Q1

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -7 & 8 \end{bmatrix}$$

Solution:-

$$A - m = 0$$

$$\begin{vmatrix} (-\lambda) & 1 & 0 \\ 0 & (-\lambda & 1 \\ 4 & -17 & (8-\lambda) \end{vmatrix} = 0$$

$\therefore (-\lambda)((-\lambda) \times (8-\lambda) - 1 \times (-17)) - 1(0 \times (8-\lambda) - 1 \times 4) + 0(0 \times (-17) - (-\lambda) \times 4 = 0$

$\therefore (-\lambda((-8\lambda + \lambda^2) - (-17)) - 1(0-4) + 0(0-(-4\lambda) = 0$

$\therefore (-\lambda)(17 - 8\lambda + \lambda^3) - 1(-4 + 0(4\lambda) = 0$

$\therefore (-17\lambda + 8\lambda^2 - \lambda^3) - (-4) + 0 = 0$

$\therefore (-\lambda^3 + 8\lambda^2 - 17\lambda + 4) = 0$

$\therefore -(\lambda - 4)(\lambda - 0.26794919)(\lambda - 3.73205081) = 0$

$\therefore (\lambda - 4) = 0$ or $(\lambda - 0.26794919) =$
0 or $(\lambda - 3.73205081) = 0$

The eigenvalues of the Matrix A are given by $\lambda = 0.26794919$
$3.73205081, 4$

Ans

Q2

$$\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

Solution:-

$$\text{Here } A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

find eigenvalues of the matrix A

$A - \lambda I = 0$

$$\begin{vmatrix} (-\lambda) & 0 & -2 \\ 1 & (2-\lambda) & 1 \\ 1 & 0 & (3-\lambda) \end{vmatrix} = 0$$

$\therefore (-\lambda)((2-\lambda)\times(3-\lambda)-1\times 0)-0(1\times(3-\lambda)-1\times 1)+(-2\times 1\times 0-(2-\lambda)\times 1)=0$

$\therefore (-\lambda)((6-5\lambda^2)-0)-0((3-\lambda)-1)-(2-\lambda))=0$

$\therefore (-\lambda)(6-5\lambda+\lambda^2)-0(2-\lambda)-2(-2+\lambda)=0$

$\therefore (-6\lambda+5\lambda^2-\lambda^3)-0-(-4+2\lambda)=0$

$\therefore (-\lambda^3+5\lambda^2-8\lambda+4)=0$

$\therefore (\lambda-1)(\lambda-2)(\lambda-2)=0$

$\therefore (\lambda-1)=0$ or $(\lambda-2=0$ or $(\lambda-2)=0$

The eigenvalues of the matrix A are given by $\lambda = 1, 2$

2 Eigenvalues for $\lambda = 2$

The eigenvalues compose the colum of matrix P

$$P = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

The diagonal matrix D is composed of the eigenvalues

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Now find P

$$P = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= 2\times\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 0\times\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - 1\times\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= -2\times(1\times1 - 0\times0) + 0\times(1\times1 - 0\times1)$$
$$-1\times(1\times0 - 1\times1)$$

$$= -2\times(1*0) + 0\times(1+0) - 1\times(0-1)$$

$$= -2\times(1) + 0\times(1) - 1\times(-1)$$

$$= -2 + 0 + 1$$

$$= -1$$

Ans

Q3

$$V1 = (1, 2, 3)$$
$$V2 = (5, 6, -1)$$
$$V3 = (3, 2, 1)$$

Solution:

Here $A = (1, -2, 3)$
$B = (5, 6 -1)$
$C = (3, 2, 1)$

The vectors ABC are linearly dependent if their determinant is zero i.e $D = 0$

$$D = \begin{bmatrix} 1 & -2 & 3 \\ 5 & 6 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= 1 \times \begin{vmatrix} 6 & -1 \\ 2 & 1 \end{vmatrix} + 2 \times \begin{vmatrix} 5 & -1 \\ 3 & 1 \end{vmatrix} + 3 \times \begin{vmatrix} 5 & 6 \\ 3 & 2 \end{vmatrix}$$

$= 1 \times (6 \times 1 - (-1) \times 2) + 2 \times (5 \times 1 - (-1) \times 3) + 3 \times (5 \times 2 - 6 \times 3)$

$= 1 \times (6+2) + 2 \times (5+3) + 3 \times 10 - 18)$

$= 1 \times (8) + 2 \times (8) + 3 \times (-8)$

$= 8 + 16 - 24$

$= 0$

Since $D = 0$ So vectors A, B, C are linearly dependent.

Ans

Q4

Definition:- A vector space is a set V on which two operations + and are defind. called vector addition and scalar multiplication The

operation + (vector addition)
must satisfy the following conditions:
closure: if u and v are any vectors in V, then the sum $u+v$ belongs to V

(1) commutative law: for all vectors u and v in V = $u+v=v+u$

(2) Associative law: for all vectors u = v = w in V = $u+(v+w)=(u+v)+w$

(3) Additive identity: The set V contains an addititive identity element = denoted by 0 = such that for any vector v in V = $0+v=v$ and $v+0=v$.

(4) Additive inverses: for each vector v in V = the equations $v+x=0$ and $x+v=0$ have a solution x

in V. Called an addittive inverse of V. and denoted by $-V$.

The operation. (Scalar multiplication) is defind between real numbers (or scalars) and vectors, and must satisfy the following conditions:

Closure: if V in any vector in V. and C is any real number. than the Product $C \cdot V$ belongs to V

(5) Distributive law: for all real number C and all vectors U. V in V. $C \cdot (U+V) = C \cdot U + C \cdot V$

(6) Distributive law: for all real number C. d and all vectors V in V. $(C+d) \cdot V = C \cdot V + d \cdot V$

(7) Associative law: for all real number c, d and all vectors V in V: $c\cdot(d\cdot V) = (cd)\cdot V$

(8) Unitary law: for all vectors V in V: $1\cdot V = V$