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Structural Analysis-II (Final Term Paper)

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—{QNo. 1}—

Analyze the beam shown in Figure 1 by stiffness method. Assume EI is constant.

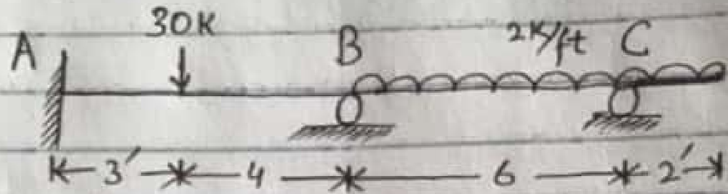
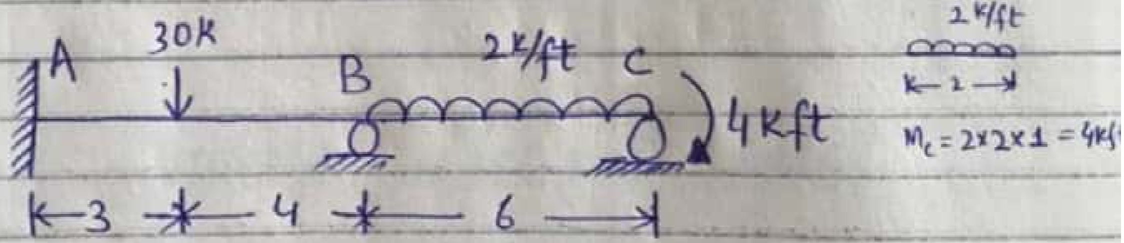
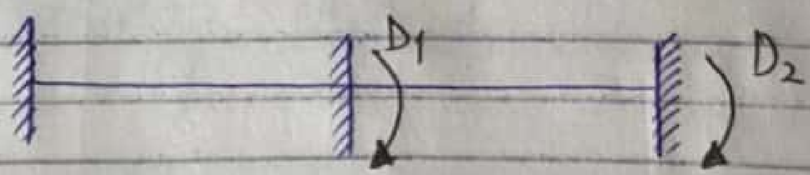


Figure 1

Step 1 : K.I = 2° (Neglecting Axial Effects)

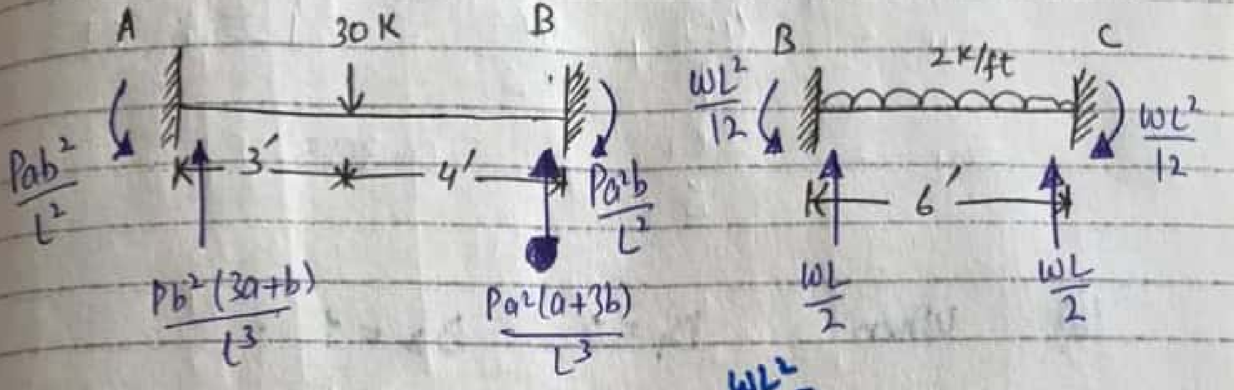
Step 2 : Select the unknown joint displacement.

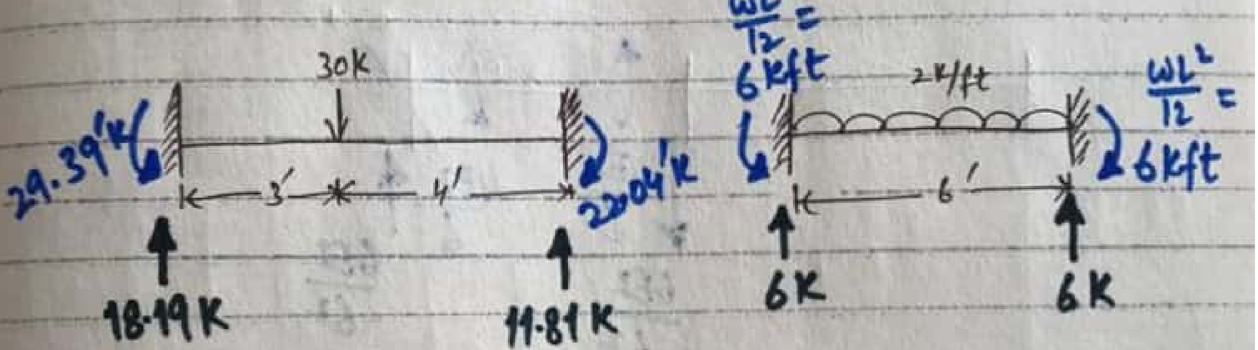




$$[D] = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad [AD] = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step 3: Compute [ADL] matrix.





$ADL_1 = 22.04'K - 6'K$

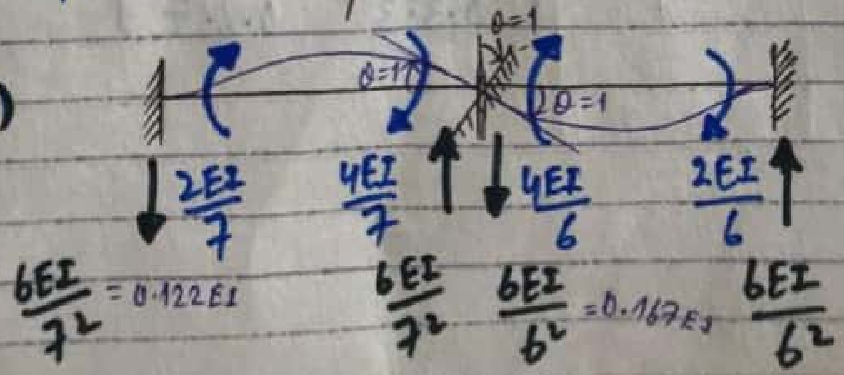
$ADL_1 = 16.04'K$

$ADL_2 = 6'K$

So,
$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} 16.04 \\ 6 \end{bmatrix}$$

Step 4: Compute [S] matrix.

(i)

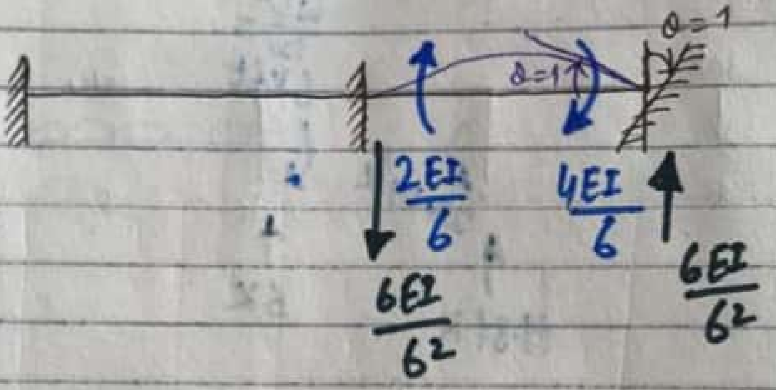


When $D_1 = 1$, $D_2 = 0$

$$S_{11} = \frac{4EI}{7} + \frac{4EI}{6} = 1.238\ EI$$

$$S_{21} = \frac{2EI}{6} = 0.333\ EI$$

(ii) When $D_1 = 0$, $D_2 = 1$.



$$S_{12} = \frac{2EI}{6} = 0.333\ EI$$

$$S_{22} = \frac{4EI}{6} = 0.667\ EI$$

Stiffness Matrix $[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

$$[S] = \begin{bmatrix} 1.238 & 0.333 \\ 0.333 & 0.667 \end{bmatrix} EI$$

Step 5: Compute the values of D_1 & D_2

$$[AD] = [ADL] + [S][D]$$

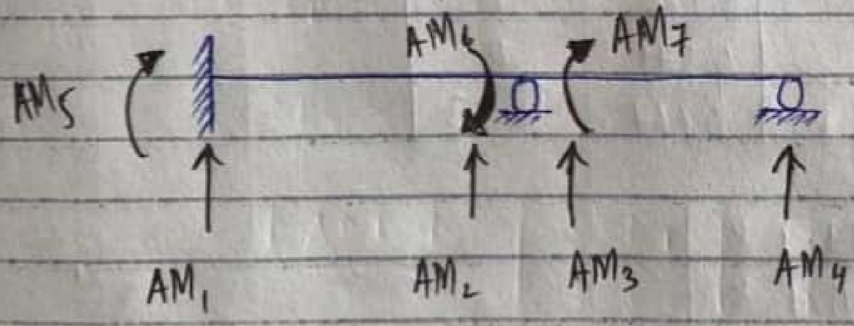
$$[D] = [S]^{-1}\,[AD - ADL]$$

$$[D] = \left(\begin{bmatrix} 1.238 & 0.333 \\ 0.333 & 0.667 \end{bmatrix} EI \right)^{-1} \begin{bmatrix} 0 - (16.04) \\ 4 - (6) \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} 0.933 & -0.466 \\ -0.466 & 1.732 \end{bmatrix} \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$[D] = \frac{1}{EI} \begin{bmatrix} -14.03 \\ 4.01 \end{bmatrix}$$

Step 6: Compute Member End Actions.



$$[AM] = [AML] + [AMD][D]$$

[AML] ;

$$[AML] = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} = \begin{bmatrix} 18.19\ K \\ 11.81\ K \\ 6\ K \\ 6\ K \\ -29.39'K \\ 22.04'K \\ -6'K \end{bmatrix}$$

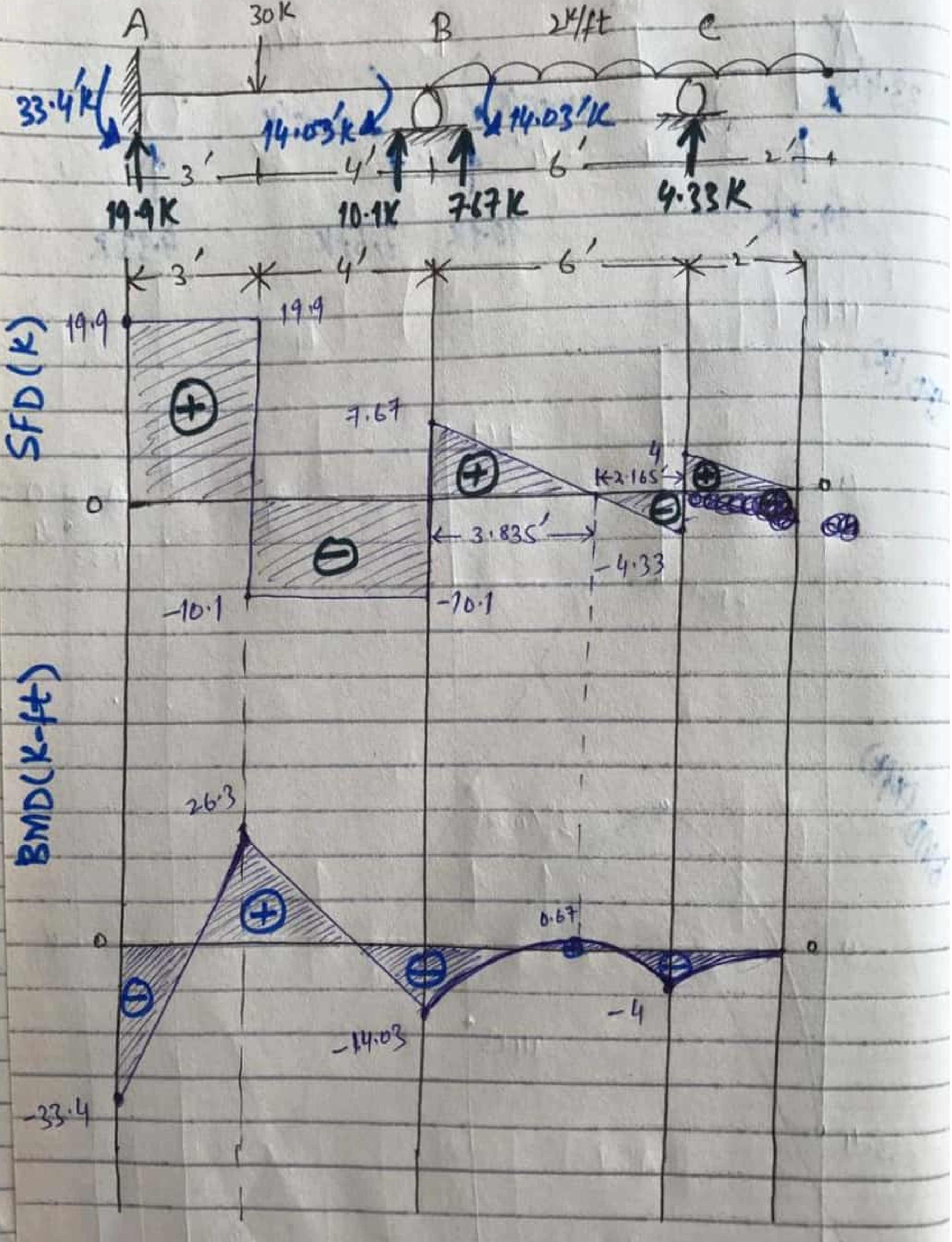
[AMD] ;

$$[AMD] = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{62} \\ AMD_{71} & AMD_{72} \end{bmatrix} = EI \begin{bmatrix} -0.122 & 0 \\ 0.122 & 0 \\ -0.167 & -0.167 \\ 0.167 & 0.167 \\ 0.286 & 0 \\ 0.571 & 0 \\ 0.667 & 0.333 \end{bmatrix}$$

Now; $[AM] = [AML] + [AMD][D]$

$$[AM] = \begin{bmatrix} 18.19 \\ 11.81 \\ 6 \\ 6 \\ -29.39 \\ 22.04 \\ -6 \end{bmatrix} + \begin{bmatrix} -0.122 & 0 \\ 0.122 & 0 \\ -0.167 & -0.167 \\ 0.167 & 0.167 \\ 0.286 & 0 \\ 0.571 & 0 \\ 0.667 & 0.333 \end{bmatrix} \cancel{EI} \times \frac{1}{\cancel{EI}} \begin{bmatrix} -14.03 \\ 4.01 \end{bmatrix}$$

$$[AM] = \begin{bmatrix} 19.9\ k \\ 10.1\ k \\ 7.67\ k \\ 4.33\ k \\ -33.4\ ' k \\ 14.03\ ' k \\ -14.03\ ' k \end{bmatrix}$$



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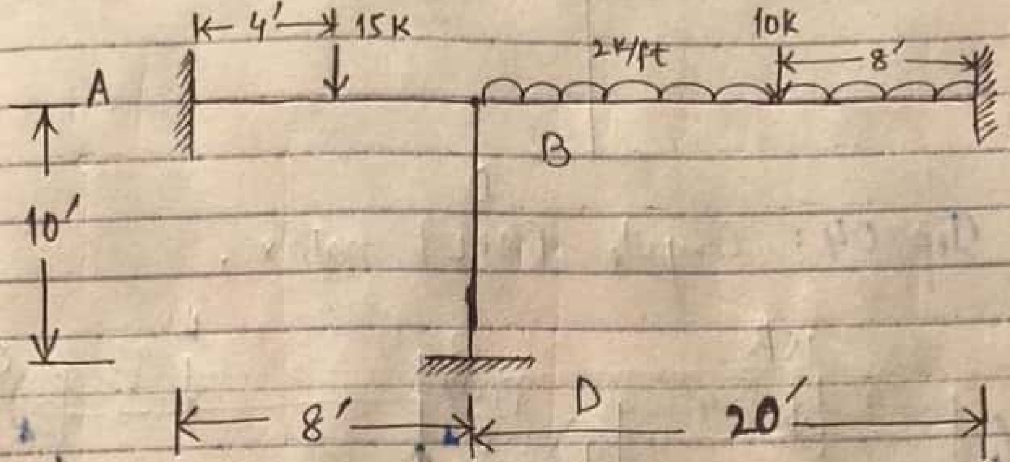
THE END OF
PROBLEM # 01.

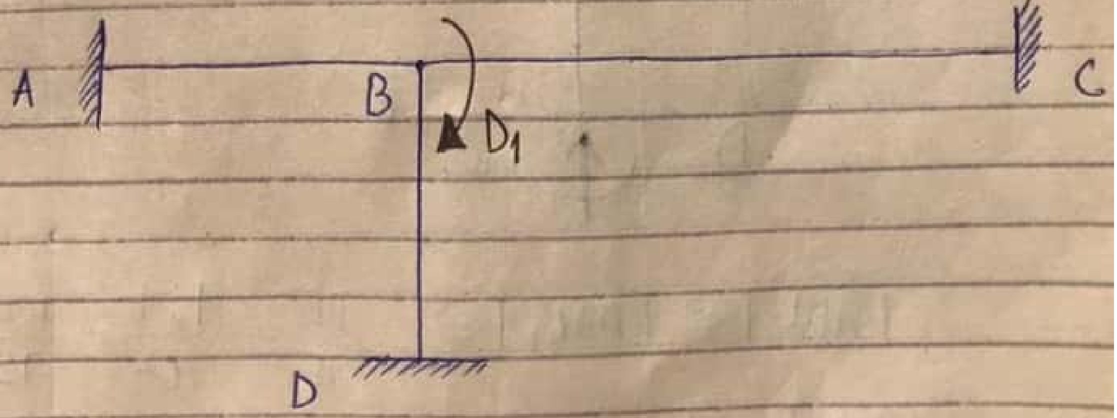
—{ Q No. 3 }—

• Analyze the rigid joint frame shown in figure 02, by stiffness method. Assume EI is constant.



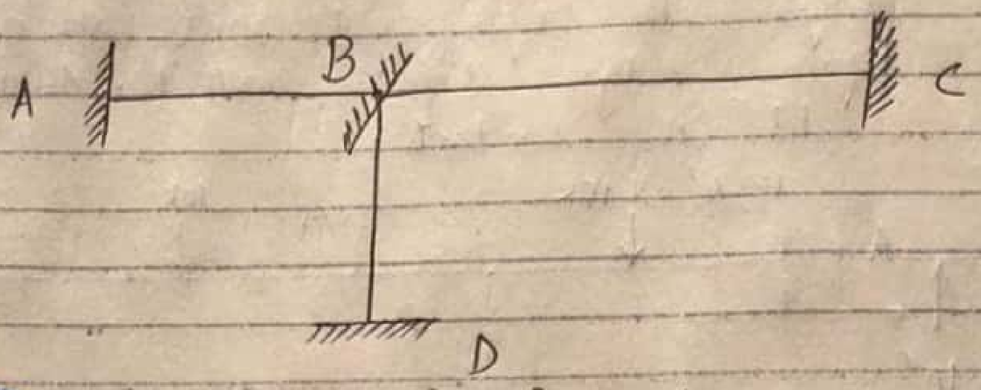
Step 01 : K.I = 1 (Neglecting Axial Effects)

• **Step 02 :** Selection of redundant joint displacements and assign coordinates at those locations, Also compute [AD] matrix.

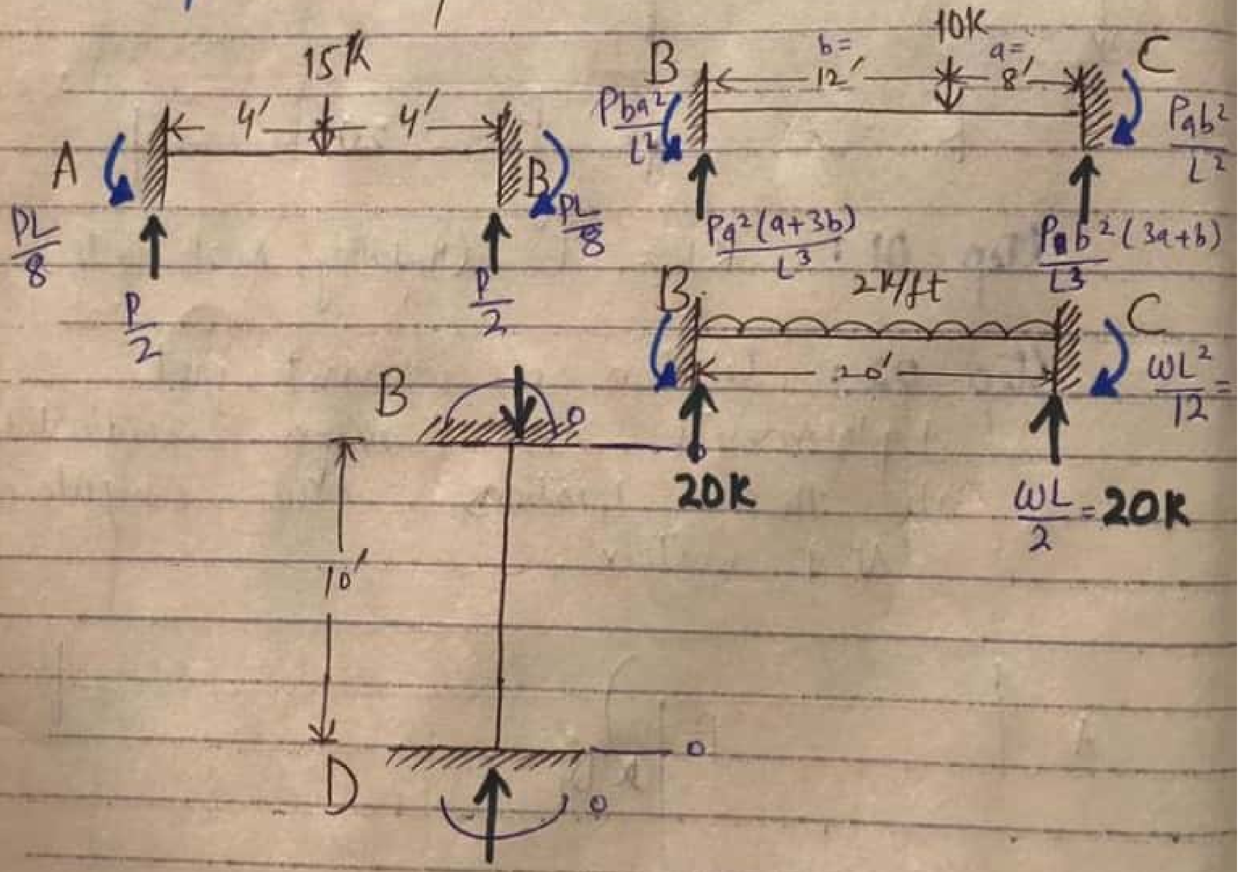


$[D] = [D_1] = [?]$, $[AD] = [AD_1] = [0]$

Step 03: Restrain all the DOF to get the "restrained" structure.



Step 04: Compute [ADL] matrix.



$$[ADL] = [ADL_1] = \frac{PL}{8} - \frac{Pa^2b}{L^2} - \frac{wL^2}{12}$$

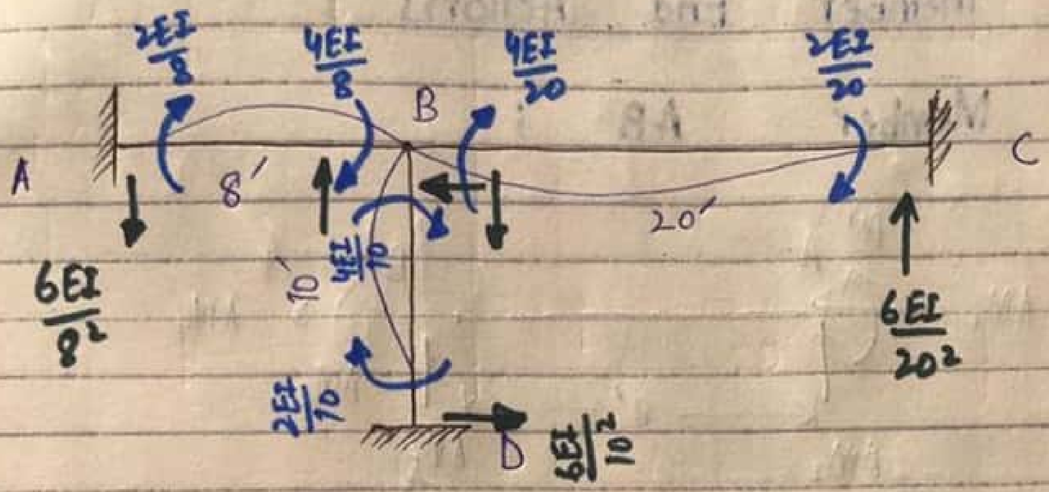
$$[ADL] = \left[\frac{15\times 8}{8} - \frac{10\times(8)^2(12)}{(20)^2} - \frac{2(20)^2}{12} \right]$$

$$[ADL] = [15 - 19.2 - 66.67]$$

$[ADL] = [-70.87 \text{ kft}]$

Step 05: Compute Stiffness Matrix [S].

$D_1 = 1.$



$$S_{11} = \frac{4EI}{8} + \frac{4EI}{20} + \frac{4EI}{10}$$

$$S_{11} = 1.1\ EI$$

$$[S] = [S_{11}] = [1.1]\ EI$$

Step 06: Compute the value of D.

$$[AD] = [ADL] + [S][D]$$

$$[D] = [S]^{-1}[AD - ADL]$$

$$[D] = [1.1\ EI]^{-1}[0 - (-70.87)]$$

$$[D] = \frac{1}{1.1\ EI}[70.87]$$

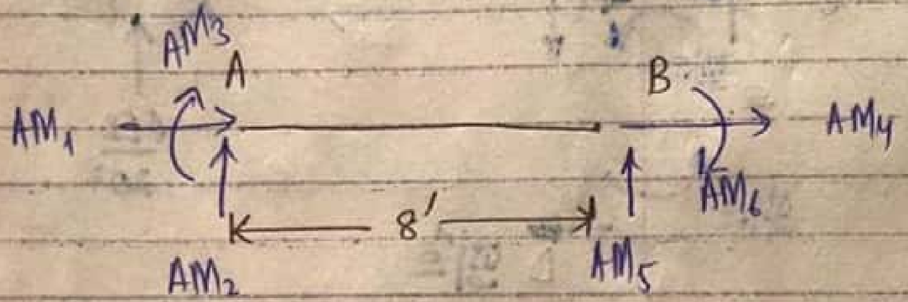
$$[D] = \frac{64.427}{EI}$$

or

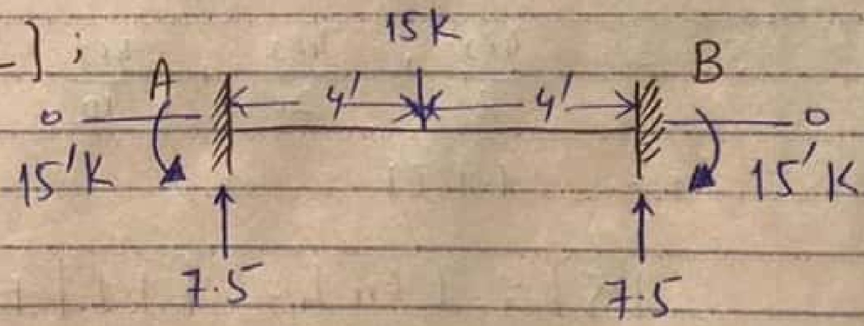
$$[D] = \frac{1}{EI}[64.427]$$

"Member End Actions"

Member AB ;

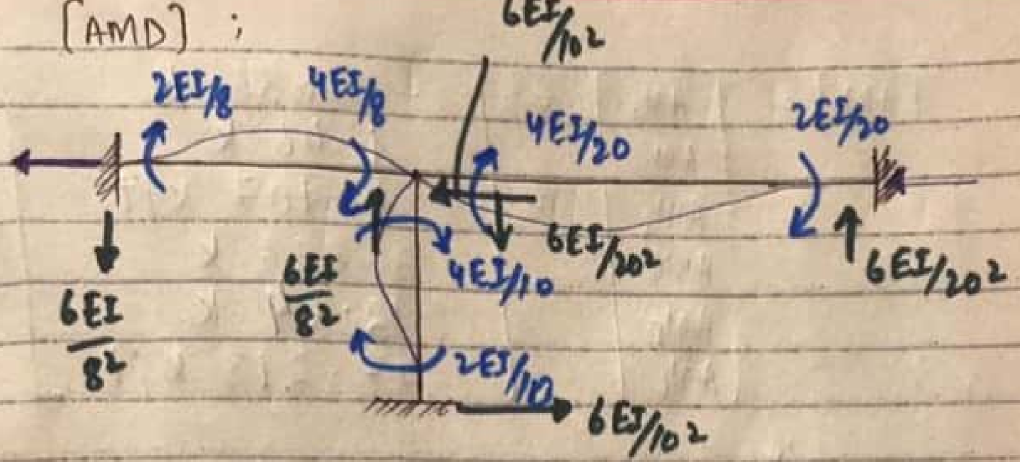


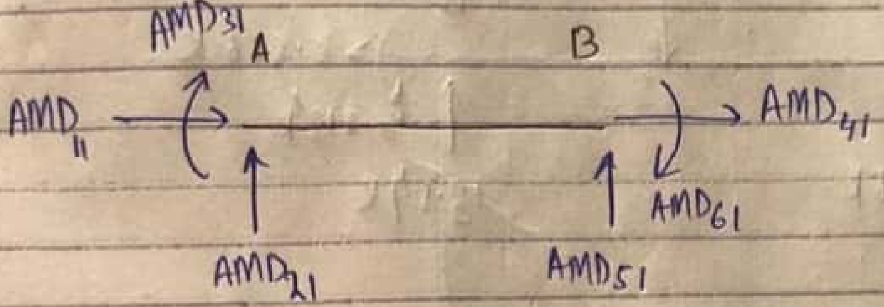
For [AML] ;



$$[AML] = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 7.5\,K \\ -15'K \\ 0 \\ 7.5\,K \\ 15'K \end{bmatrix}$$

For [AMD] ;



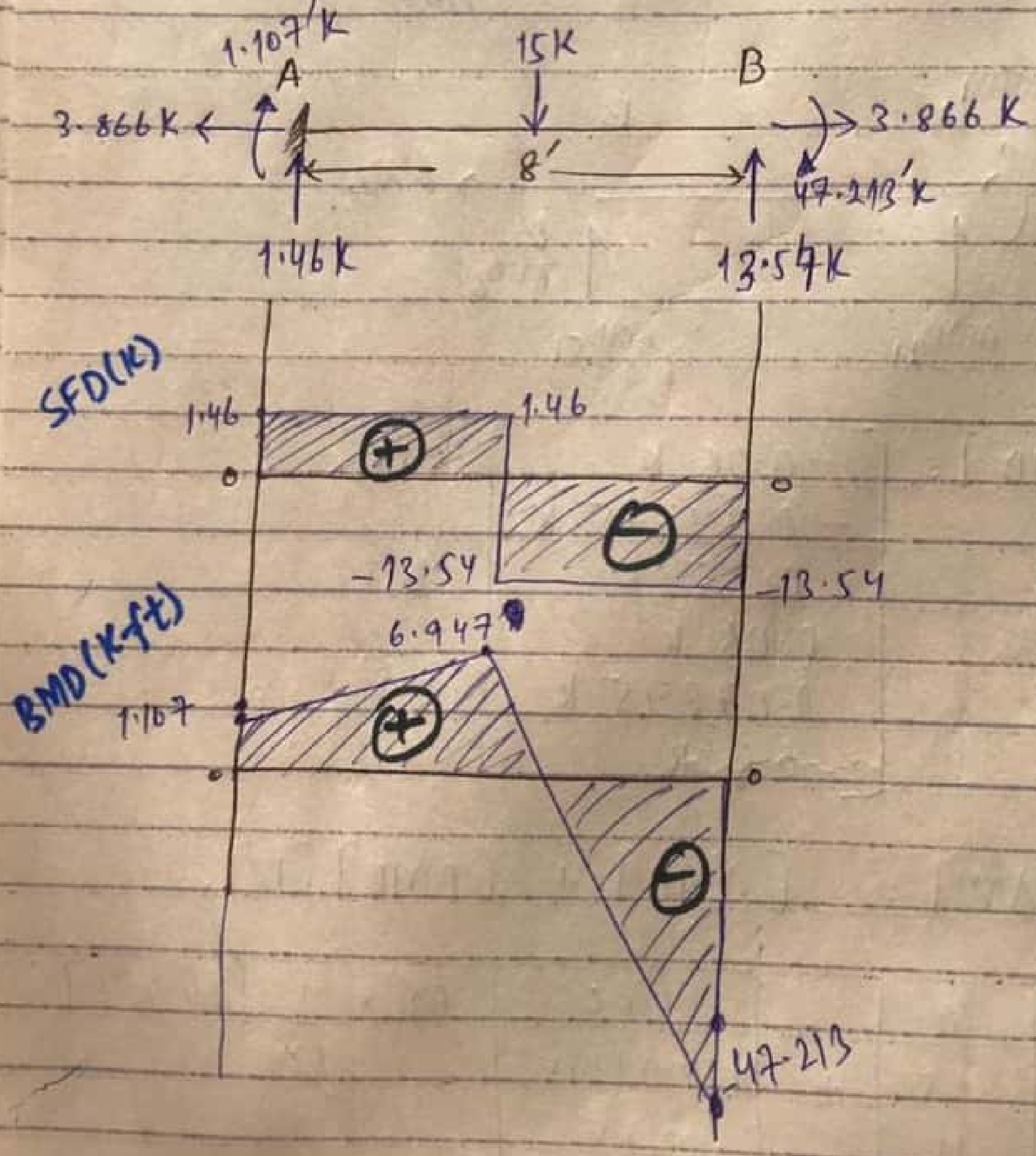


$$[AMD] = \begin{bmatrix} -0.06\,k \\ -0.09375\,k \\ 0.25'\,k \\ 0.06\,k \\ 0.07875\,k \\ 0.5'\,k \end{bmatrix} EI$$

$$[AM] = [AML] + [AMD][D]$$

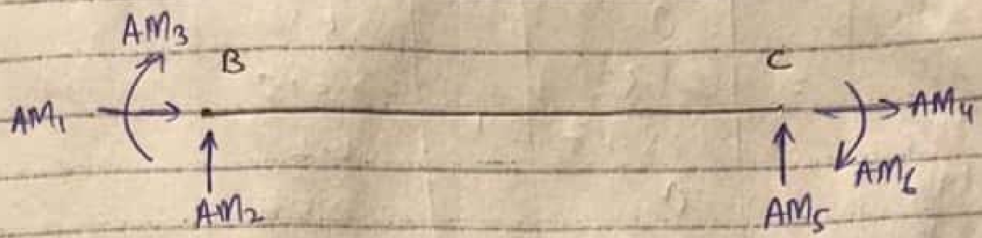
$$[AM] = \begin{bmatrix} 0 \\ 7.5 \\ -15 \\ 0 \\ 7.5 \\ 15 \end{bmatrix} + \begin{bmatrix} -0.06 \\ -0.09375 \\ 0.25 \\ 0.06 \\ 0.09375 \\ 0.5 \end{bmatrix} \cancel{EI} \times \frac{1}{\cancel{EI}} [64.427]$$

$$[AM] = \begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} -3.866 \text{ K} \\ 1.46 \text{ K} \\ 1.107 \text{ Kft} \\ 3.866 \text{ K} \\ 13.54 \text{ K} \\ 47.213 \text{ Kft} \end{bmatrix}$$

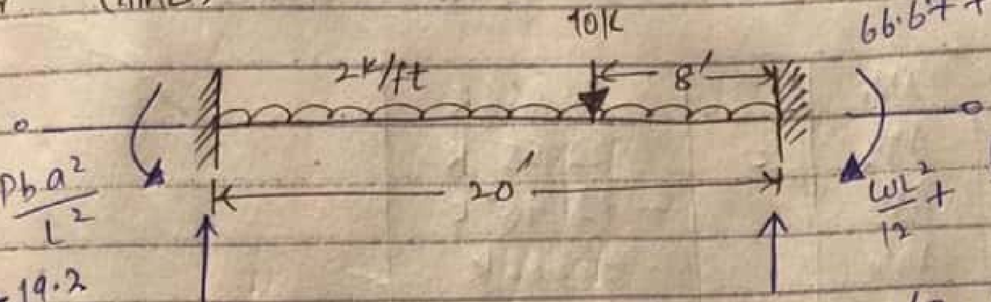


22.55
~~27.45~~

Member BC ;



For [AML] ;



$\frac{wL^2}{12} + \frac{Pba^2}{L^2}$ $= 66.67 + 19.2$

$\frac{wL^2}{12} + \frac{Pab^2}{L^2}$ $66.67 + 28.8$

$\frac{wL}{2} + \frac{Pa^2(a+3b)}{L^3} = 20 + 3.52 = 23.52\text{ K}$

$\frac{wL}{2} + \frac{Pb^2(3a+b)}{L^3} = 20 + 6.48 = 26.48\text{ K.}$

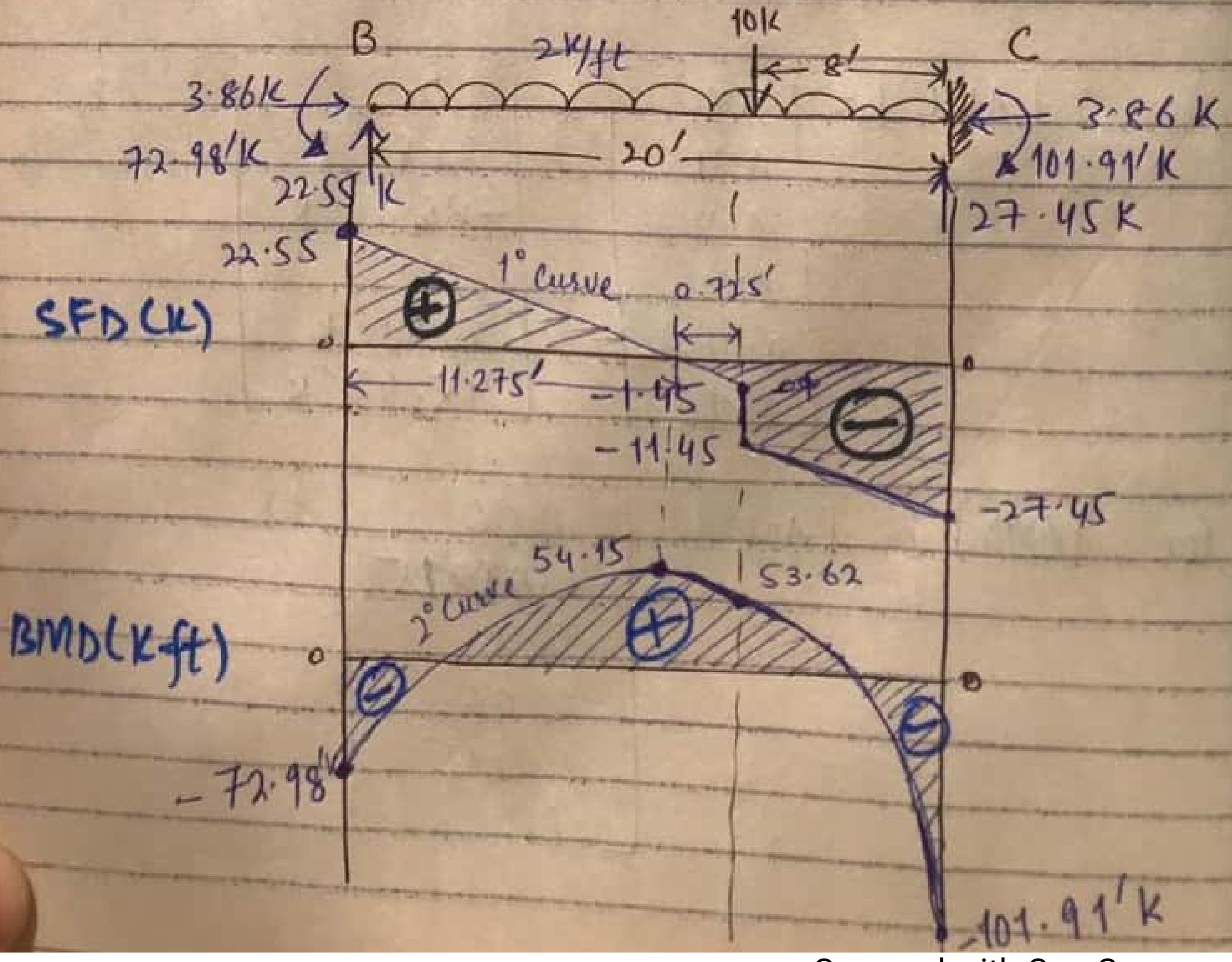
$$[AML] = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 23.52\text{ K} \\ -85.87'\text{K} \\ 0 \\ 26.48\text{ K} \\ 95.47'\text{K} \end{bmatrix}$$

For [AMD] ; $[AMD] = \begin{bmatrix} 0.06 \\ -0.015 \\ 0.2 \\ -0.06 \\ 0.015 \\ 0.1 \end{bmatrix} EI$

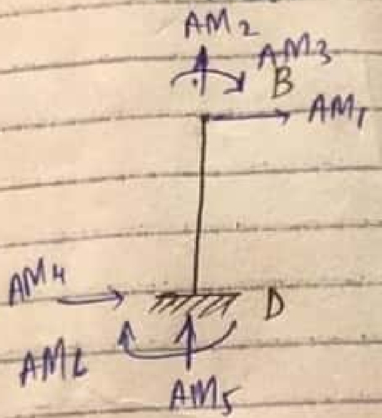
$[AM] = [AML] + [AMD] \cdot [D]$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 23.52 \\ -85.87 \\ 0 \\ 26.48 \\ 95.47 \end{bmatrix} + \begin{bmatrix} 0.06 \\ -0.015 \\ 0.2 \\ -0.06 \\ 0.015 \\ 0.1 \end{bmatrix} \cancel{EI} \times \frac{1}{\cancel{EI}} [64.42$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} 3.866K \\ 22.55\ K \\ -72.98'K \\ -3.866K \\ 27.45\ K \\ 101.91'K \end{bmatrix}$$

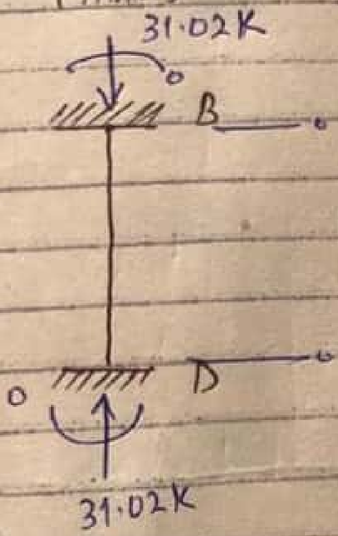


Member BD ;



$$[AM] = \begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

For [AML] ;



$$[AML] = \begin{bmatrix} 0 \\ -31.02K \\ 0 \\ 0 \\ 31.02K \\ 0 \end{bmatrix}$$

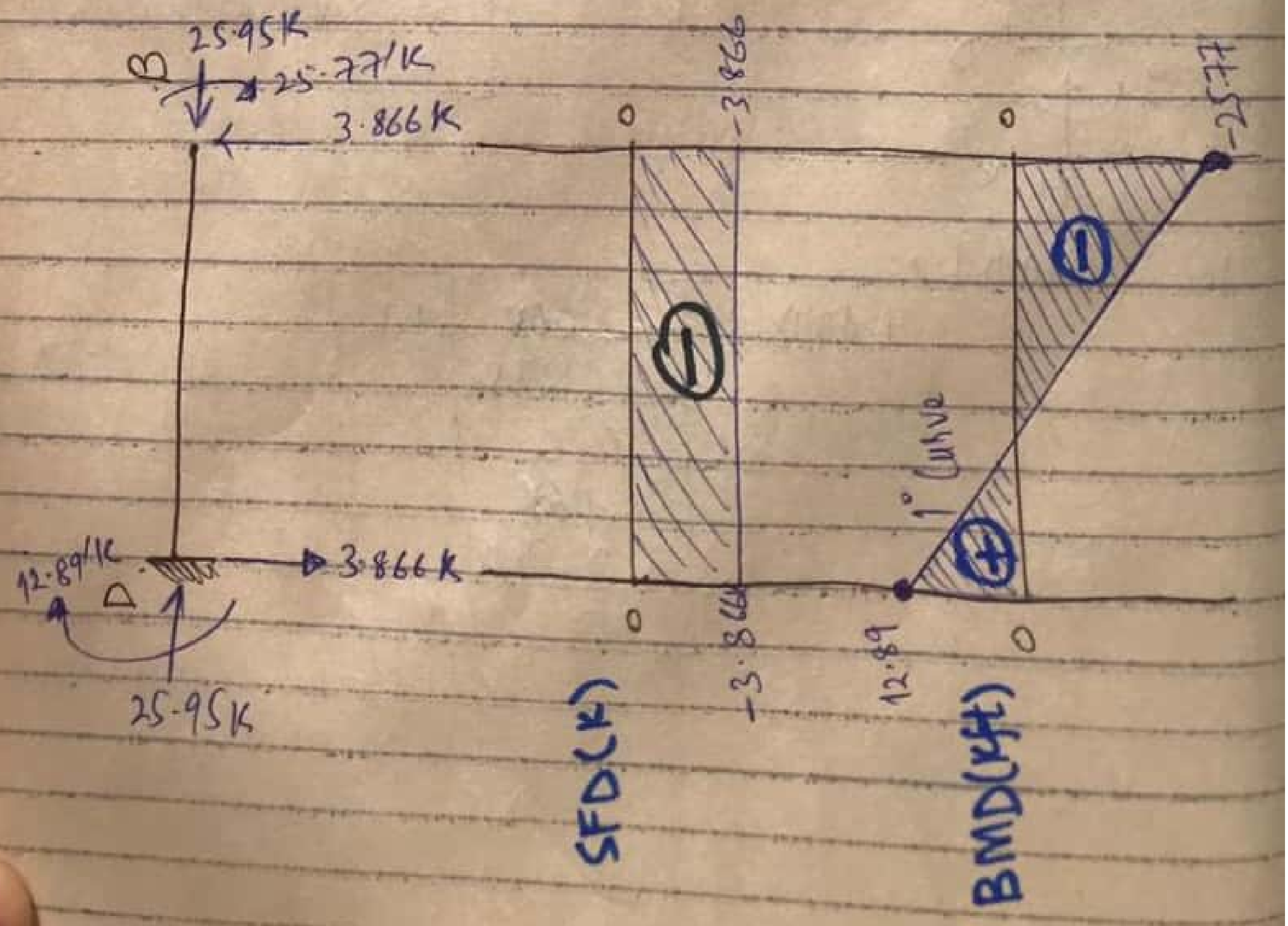
For [AMD] ;

$$[AMD] = \begin{bmatrix} -0.06 \\ 0.07875 \\ 0.4 \\ 0.06 \\ -0.07875 \\ 0.2 \end{bmatrix} EI$$

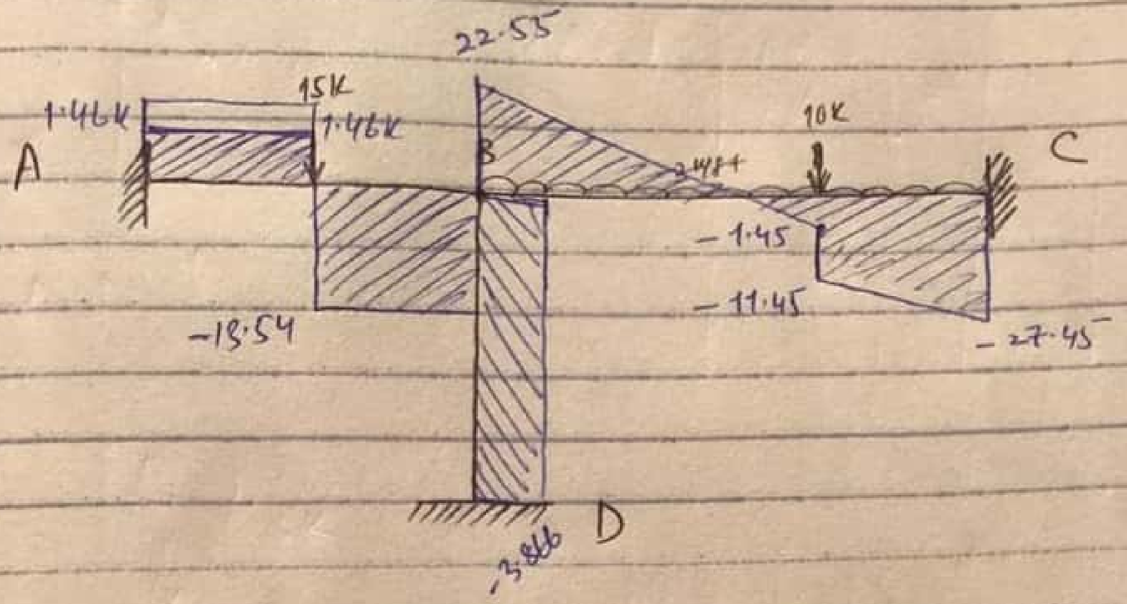
$[AM] = [AML] + [AMD][D]$

$$[AM] = \begin{bmatrix} 0 \\ -31.02 \\ 0 \\ 0 \\ 31.02 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.06 \\ 0.07875 \\ 0.4 \\ 0.06 \\ -0.07875 \\ 0.2 \end{bmatrix} EI \times \frac{1}{EI} (64.427$$

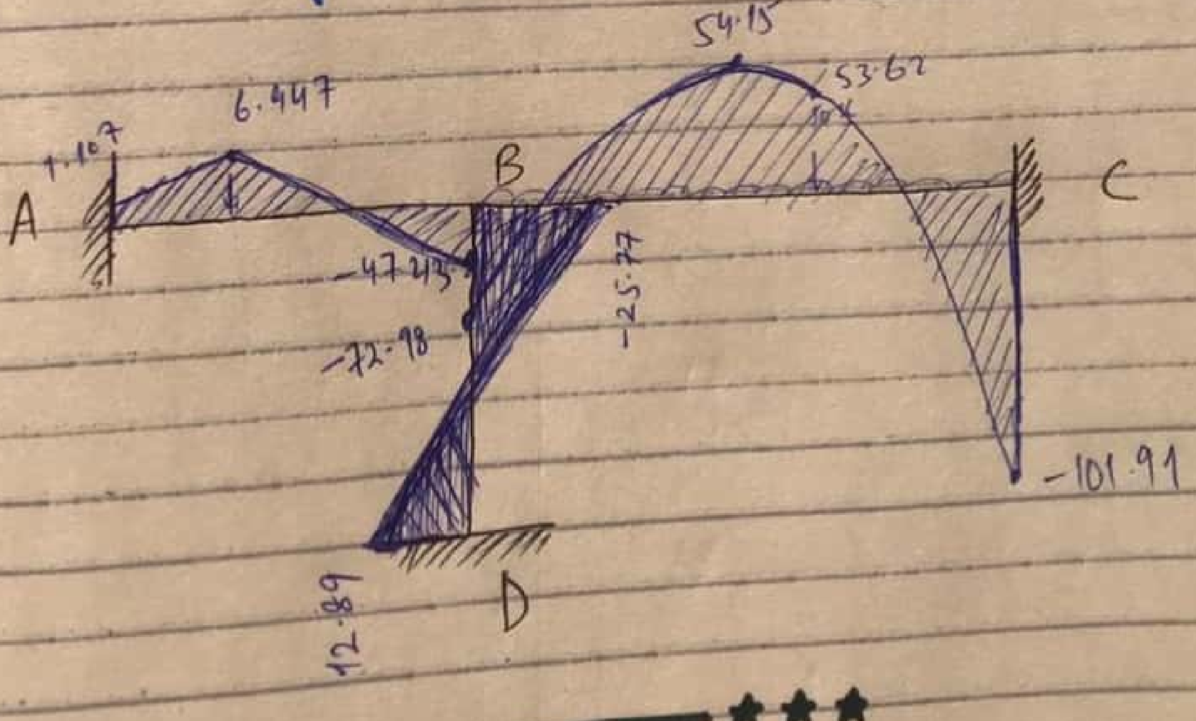
$$[AM] = \begin{bmatrix} -3.866K \\ -25.95K \\ 25.77'K \\ 3.866K \\ 25.95K \\ 12.89'K \end{bmatrix}$$



Shear Force Diagram (SFD (k)).



Bending Moment Diagram (BMD (kft)).



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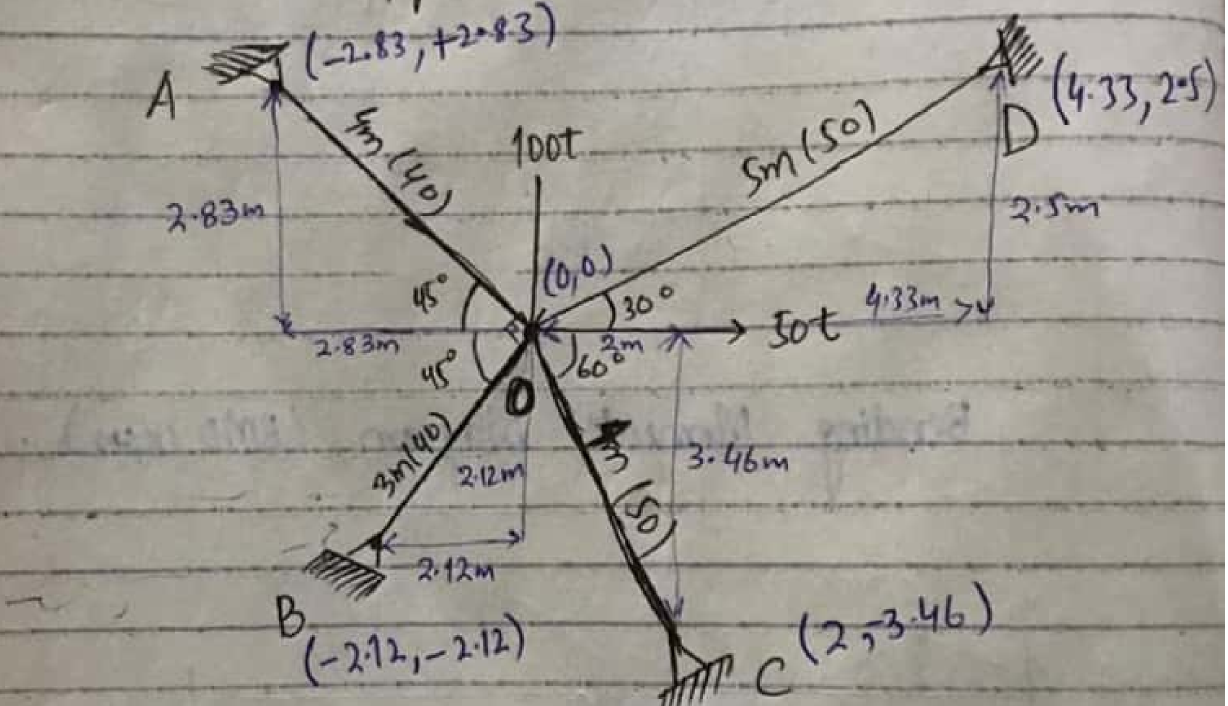
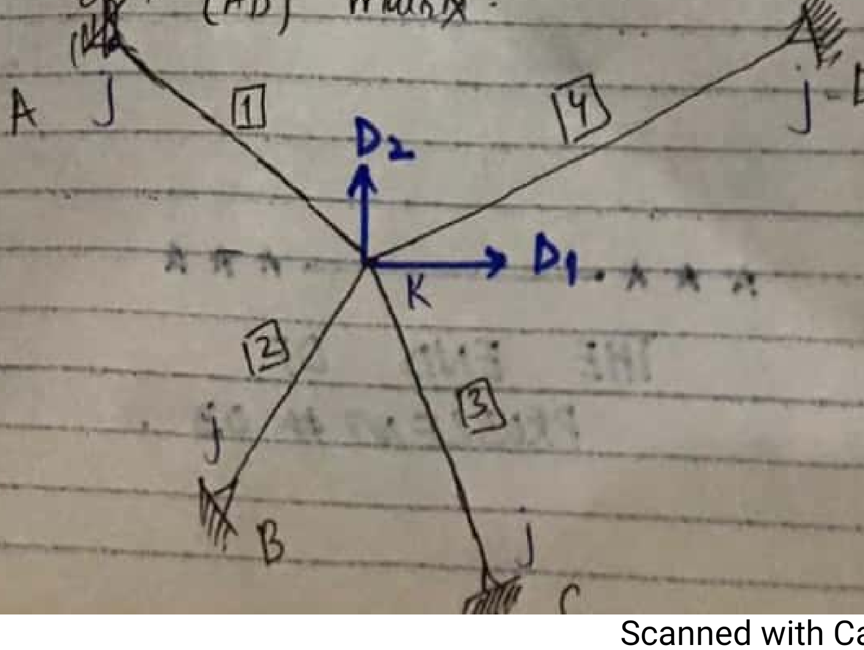
THE END OF PROBLEM # 03.

—{ Q No. 2 }—

Analyze the pin-jointed frame shown by, using stiffness method. Length of the members in 'm' and cross-sectional area of the members in 'cm^2' are shown in figure 3. Take $E = 2000\ t/cm^2$.



Step 01: Identify the unknown joint displacements and compute the values of [AD] matrix.

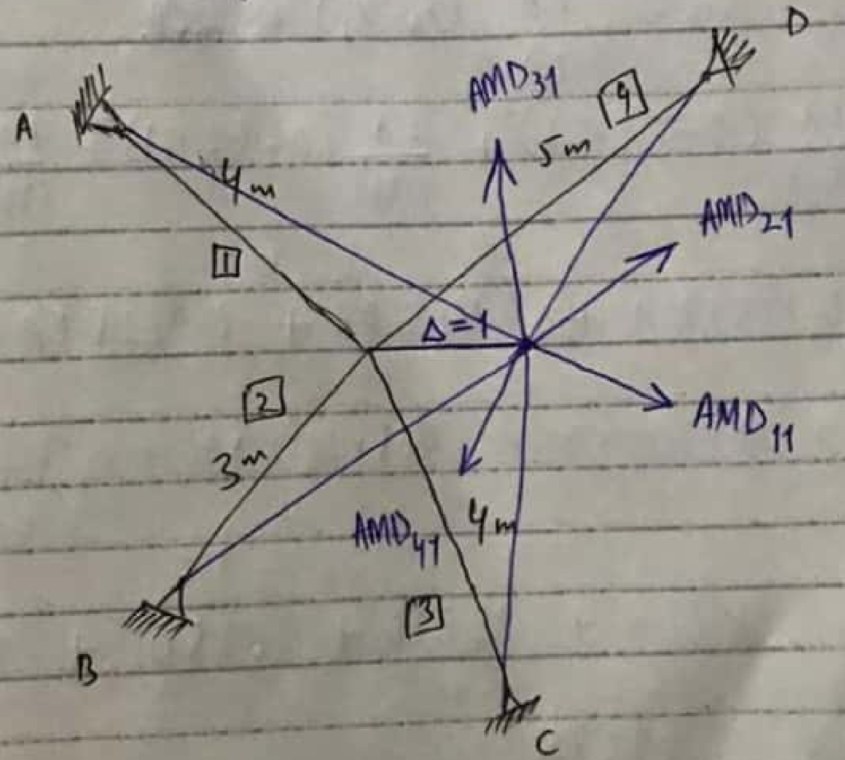


$$[D] = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[AD] = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step 02 : Computation of [AMD] and [S].

(i) When $D_1 = 1$, $D_2 = 0$.



$$AMD_{11} = \frac{EA}{L^2}(x_k - x_j) = \frac{EA}{(4)^2}\left(0-(-2.83)\right) = 0.1769\,EA$$

$$AMD_{11} = 0.1769 \times 2000\ \frac{t}{cm^2} \times 40\ cm^2 = 14152\ t.$$

$$AMD_{21} = \frac{EA}{L^2}(x_k - x_j) = \frac{EA}{(3)^2}\left(0-(-2.12)\right) = 0.2356\,EA$$

$$AMD_{21} = 0.2356 \times 2000 \times 40 = 18848\ t.$$

$$AMD_{31} = \frac{EA}{L^2}(x_k - x_j) = \frac{EA}{(4)^2}(0-2) = -0.125\,EA$$

$$AMD_{31} = -0.125 \times 2000 \times 50 = -12500\,t$$

$$AMD_{41} = \frac{EA}{L^2}(x_k - x_j) = \frac{EA}{(5)^2}(0-4.33) = -0.1732\,EA$$

$$AMD_{41} = -0.1732 \times 2000 \times 50 = -17320\,t$$

Also ;

$$S_{11} = \frac{EA}{L^3}(x_k - x_j)^2$$

$$S_{11} = \frac{EA}{(4)^3}(0-(-2.83))^2 + \frac{EA}{(3)^3}(0-(-2.12))^2 + \frac{EA}{(4)^3}(0-2)^2 + \frac{EA}{(5)^3}(0-4.33)^2$$

$$S_{11} = 0.1251\,EA + 0.1665\,EA + 0.0625\,EA + 0.15\,EA$$

$$S_{11} = 0.1251 \times 2000 \times 40 + 0.1665 \times 2000 \times 40 + 0.0625 \times 2000 \times 50 + 0.15 \times 2000 \times 50$$

$$S_{11} = 10008 + 13320 + 6250 + 15000$$

$$S_{11} = 44578$$

$$S_{21} = \frac{EA}{L^3}(x_k - x_j)(y_k - y_j)$$

$$S_{21} = \frac{EA}{(4)^3}(0-(-2.83))(0-2.83) + \frac{EA}{(3)^3}(0-(-2.12))(0-(-2.12)) + \frac{EA}{(4)^3}(0-2)(0-(-3.46)) + \frac{EA}{(5)^3}(0-4.33)(0-2.5)$$

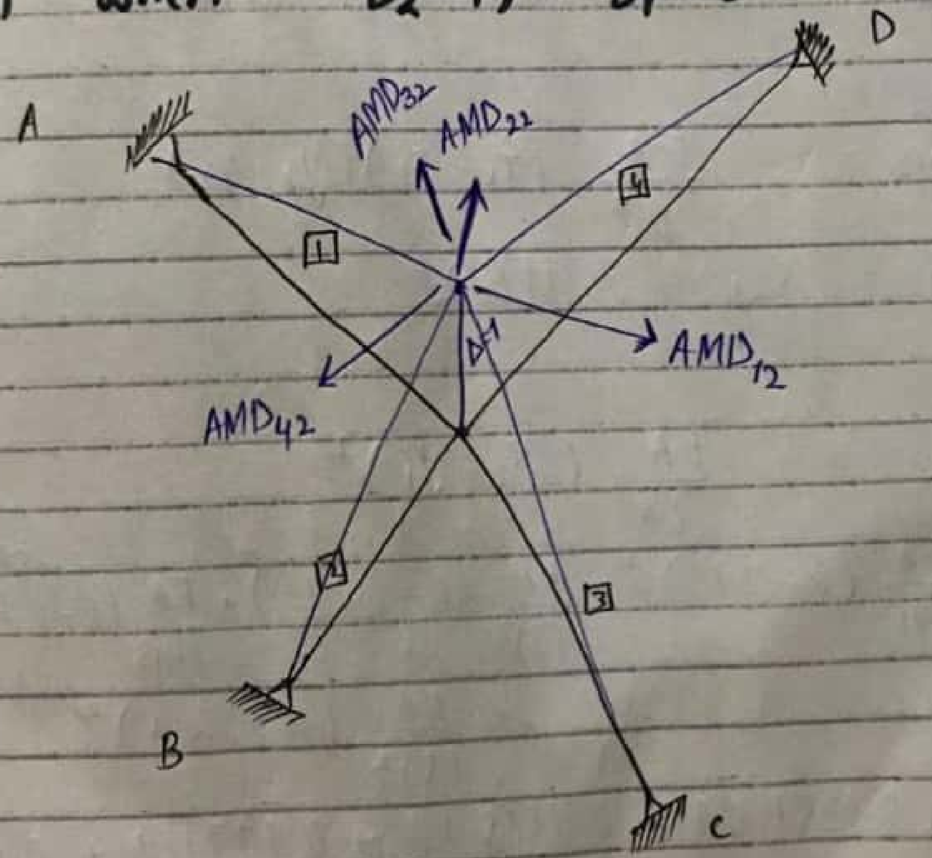
$S_{21} = -0.1251\,EA + 0.1665\,EA - 0.1081\,EA + 0.0866\,EA$

$S_{21} = -0.1251 \times 2000 \times 40 + 0.1665 \times 2000 \times 40 - 0.1081 \times 2000 \times 5$
$+ 0.0866 \times 2000 \times 50$

$S_{21} = -10{,}008 + 13{,}320 - 10{,}810 + 8660$

$S_{21} = 1162$

(ii) When $D_2 = 1$, $D_1 = 0$.



$AMD_{12} = \frac{EA}{l^2}(y_k - y_j) = \frac{EA}{(4)^2}(0 - 2.83) = -0.1769\,E$

$AMD_{12} = -0.1769 \times 2000 \times 40 = -14152.$

$AMD_{22} = \frac{EA}{L^2}(y_k - y_j) = \frac{EA}{(3)^2}(0-(-2.12)) = 0.2356EA$

$AMD_{22} = 0.2356 \times 2000 \times 40 = 18,848$.

$AMD_{32} = \frac{EA}{L^2}(y_k - y_j) = \frac{EA}{(4)^2}(0-(-3.46)) = 0.2162\ EA$

$AMD_{32} = 0.2162 \times 2000 \times 50 = 21620$

$AMD_{42} = \frac{EA}{L^2}(y_k - y_j) = \frac{EA}{(5)^2}(0-2.5) = -0.1\ EA$

$AMD_{42} = -0.1 \times 2000 \times 50 = -10,000$.

Also ;

$S_{12} = \frac{EA}{L^3}(x_k - x_j)(y_k - y_j)$

$S_{12} = S_{21} = 1162$

$S_{22} = \frac{EA}{L^3}(y_k - y_j)^2$

$S_{22} = \frac{EA}{(4)^3}(0-2.83)^2 + \frac{EA}{(3)^3}(0-(-2.12))^2 + \frac{EA}{(4)^3}(0-(-3.46))^2 + \frac{EA}{(5)^3}(0-2.5)^2$

$S_{22} = 0.1251\ EA + 0.1665\ EA + 0.1871EA + 0.05\ EA$

$S_{22} = 0.1251 \times 2000 \times 40 + 0.1665 \times 2000 \times 40 + 0.1871 \times 2000 \times 50 + 0.05 \times 2000 \times 50$

$S_{22} = 47,038$.

$$[AMD] = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \end{bmatrix}$$

$$[AMD] = \begin{bmatrix} 14152 & -14152 \\ 18848 & 18848 \\ -12500 & 21620 \\ -17320 & -10,000 \end{bmatrix}$$

And ;

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$[S] = \begin{bmatrix} 44578 & 1162 \\ 1162 & 47038 \end{bmatrix}$$

Step 03 : Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement

$$[AD] = [S][D]$$

or

$$[D] = [S]^{-1}[AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 44578 & 1162 \\ 1162 & 47038 \end{bmatrix}^{-1} \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.00002245 & -0.000000555 \\ -0.000000555 & 0.00002127 \end{bmatrix} \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.00178 \\ -0.00215 \end{bmatrix}$$

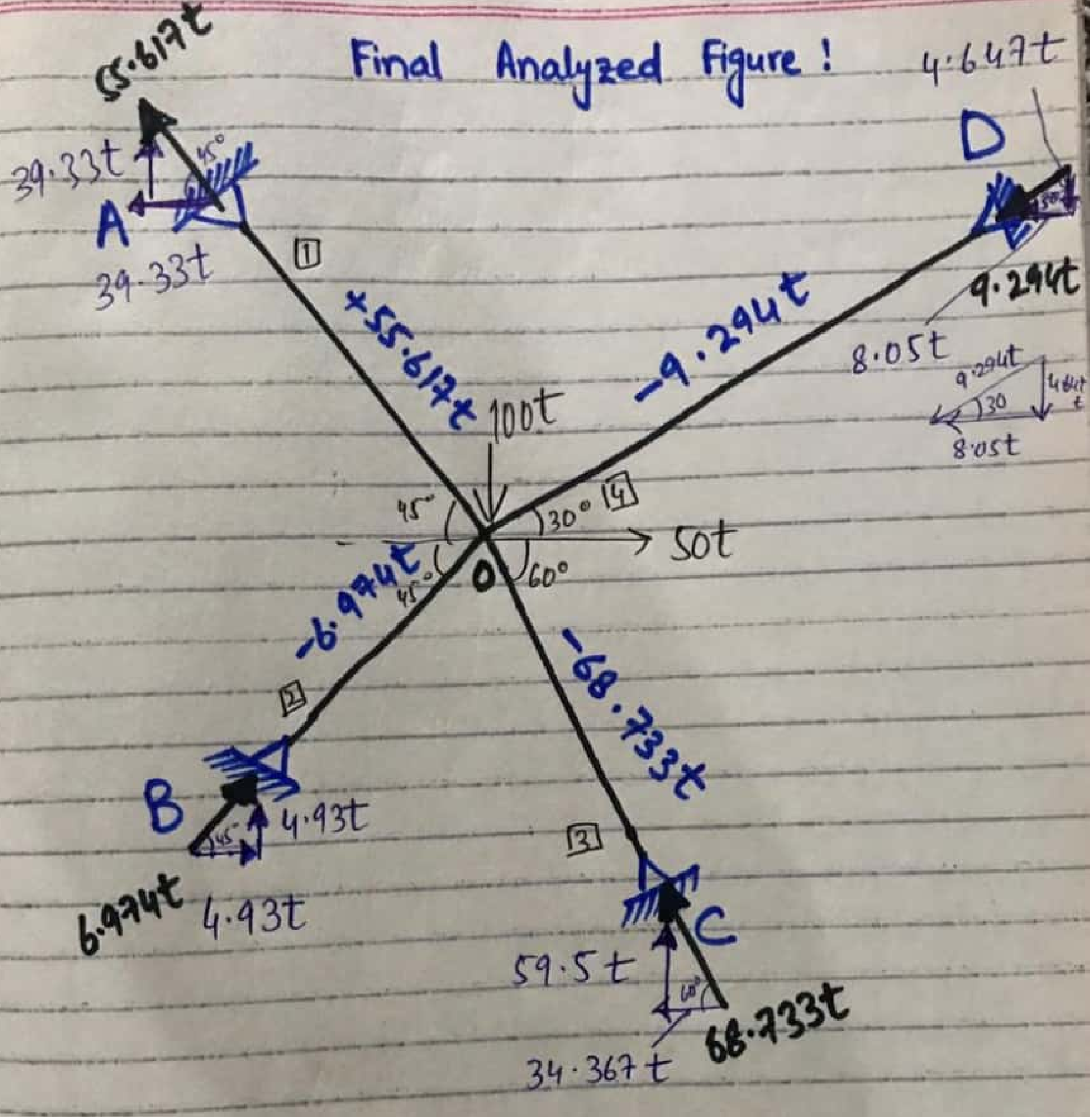
Step 04 : Member Actions ;

$$[AM] = [AMD][D]$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 14152 & -14152 \\ 18848 & 18848 \\ -12500 & 21620 \\ -17300 & -10,000 \end{bmatrix} \begin{bmatrix} 0.00178 \\ -0.00215 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 55.617\ t \\ -6.974\ t \\ -68.733\ t \\ -9.294\ t \end{bmatrix}$$

Final Analyzed Figure !



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THE END OF PROBLEM #02.