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Module	Summer
Subject	Linear Algebra
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Q No - 1

(Part(a))

$A(2,-2,1)$ $B(-1,0,3)$ and

$(5,-3,4)$

$\bar{AB} = (-1-2,\ 0+2,\ 3-1)$, $\bar{BC} = (5+1,\ -3+0,\ 1-3)$

$\bar{AB} = (-3, 2, 2)$, $\bar{BC} = (6, -3, 1)$

$$\bar{AB} \times \bar{BC} = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 6 & -3 & 1 \end{vmatrix} \Rightarrow \hat{i}(2+6) - j(-3-12) + k(9-12)$$

$$\Rightarrow 8\hat{i} + 15\hat{j} - 3k$$

The Equation of plane is

$$ax + by + cz = d$$

$a = 8$, $b = 15$ and $c = -3$

$$8x + 15y - 3z = d \rightarrow (A)$$

Ply B pair in (A)

$$8(-1) + 15(0) - 3(3) = d \Rightarrow -8 - 9 = d \Rightarrow -17$$

so

$$\boxed{8x + 15y - 3z = -17}$$

$$\boxed{8x + 15y - 3z + 17 = 0}$$

②

Q No Q1 (Part b)

$x = 2-3t$, $y = 3+t$, $z = 2-4t$

$P = (2, 3, 2)$ } $v = (-3, 1, -4)$

$A = (x, y, z)$

$\bar{PA} = (x-2, y-3, z-2)$

so

$v \cdot \bar{PA} = -3(x-2) + 1(y-3) - 4(z-2) = 0$

$\Rightarrow -3x + 6 + y - 3 - 4z + 8 = 0$

$-3x + y - 4z + 11 = 0$

$$\boxed{-3x + y - 4z = -11}$$

Q No 2 :- $L(x,y) = (x+1, y, x+y)$

illustrate That L is The linear transformation.

$L(x,y) = (x+1, y, x+y)$

let $u = (x_1, y_1)$ $v = (x_2, y_2)$

$u+v = (x_1, x_2) + (y_1, y_2)$

$u+v = (x_1 + x_2, y_1 + y_2)$

$L(u+v) = L(x_1 + x_2, y_1 + y_2)$

$L(u+v) = (x_1 + x_2 + 1, y_1 + y_2, x_1 + x_2 + y_1 + y_2) \rightarrow (1)$

Given That $u = (x_1, y_1)$

$L(u) = L(x_1, y_1) = (x_1 + 1, y_1, x_1 + y_1)$

$L(v) = L(x_2, y_2) = (x_2 + 1, y_2, x_2 + y_2)$

$L(u) + L(v) = (x_1 + x_2 + 2, y_1 + y_2, x_1 + x_2 + y_1 + y_2) \rightarrow (ii)$

sin $1 \neq 2$

so Not L.T.

Q NO 3

using The matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

Then inteupart to decode The massage:

77, 54, 38, 71, 49, 29, 68, 51, 33, 76, 48, 40, 86, 53, 52.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

To decode The above message we have to break it into five vectors in R^3:

$$\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix}$$

so solve The equation.

$$L(x_1) = \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = Ax_1$$

for x_1 since A is non singular.

$$x_1 = A^{-1}\begin{bmatrix}77\\54\\38\end{bmatrix} = \begin{bmatrix}0 & 1 & -1\\2 & -2 & -1\\-1 & 1 & 1\end{bmatrix}\begin{bmatrix}77\\54\\38\end{bmatrix} = \begin{bmatrix}16\\8\\15\end{bmatrix}$$

similerly

$$x^2 = A^{-1}\begin{bmatrix}71\\49\\29\end{bmatrix} = \begin{bmatrix}20\\15\\7\end{bmatrix}$$

$$x3 = \begin{bmatrix}68\\51\\33\end{bmatrix} = \begin{bmatrix}18\\1\\16\end{bmatrix}$$

$$x4 = \begin{pmatrix}76\\48\\40\end{pmatrix} = \begin{pmatrix}8\\16\\12\end{pmatrix}$$

$$x_5 = \begin{pmatrix}86\\53\\52\end{pmatrix} = \begin{pmatrix}1\\14\\19\end{pmatrix}$$

using our correspondence between letters and numbers, we received The following message

PHOTOGRAPH PLANS.

Answer.

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Q No 4 :-

Point $(-1, 3, 2)$ and vector

$n = (0, 1, -3)$

$A = (x, y, z)$

$n = 0\hat{i} + 1\hat{j} - 3\hat{k} = (\overset{a}{0}, \overset{b}{1}, \overset{c}{-3})$

$\text{Point} = (-1, 3, 2) = (x_0, y_0, z_0)$

$= a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$0(x+1) + 1(y-3)(z-2) = 0$

$y - 3 - 3z + 6 = 0$

$y - 3z + 3 = 0$

$\boxed{y - 3z = -3}$

Q no 5 :-

Find Eigen values of matrix & Eigen vectors of matrix

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

solution :-

we know that $Ax = \lambda x$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

Then

$$x_1 + x_2 = \lambda x_1 \rightarrow (i)$$

$$-2x_1 + 4x_2 = \lambda x_2 \rightarrow (ii)$$

so

$$x_1 - \lambda x_1 + x_2 = 0$$

$$= (1-\lambda) x_1 + x_2 = 0$$

& $-2x_1 + 4x_2 - \lambda x_2 = 0$

$$= -2x_1 + (4-\lambda) x_2 = 0$$

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$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Characteristic equation

$$\begin{vmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(4-\lambda)+2 = 0$$

$$4-\lambda-4\lambda+\lambda^2+2 = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda(\lambda-3) - 2(\lambda-3) = 0$$

$$(\lambda-3)(\lambda-2) = 0$$

$$\lambda-3=0 \quad \lambda-2=0$$

$$\lambda = 3 \quad \lambda = 2$$

are eigen value.

Now find eigen vectors of $\lambda_1 = 3$
put in ① & 2

Then $x_1 + x_2 = 3x_1 \rightarrow$ ①

$= -2x_1 + x_2 = 0$

$\Rightarrow 2x_1 - x_2 = 0$

$\Rightarrow -2x_1 + 4x_2 = 3x_2 \rightarrow$ (ii)

$\Rightarrow -2x_1 + x_2 = 0$

$\Rightarrow 2x_1 - x_2 = 0.$

$x_1 = \frac{1}{2} x_2$

let $x_2 = \gamma$

where $\gamma \neq 0$

so $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\gamma \\ \gamma \end{bmatrix}$

eigen vector for $\lambda_2 = 2$ put

in i Eq, 2

$x_1 + x_2 = 2x_1 \rightarrow$ (i)

$-2x_1 + 4x_2 = 2x_2 \rightarrow$ (ii)

$= -x_1 + x_2 = 0 \rightarrow$ (i).

$\Rightarrow x_1 - x_2 = 0$

$= x_1 = x_2$

$= -2x_1 + 4x_2 = 2x_2 \rightarrow$ (ii)

$= -2x_1 + 2x_2 = 0.$

$\Rightarrow x_1 - x_2 = 0$

$x_1 = x_2$

$x_1 = \gamma$ then $x_2 = \gamma$

so $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \gamma \\ \gamma \end{bmatrix}.$