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Course :- DSP
Assignment :- Final Exam.
Module :- 6Th.
Date :- 27/06/2020.

x ~~*~~ x

Q1:-
a)-
Ans:- As we are given The condition starting or initial conditions are $y(-1) = y(-2) = 0$.
And considering
$$x(n) = z(n).$$
So:-
$$y(n) = 4y(n-1) + 4y(n-2) = z(n) - z(n-1).$$
The characteristic eq is:-
$$\lambda^2 - 4\lambda + 4 = 0.$$
$$\lambda = 2,2 \quad \text{Hence}$$
$$y_h(n) = c_1 2^n + C_2 n 2^n.$$
The particular solution is
$$y_p(n) = k(-1)^n u(n).$$
substituting The equation we obtain
$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2)$$
$$= (-1)^n u(n) - (-1)^{n-1} u(n-1).$$
For $n=2$, $k(1+4+4) = 2$
$\Rightarrow k = 2/9$ The total sol. is
$$y(n) = \left[c_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n \right] u(n)$$
From initial conditions, we obtain $y(0) = 1$, $y(1) = 2$,
Then;

$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow \{c_1 = \frac{7}{9}\}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2.$$

$$\{c_2 = \frac{1}{3}\}$$

x ——— ★ ——— x

Q1
b).-
Ans:-

Consider the difference equation

$Y(n) - 0.7y(n-1) + 0.14(n-2) = 2x(n) - x(n-2)$

To obtain homogenous equation

$x(n) = 0.$

$Y(n) - 0.7y(n-1) + 0.1y(n-2) = 0.$

Determining the sol.

$Y_h(n) = \lambda^n.$

Substitute the solution

$\lambda^n - 0.7\lambda^{n-1} + 0.1\lambda^{n-2} = 0.$

$\lambda^{n-2}(\lambda^2 - 0.7\lambda + 0.1) = 0.$

$\lambda^2 - 0.7\lambda + 0.1 = 0.$

Therefore the roots are

$(\lambda - 0.5)(\lambda - 0.2) = 0.$

So:-

$(\lambda = \frac{1}{2}, \frac{1}{5})$

Hence general form of Sol.

$Y_h(n) = c_1(\lambda_1)^n + c_2(\lambda_2)^n.$

$Y(n) = c_1(0.2)^n + c_2(0.5)^n \longrightarrow (1)$

As:- $\lambda = \frac{1}{2}, \lambda = \frac{1}{5}$ Then

$Y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1^n}{5}.$

with $x(n) = \delta(n)$ we have

$Y(0) = 2.$

$Y(1) - 0.7y(0) = 0$

$y(1) = 1.4$

Hence $\quad c_1 + c_2 = 2.$

$$\frac{1}{2}c_1 + \frac{1}{5} = 1.4 = \left(\frac{7}{5}\right)$$

Now:-

$$C_1 + \frac{2}{5}C_2 = 14/5$$

These eq - yield

$$c_1 = \frac{10}{3}, \quad c_2 = -\frac{4}{3}.$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n).$$

Now step - response is

$$s(n) = \sum_{k=/0}^{n} h(n-k)$$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

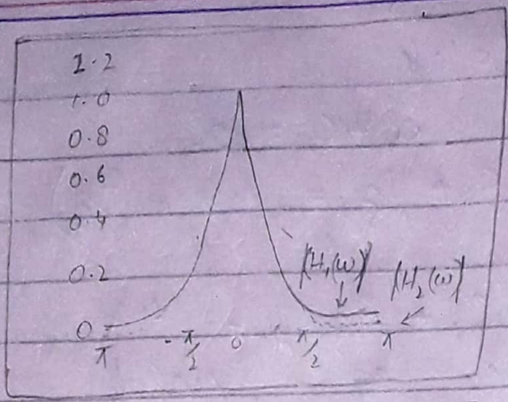
$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= s_n = \frac{10}{3}\left(\frac{1}{2}\right)^n\left(2^{n+1}-1\right)(u(x)) - \frac{1}{3}\left(\frac{1}{3}\right)^n\left(5^{n+1}-1\right)$$

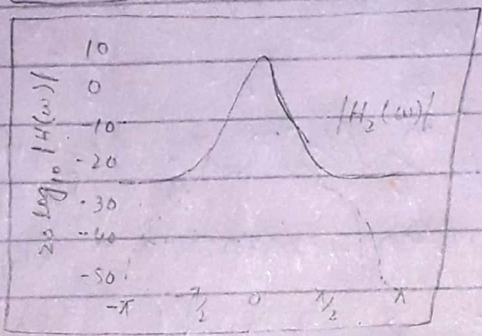
$u(x)$

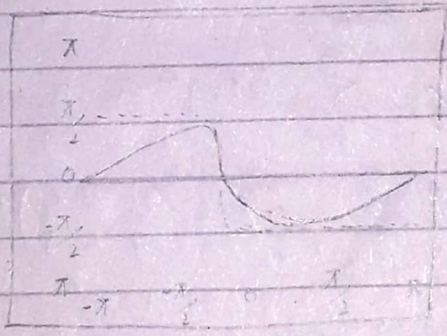
Q3:-

a) Ans:-



$|H(\omega)|$





$\Theta(\omega)$.

$H(0) = 1$.

$$\left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{1}{2},$$

statisfying the conditions.

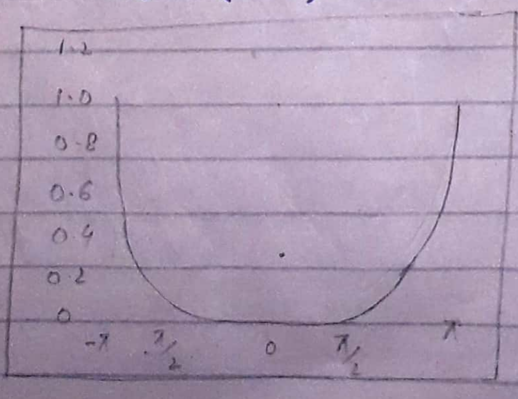
Solution:-

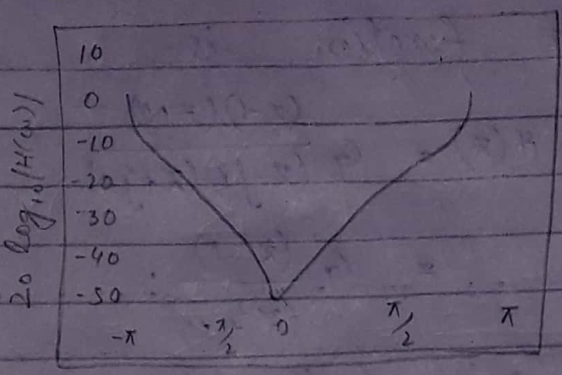
At $\omega = 0$ we have

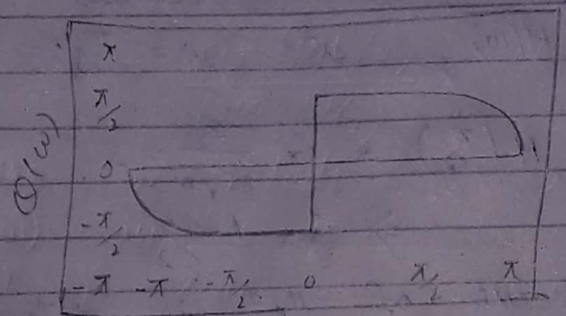
$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

So:-

$$b_0 = (1-p)^2$$







At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4) + jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2} + jp\sqrt{2})^2}$$

Hence :-

$$= \frac{(1-p)^4}{[(1-p/\sqrt{2})^2 + (p^2\sqrt{2})^2}$$

$$= \left(\tfrac{1}{2}\right) \text{ Ans}$$

x ——— * ——— x

Q3:-
b)
Ans:- Clearly the filter must poles at:

$$P_{1,2} = re^{\pm j\pi/2}$$

and zeros at $z=1$ & $z=-1$,
Then:-

System function is

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}.$$

$$= G\frac{(z^2-1)}{(z^2+r^2)}.$$

The gain factor becomes:- by $H(\omega)$ at $\omega = \pi/2$.
Thus

$$H(\pi/2) = G\frac{2}{1-r^2} = 1$$

$$\Rightarrow G = 1-r^2/2$$

The value of "r" is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$
Thus:-

$$|H(4\pi/9)|^2 = \frac{(1-r^2)^2}{4}\frac{2-2\cos(8\pi/9)}{1+r^2+2r^2\cos(8\pi/9)}.$$

$$= 1/2$$

or equivalently:-

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

So:-
The value of $r^2 = 0.7$ satisfies the equation. Therefore the system function for the desired filter is

$$\left\{H(z) = 0.15\frac{1-z^{-2}}{1+0.7z^{-2}}\right\}$$

Q4:-
a)-
Ans:-

P.T.O →

The fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{(\sin \omega/2)} e^{-j\omega(L-1)/2}$$

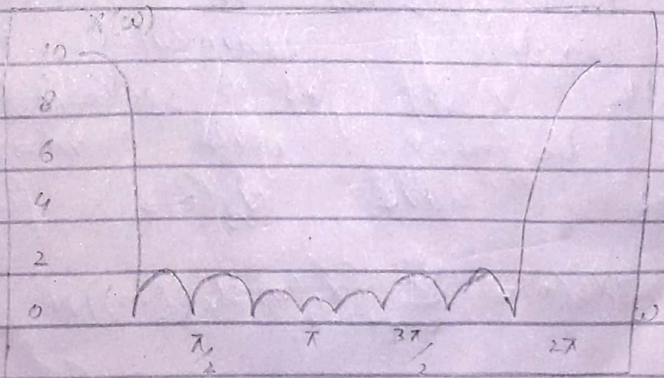
The magnitude & phase of $X(\omega)$ are $L = 10$. The L-point DFT of $x(n)$ is simply $X(\omega)$ at set of N equally spaced frequencies $\omega_k = 2\pi k/N$, $k = 0, 1, \cdots N-1$

Hence

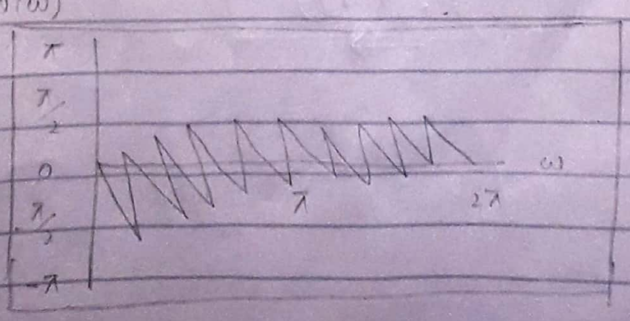
$$X(k) = \frac{1-e^{-j2\pi kL/N}}{1-e^{-j2\pi k/N}}$$

$\therefore k = 0, 1, \cdots\cdots N-1.$

$$= \frac{\sin(\pi kL/N)}{(\sin \pi k/N)} e^{-j\pi k(L-1)N}.$$



Θ(ω)



If "N" is selected such that $N = L$, Then DFT becomes

$$X(k) = \begin{cases} L & k = 0 \\ 0 & k = 1, 2, \ldots L-1 \end{cases}$$

If we wish to have a better picture, we must evaluate $X(\omega)$ at more closely spaced frequencies, say $\omega_k = 2\pi k/N$ where $N > L$.

x ——— x ——— x

Q4:-
b).
Ansn

Each sequence consist of four non-zero points For The purpose of illustrating The operation involved in circular convolution is doubled to graph each sequence as point on a circle.

Now $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$ as specified by begining with $m=0$ we have $x_3(0) =$

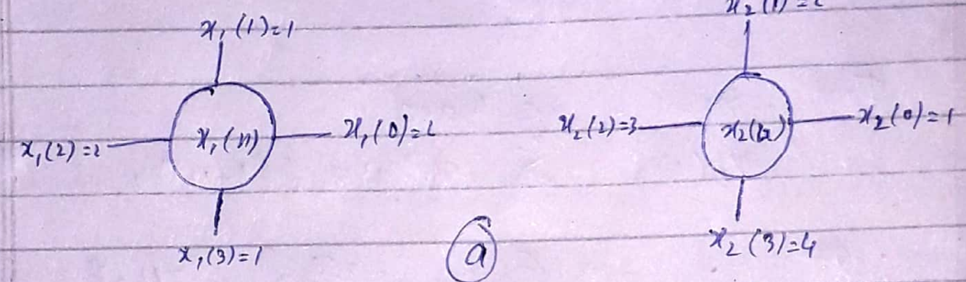
$$x_3(0) = \sum_{m=0}^{3} x_1(n)\, x_2((-n))N.$$

P.T.O
→

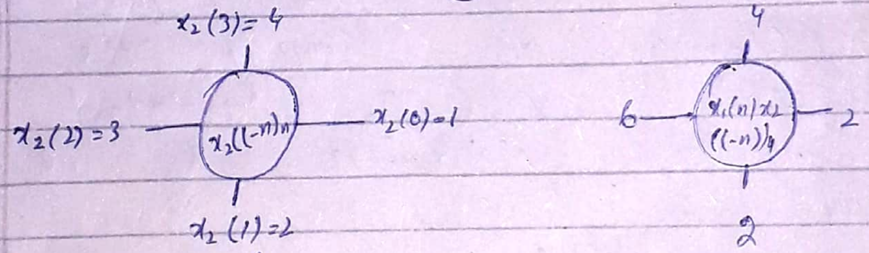
(8)(a)

$x(-n)_4$ is simply the sequence $x_2(n)$ folded & graphed on a circle as illustrated.
Finally, we sum the values in the product sequence to obtain

$$x_3(0) = 14$$

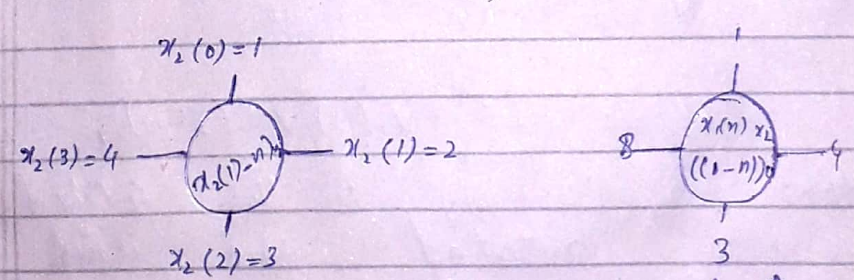


(a)



Folded sequence

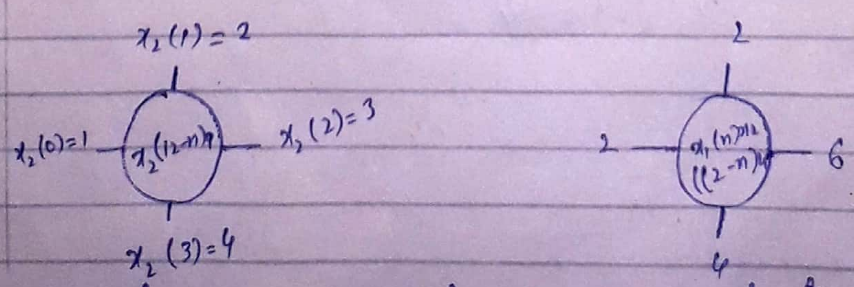
(b)



Folded sequence rooted by unit in time.

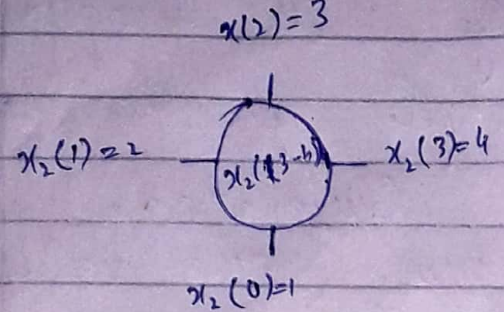
product sequence

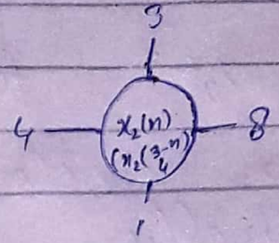
(c)



Folded sequences rotated by unit in time (d)

product square





product sequence

Folded sequence rotated by thee unit in time.

For $m = 1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4.$$

It is easily verified that $x_2((1-n))_4$ is simply the sequence $x_2((-n))_4$ rotated counter clockwise by one unit in time

$$x_3(1) = 6$$

For $m = 2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

Now $x_2((2-n))_4$ is the folded sequence rotated two unit of time in the counter-clockwise direction. The product sequence $x_1(n)\, x_2((2-n))_4$

$$x_3(2) = 14$$

For $m=3$ we have

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$

The folded sequence $x_2((-n))_4$ is now rotated by three unit in time to yield $x_2((3-n))_4$ & the resultant sequence is "xied" by $x_1(n)$ to yield the product sequence $x_3(3) = 16$.

We observe that if the computation above is continued beyond $m=3$, we simply repeat the sequence of 4 values obtained above.

Therefore the circular convolution of the 2 sequences $x_1(n)$ & $x_2(n)$ yield the sequence

$$x_3(n)\ \{\underset{\uparrow}{14}, 16, 14, 16\}.$$

Q2:-
b)-
Ans:- We have :-

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}} dz$$

$$= \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z-a}$$

where "c" is a circle at radius greater than $|a|$. We shall evaluate this integral using $f(z) = z^n$.

We get 2 cases.

1)- If $n \geq 0$, $f(z)$ has only zeros & hence no poles inside c. The only pole inside C is $z = a$. Hence

$$x(n) = f(z_0) = a^n = n \geq 0.$$

2)- If $n < 0$, $f(z) = z^n$ has an n^{th} order pole at $z = 0$ which is also inside C. Thus there are contributions from both poles. For $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j} \oint_c \frac{1}{z(z-a)} dz$$

$$= \frac{d}{dz}\left(\frac{1}{z-a}\right)\Bigg|_{z=0} + \frac{1}{z^2}\Bigg|_{z=a} = 0.$$

By continuing in the same way we can show that $x(n) = 0$ for $n < 0$.

Thus

$$\{x(n) = a^n u(n)\}$$

x ——— * ——— x

Q2:-
1).
Ans:-

The z-transform is

$$X(Z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}.$$

The expression is written as:-

$$X(Z) = \frac{1}{\left(1-\frac{2}{Z}\right)\left(1-\frac{1}{Z}\right)^2}.$$

$$= \frac{1}{\left(\frac{Z-2}{Z}\right)\left(\frac{Z-1}{Z}\right)^2}$$

$$= \frac{1}{\frac{(Z-2)(Z-1)^2}{Z^2}}.$$

$$= \frac{Z^3}{(Z-2)(Z-1)^2}. \rightarrow (1).$$

$X(z)$ is a simple pole at $P_1 = 2$ & a double $P_2 = P_3 = 1$

Then partial expansion:-

$$X(Z) = \frac{Z^3}{(Z-2)(Z-1)^2} = \frac{A_1}{Z-2} + \frac{A_2}{(Z-1)} + \frac{A_3}{(Z-1)^2}.$$

We have to find the co-efficients A_1, A_2, A_3.

Sol:- xing b.s by $(Z-2)$ & evaluate the result $Z=-2$.

$$(Z-2)X(Z) = A_1 + \frac{Z-2}{Z-1}A_2 + \frac{Z-2}{(Z-1)^2}A_3.$$

which are evaluated at $Z=2$

$$A_1 = (z-2)\, X(z) / z \Big|_{z=2.}$$

$(A_1 = 4)$

$$A_2 = A_1 + \frac{z-2}{z-1}$$

$A_2 = -3$

$$A_3 = A_1 + \frac{z-2}{z-1} A_2$$

$= -1$

Hence $\{x(n) = [4(2)^n - 3 - n]\, u(n)\}$ Ans