7859

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Subject = Differential eq

semestr = 07

7859 ①

# "Question No 1"

①. $\frac{dy}{dt}$ $e^{y-t}$ $\sec(y)(1+t^2)$ $y(0)=0$

Solution:-

$y(0)=0$ so $x=0$ and $y=0$

$dy = e^{y}.e^{-t} \sec(y)(1+t^2)\,dt$

$\frac{1}{e^{y}.\sec(y)}dy = (1+t^2)e^{-t}\,dt$

As ~~$\cos(y)e^{-t}dt$~~

$\cos(y) = \frac{1}{\sec(y)}$

$= \int e^{-y}\cos y\,dy = \int(1+t^2)e^{-t}\,dt$

using integration by Parts

$e^{-y}\int\cos y\,dx - \int\left(\int\cos y . \frac{d}{dy}e^{-y}\right. =$

$(1+t^2)\int e^{-t} - \int\left(\int e^{-t}.\frac{d}{dt}(1+t^2)\right) \Rightarrow$ ⓘ

L. H. S

$$= e^{-y} \int \cos y \, dx - \int \left( \int \cos y \cdot \frac{d}{dy} e^{-y} \right)$$

$$= e^{-y} \sin y - \int ( \sin y \cdot e^{-y} (-1))$$

$$= e^{-y} \sin y + \int ( \sin y \cdot e^{-y} )$$

$$= e^{-y} \sin y + \int ( e^{-y} \sin y )$$

again using integration by Parts

$$= e^{-y} \sin y + e^{-y} (-\cos y) - \int \left( \int \sin y \frac{d}{dy} e^{-y} \right)$$

$$= e^{-y} \sin y + e^{-y} (-\cos y) - \int \left( -\cos y \frac{e^{-y}}{-1} \right)$$

$$= e^{-y} \sin y - e^{-y} \cos y - \int ( \cos y \, e^{-y} )$$

since $\int ( \cos y \, e^{-y} ) =$ L.H.S

7859

Since it is again same to the first one so L.H.S will beome

$$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S$$

$$2L.H.S = e^{-y}(\sin y - \cos y)$$

$$L.H.S = \frac{e^{-y}(\sin y - \cos y}{2}$$

Now taking R.H.S

$$\int (1+t^2)e^{-t}\,dt$$

$$(1+t^2)\int e^{-t} - \int\left(\int e^{-t} \cdot \frac{d}{dt}(1+t^2)\right)$$

$$-(1+t^2)e^{-t} - \int(-e^{-t}(2t))$$

$$-(1+t^2)e^{-t} + \int((2t)e^{-t})$$

using integration by part

$$-(1+t^2)e^{-1} + (2t\int e^{-t} - \int\left(\int e^{-t}\frac{d}{dt}2t\right))$$

$-(1+t^2)\,e^{-t} + (-2t\,e^{-t} - \int c\,-e^{-t}\,2)$

$-(1+t^2)\,e^{-t}\,(-2t\,e^{-t} + \int c\,2e^{-t}))$

$-(1+t^2)\,e^{-t} + (-2te^{-t} - 2e^{-t}) + c$

$\Rightarrow -(1+t^2)e^{-t} - 2te^{-t} - 2e^{-t} + c$

$= -e^{-t} - e^{-t}t^2 - 2te^{-t} + c$

$\Rightarrow -(t^2+2t+3)e^{-t} + c = R.H.S$

Now take L.H.S = R.H.S

$$\frac{e^{-y}(\sin y - \cos y)}{2} = -(t^2+2t+3)e^{-t} + c$$

we know that

$x=0$ and $y=0$

Put it above

$\Rightarrow \frac{1}{2}(0-1) = -3 + c$

2859 (5)

$c = 5/2$

Put value of $c$

$$\boxed{\frac{e^{-y}}{2}(\sin y - \cos y) = -(t^2 + 2t + 3)e^{-t} + 5/2}$$

## "Question No 2"

$$Q_2 = \left(\sqrt{x+y} + \sqrt{x-y}\right)dx - \left(\sqrt{x+y} - \sqrt{x-y}\right)dy = 0$$

$$\frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} \longrightarrow (1)$$

This is homogeneous Diffrenetial equation in $x$ and $y$ to solve this put

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Thus equation one become

$$v + x\frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1+\cancel{v} + 1-\cancel{v} + 2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}\left(1+\sqrt{1-v^2}\right)}{\cancel{2}v}$$

$$v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \frac{dx}{x}$$

taking integral on B./s

$$\int \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

Put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \frac{1}{2}(1-v^2)^{1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int -\frac{dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln(1+\sqrt{1-v^2}) = \ln cx$$

$$\cancel{\ln}(1+\sqrt{1-v^2}) = \cancel{\ln}(cx)^{-1}$$

$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$1+\sqrt{1-\frac{y^2}{x}} = \frac{1}{cx}$$

$$1 + \frac{\sqrt{x^2 - y^2}}{x_2} = \frac{1}{cx}$$

$$x + \sqrt{x^2 - y^2} = \frac{1}{c}$$

$$x + \sqrt{x^2 - y^2} = C_1 \quad \because \frac{1}{c} = C_1$$

which is reaurired solution.

# "Question no 3"

$Q = (D^4 + D^2)\,y = 3x^2 + 4\sin x - 2\cos x$

Solution:

$(D^4 + D^2)\,y = 3x^2 + 4\sin x - 2\cos x$

$\Rightarrow \quad f(D)\cdot y = f(x)$

As it is non-homogenus linear eq

$y = y_c + y_p \quad - (i)$

Complemetry solution $y_c$

$D^4 + D^2 = 0 \quad \Rightarrow \quad D^2(D^2+1) = 0$

Either $D^2 = 0 \quad \Rightarrow \quad \boxed{D = 0}$

$D^2 + 1 = 0 \quad \Rightarrow \quad D^2 = -1$

$D = \sqrt{-1} = \boxed{D = i}$

Roots are equal and complex

$y_c = c_1 e^{0x} + e^{0x}(c_2\cos x + c_3 \sin x)$

$y_c = c_1 + c_2 \cos x + c_3 \sin x$

$$y_p = \frac{1}{f(D)} f(x)$$

$$y_p = \frac{1}{D^4 + D^2} (3x^2 + 4 \sin x - 2 \cos x)$$

$$= \frac{3x^2}{D^4 + D^2} + \frac{4 \sin x}{D^4 + D^2} - \frac{2 \cos x}{D^4 + D^2}$$

$f(D) = D^4 + D^2$

at $D = 0 \Rightarrow f(D) = 0$

so $f'(D) = 4D^3 + 2D$

Now also for $D = 0 \Rightarrow f'(D) = 0$

again differenting

$f''(D) = 12D + 2$

So for $D = 0$

$f''(D) = 12(0) + 2 = 2$

so replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

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7859

$$\rightarrow \quad y_p = \frac{x^2\, 3x^2}{12D + 8} + \frac{x^2}{12D+8} \cdot 4\sin x - \frac{x^2}{12D+8} \cdot x\cos x$$

Putting $D = 0$ in all

$$y_p = \frac{x^2\, 3x^2}{12(0)+8} + \frac{x^2 \cdot 4\sin x}{12(0)+8} - \frac{2x^2 \cos x}{12(0)+8}$$

$$y_p = \frac{3x^4}{8} + \frac{4x^2 \sin x}{8} - \frac{2x^2 \cos x}{8}$$

$$= \frac{3}{8} x^4 + 2x^2 \sin x - x^2 \cos x$$

so Putting in equation (i)

$$y = C_1 + C_2 \cos + C_3 \cos x + \tfrac{3}{2} x^4 + 2x^2 \sin x - x^2 \cos x$$

$$\boxed{y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \tfrac{3}{2} x^4}$$

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