

ID# 12724

Final Term

Date = 30/6/2020

Q3 :- Given data :-

$$2x + 2y + 4z = 18$$

$$x + 3y + 2z = 13$$

$$3x + y - 3z = 14$$

Sol:-

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & -3 & 14 \end{array} \right] R_1 = \frac{1}{2} R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & -3 & 14 \end{array} \right] \begin{array}{l} R_2 = R_2 - R_1 \\ R_3 = 3R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 4 \\ 0 & -2 & -9 & -13 \end{array} \right] R_2 = \frac{1}{2} R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -9 & -13 \end{array} \right] R_3 + 2R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -9 & -9 \end{array} \right] \begin{array}{l} R_1 = R_1 - R_2 \\ R_3 = \frac{-1}{9} R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right] R_1 = 4R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

=>

$$x = 3$$

$$y = 2$$

$$z = 1$$

ANS

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Farhan Shah

Q.06 \Rightarrow Reduce the matrix

30/6/2020

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Sol.:-

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix} \begin{array}{l} R_2 - 3R_1 \\ R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3 \end{bmatrix} \begin{array}{l} \\ -2R_3 - R_2 \end{array}$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} C_2 - 3C_1 \\ C_1 - 4C_1 \\ C_4 - 3C_1 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \text{Swip } C_4 \text{ with } C_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \text{Divid } C_2 \text{ by } -6 \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{Ans}$$

Rank = 2

Question No 2

Find the Inverse of $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4 \\ 5 & -2 & 7 \end{bmatrix}$

by adjoint Method.

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

by this formula we find $|A|$.

$$\begin{aligned} |A| &= 3 \begin{vmatrix} -1 & 4 \\ -2 & 7 \end{vmatrix} - 4 \begin{vmatrix} 2 & 4 \\ 5 & 7 \end{vmatrix} + 5 \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} \\ &= 3(-7+8) - 4(14-20) + 5(-4+5) \\ &= 3 + 24 + 5 \end{aligned}$$

$$|A| = 32$$

Now adj A by Co-factor Method

Directly by Rough work

$$\text{adj } A = \begin{bmatrix} 1 & 6 & 1 \\ -38 & -4 & 26 \\ 21 & -2 & -9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -4 \\ 0 & 1 & -2 \end{bmatrix}$$

$R_2/2$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$R_3 - R_2$

Rank of $A = 3$

Now Rank of AB

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 2 & 8 & 8 \\ 5 & 0 & -5 & 10 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} -1 & -1 & 0 & 0 \\ 0 & 2 & 8 & 8 \\ 0 & 5 & -10 & 10 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 2 & 8 & 8 \\ 0 & 0 & -5 & 10 \end{array} \right]$$

So the Rank of $AB = 3$
So the Question is Consistent.

Question No 1

$$\begin{aligned}x_1 - 3x_2 + x_3 &= 0 \\ -2x_2 - 8x_3 &= 0 \\ 5x_1 - 5x_3 &= 0\end{aligned}$$

Sol

To Find A

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & 5 \end{bmatrix} \quad R_3 + (-5R_1)$$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 2 & -8 \\ 0 & 1 & -2 \end{bmatrix} \quad R_3 / 5$$

adj a

$$\begin{bmatrix} 1 & -38 & 21 \\ 6 & -4 & -2 \\ 1 & 26 & -9 \end{bmatrix}$$

by formula

$$A^{-1} = \frac{1}{32} \begin{bmatrix} 1 & -38 & 21 \\ 6 & -4 & -2 \\ 1 & 26 & -9 \end{bmatrix}$$

Ans