, Rectangular Beam Analysis is required So, The Design Moment formula will be



$$M_d = \phi \times f_y \times A_{st} \times (d - a/2)$$

CASE - II :



From the figure,
$a > h_f$
So in this, special beam analysis ie, T-Beam Analysis is required.
So, the required Design Moment will be,

$$M_d = \phi \times \left[A_s \times f_y \times \left(d - \frac{h_f}{2}\right) + (A_s - A_{st}) \times f_y \times (d - a/2)\right]$$

QUESTION: 05

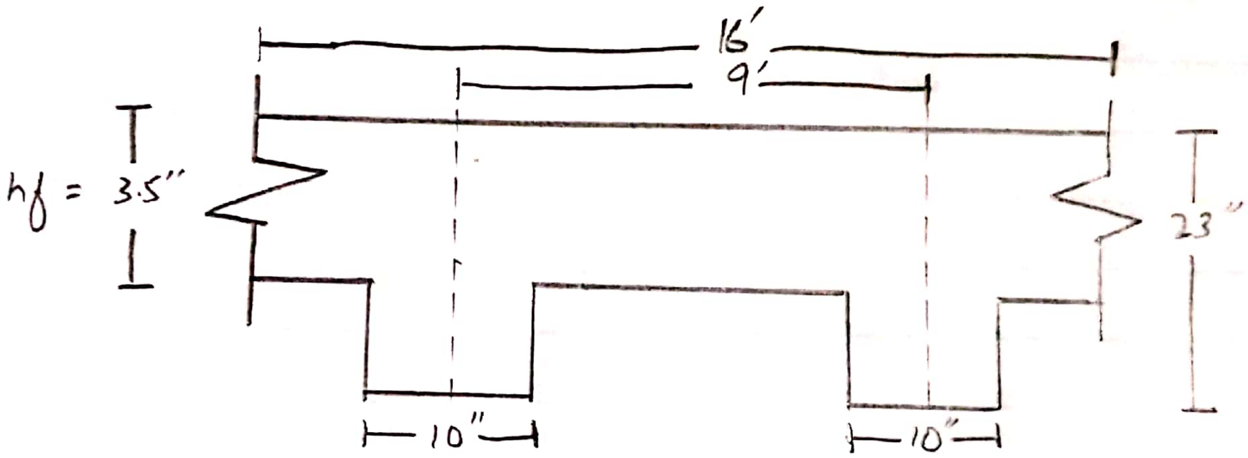
A floor system consist of 3.5" concrete slab supported by 16' simple span spaced at 9' c/c, the beam having a web width of 10" and effective depth of 18" and total height is 23". Calculate the necessary flexural reinforcement if the factored applied moment is 5800 kip-inch. Use f'c = 3 ksi and fy = 60 ksi.

GIVEN:

Height of flange (h_f) = 3.5"

c/c distance = 9'
length / span of the beam = 16'
Web width (b_w) = 10"
Effective depth (d) = 18"
Height (h) = 23"
Total factored moment (M_u) = 5800 kip-inch
f'_c = 3 ksi;
f_y = 60 ksi;

SOLUTION:



STEP: 1

Calculate the effective width (bc) for T-beam

1- $16(h_f) + b_w = 16(3.5)+10 = 66"$
2- c/c distance = $9 \times 12 = 108"$
3- Span/4 = $\frac{16}{4} \times 12 = 48"$

Selecting the least value of be as,
$b_e = 48"$

STEP: 2

Check whether Rectangular or T-beam Analysis is required.

Trial # 0.1: Let $a = h_f = 3.5''$

$$A_{st} = \frac{M_u}{\phi \times f_y \times (d - a/2)} = \frac{5800}{0.90 \times 60 \times (18 - 3.5/2)} = 6.61 \text{ in}^2$$

Trial # 0.2:

$$a = \frac{A_{st} \times f_y}{0.85 \times f'_c \times b_c}$$

$$a = \frac{6.61 \times 60}{0.85 \times 3 \times 48} = 3.2''$$

and $A_{st} = 6.655 \text{ in}^2$ $\Rightarrow 3.2'' < 3.5''$

So Rectangular Beam Design is Required!

Trial # 03: $a = 3.21''$

and $A_{st} = \dfrac{5800}{0.90 \times 60\,(18 - 3.21/2)} = 6.55 \text{ in}^2$

So Area of steel is 6.55 in^2

STEP: 3:

Check ρ_{max} and ρ_{min}.

$$\Rightarrow \rho_{max} = 0.85 \times \beta \times \frac{f'_c}{f_y}\left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}\right)$$

$$= 0.85 \times 0.85 \times \frac{3}{60}\left(\frac{0.003}{0.003 + 0.005}\right) = 0.013$$

$$\Rightarrow \rho_{min} = \frac{200}{f_y} = \frac{200}{60000} = 0.003$$

$$\Rightarrow \rho = \frac{A_{st}}{b \times d} = \frac{6.55}{10 \times 18} = 0.036$$

$\rho_{min} < \rho < \rho_{max}$

$\underbrace{0.003 < 0.036 < 0.013}$

As the value of ρ_{max} is less than ρ, so we have to design it as "Doubly Reinforced Beam".

=> First we have to find the Area of steel against ρ_{max}.

$$\rho_{max} = \frac{A_{st}}{b \times d} \Rightarrow A_{st} = \rho_{max} \times (b \times d)$$

$$A_{st} = 0.013 \times (10 \times 18)$$

$$A_{st} = 2.34 \text{ in}^2$$

STEP: 4

Finding The value of M_{U2}:

By formula,

$$M_{U2} = \phi \times A_{st} \times f_y \times (d - a/2)$$

First Finding the value of "a"

$$\Rightarrow a = \frac{A_{st} \times f_y}{0.85 \times f'_c \times b} = \frac{2.43 \times 60}{0.85 \times 3 \times 6}$$

$$a = 5.72''$$

$$\Rightarrow M_{U2} = 0.90 \times 2.43 \times 60 \times (18 - 5.72/2)$$

$$M_{U2} = 1986.67 \text{ kip-inch}$$

AS $M_{U2} < M_U$

$1986.67 < 5800$

So we have to design the beam in such away that it can resist more bending moment than the applied external moment.

STEP: 5

Finding Difference in moments and Area of steel

$$M_{U1} = M_U - M_{U2}$$
$$= 5800 - 1986.67$$
$$M_{U1} = 3813.33 \text{ kip-inch}$$

By formula

$$A'_{st} = 4.56 \text{ in}^2$$

STEP: 6

Finding Total steel area.

$$A_s = A_{st} + A'_{st}$$
$$= 2.43 + 4.56 = 6.99 \text{ in}^2$$

STEP: 7

Selection of Bar:

In Tension Bar:

Let we use #8 bar

dia $= (8/8) = 1''$, Area $= \frac{\pi}{4}(1)^2 = 0.785 \text{ in}^2$

By formula

$$\text{No. of bars} = \frac{\text{Area of steel}}{\text{Area of single steel}} = \frac{6.99}{0.785} = 8.9 \approx 9$$

So 9 #8 bars.

In compression zone:

Let we use #7 bar

dia $= (7/8)''$, Area $= \frac{\pi}{4}(7/8)^2 = 0.601 \text{ in}^2$

By formula

$$\text{No. of bars} = \frac{\text{Area of steel}}{\text{Area of single bar}}$$

$$= \frac{4.56}{0.601} = 7.5 \approx 8$$

So 8 #7 bars.

STEP : 8

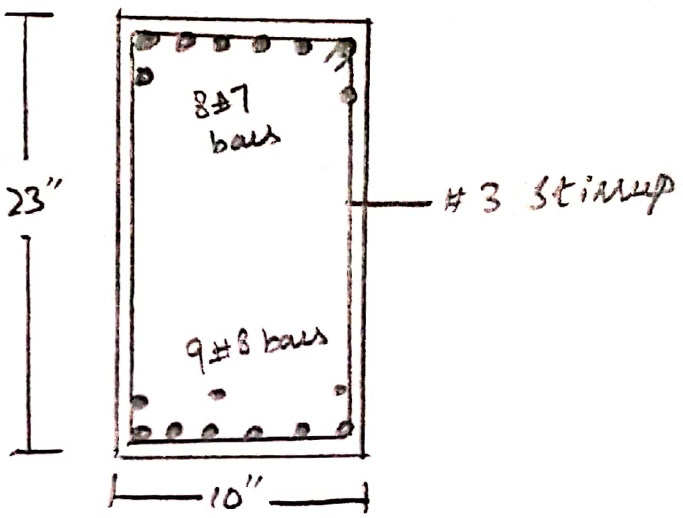
Minimum width for Accomodation of bars.

$b_{min} = (2 \times 1.5) + (2 \times 3/8) + 9(8/8) + 8(8/8)$

$= 20.75''$

As $20.75'' > 10''$

So, the bars will be placed in multiple layers.



Effective depth $(d) = 23 - 1.5 + 3/8 + 8/8 + 1/2(8/8) = 19.6''$

Effective cover $(d') = 1.5 + 3/8 + 7/8 + 1/2(7/8) = 3.18''$

STEP : 9.

Finding the Design Moment.

$M_d = \phi\,[A_s' \times f_y \times (d - d') + (A_s - A_{st}') \times f_y \times (d - a/2)]$

First $a = \dfrac{(A_s - A_{st}') \times f_y}{0.85 \times f'_c \times b} = \dfrac{(9 \times 0.785 - 8 \times 0.601) \times 60}{0.85 \times 3 \times 10} = 5.31''$

$\Rightarrow M_d = 0.90\,[(8 \times 0.601) \times 60 \times (19.6 - 3.18) + (9 \times 0.785 - 8 \times 0.601) \times 60 \times (19.6 - \frac{5.31}{2})]$

$M_d = 6328.38$

As $6328.38 > 5800 \rightarrow$ so design is ok!

QUESTION: 6

A beam is revised to developed and ultimate moment of 6000 kip-inches limited to 14x26 inch size, use f'c = 4 ksi and fy = 60 ksi. Determine flexural reinforcement assume two rows of tensile reinforcement and effective depth of beam is 22 inches.

SOLUTION:

GIVEN:

Breadth (b) = 14"

Height (h) = 26"

concrete compression strength (f'c) = 4 ksi

steel Tensile strength (fy) = 60 ksi

Ultimate Factored Moment (Mu) = 6000 kip-inches

Assume Effective cover (d') = 2.5"

STEP #1 (REINFORCEMENT RATIO)

By formula,

$$\rho_{max} = 0.85 \times \beta \times \frac{f'c}{fy} = \left(\frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}\right)$$

$$= 0.85 \times 0.85 \times \frac{4}{60} \times \left(\frac{0.003}{0.003 + 0.005}\right)$$

$$\rho_{max} = 0.0180$$

STEP #2 (Area of Steel)

As we know that,

$$\rho_{max} = \frac{Ast}{b \times d} \Rightarrow Ast = \rho_{max} \times (b \times d)$$

$\Rightarrow A_{st} = 0.0180 \times (14 \times 22) = 5.54 \text{ in}^2$

STEP # 3 (Design Moment)

By using formula

$M_{u2} = \phi \times A_{st} \times f_y \times (d - a/2)$

$\Rightarrow a = \dfrac{A_{st} \times f_y}{0.85 \times f'_c \times b} = \dfrac{5.54 \times 60}{0.85 \times 4 \times 14} = 6.98''$

so,

$M_{u2} = 0.90 \times 5.54 \times 60 \times \left(22 - \dfrac{6.98}{2}\right)$

$= 5537.4$ kip-inch

As $5537.4 < 6000$

So we have to design a section as doubly reinforced.

STEP # 4 (Difference In Moments)

$M_{u1} = M_u - M_{u2}$

$= 6000 - 5537.4$

$M_{u1} = 462.6$ kip-inches

STEP # 5 (Area of Steel)

$M_{u1} = \phi \times A'_{st} \times f_y \times (d - d')$

So area of steel in compression zone will be,

$A'_{st} = \dfrac{M_{u1}}{\phi \times f_y \times (d - d')} = \dfrac{462.6}{0.90 \times 60 \times (22 - 2.5)}$

$\rightarrow A'_{st} = 0.44 \text{ in}^2$

STEP # 6: (Total Steel Area)

$$A_s = A_{st} + A'_{st}$$
$$= 5.54 + 0.44 = 5.98 \text{ in}^2$$

STEP # 7: (Selection & No. of bars used)

1. Steel in Tension zone:

We use # 7 bars,

dia = $(7/8)'' = 0.875''$, Area = $\pi/4\,(0.875)^2$
$= 0.601 \text{ in}^2$

So,

$$\text{No of bars} = \frac{A_s}{\text{Area of sigle bar}}$$
$$= \frac{5.98}{0.601}$$

So 10 #7 bars

2. Steel in compression zone:

We use # 5 bars,

dia = $(5/8)'' = 0.625''$, Area = $\pi/4\,(0.625)^2$
$= 0.306 \text{ in}^2$

So,

$$\text{No. of bars} = \frac{A_{st}'}{\text{Area of single bar}}$$
$$= \frac{0.44}{0.306} = 1.43 \approx 2 \text{ bars}$$

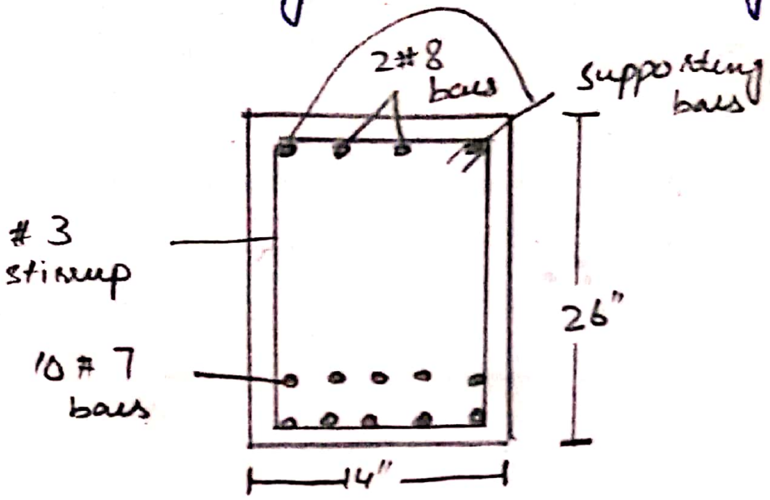
So 2 #5 bars

STEP: 8. (Minimum Width of Beam)

$b_{min} = 2(1.5) + 2(3/8) + 10(7/8) + 9(7/8)$

$b_{min} = 20.37 > 14"$

so not good in one layer.



Now,

=> Effective depth $(d) = 26 - 1.5 - 3/8 - 7/8 - \frac{1}{2}(7/8)$

$= 22.82"$

=> Effective cover $(d') = 1.5 + 3/8 + \frac{1}{2}(5/8)$

$= 2.18"$

STEP: 9 (Design Moment)

$M_d = \phi \times [A'_{st} \times f_y \times (d - d') + (A_{st} - A'_{st}) \times f_y \times (d - a/2)]$

$a = \frac{(A_{st} - A_{st}') \times f_y}{0.85 \times f'_c \times b}$

$= \frac{(10 \times 0.601 - 2 \times 0.306) \times 60}{0.85 \times 4 \times 14} = 6.80".$

$M_d = 0.90 [(2 \times 0.306) \times 60 \times (22.82 - 2.18) + (10 \times 0.601 - 2 \times 0.306) \times 60 \times (22.82 - 6.80/2)]$

$M_d = 7047.6$ kip - inches

As = 7047.6 > 6000 Design is ok!