



CALCULAS AND ANALYTICAL GEOMETRY

**NAME: MUZAMIL AHMED KHAN
STUDENT ID: 16941
DEPARTMENT: BS-SE (1ST-SEMESTER)**



COURSE: CALCULUS AND ANALYTICAL GEOMETRY
PROGRAM: BS (SE, CS)
INSTRUCTOR: MUHAMMAD ABRAR KHAN
EXAMINATION: MIDTERM ASSIGNMENT

Note: Attempt all questions. Use examples and diagrams where necessary.

Ans 1a)

①

$$\text{(a) } \frac{d}{dx} \left(\frac{2x^3 - 3x^2 + 5}{x^2 + 1} \right)$$

using Quotient rule, we have

$$= \frac{(x^2 + 1) \frac{d}{dx} (2x^3 - 3x^2 + 5) - (2x^3 - 3x^2 + 5) \frac{d}{dx} (x^2 + 1)}{(x^2 + 1)^2}$$

$$= \frac{(x^2 + 1)(6x^2 - 6x) - (2x^3 - 3x^2 + 5)(2x)}{(x^2 + 1)^2}$$

$$= \frac{6x(x^2 + 1)(x - 1) - (2x^3 - 3x^2 + 5)2x}{(x^2 + 1)^2}$$

$$= \frac{2x [3x(x^2 + 1)(x - 1) - (2x^3 - 3x^2 + 5)]}{(x^2 + 1)^2}$$

(b)

$$\frac{x^2 - 1}{(x^2 + 1)^2}$$

Again use Quotient rule,

$$= \frac{(x^2 - 1) \frac{d}{dx} (x^2 + 1)^2 - (x^2 + 1)^2 \frac{d}{dx} (x^2 - 1)}{(x^2 - 1)^2}$$

$$= \frac{(x^2 - 1) 2(x^2 + 1)(2x) - (x^2 + 1)^2 (2x)}{(x^2 - 1)^2}$$

$$= \frac{2x(x^2 + 1)^2 [2(x^2 - 1) - (x^2 + 1)]}{(x^2 - 1)^2}$$

$$= \frac{2x(x^2 + 1)^2 [2x^2 - 2 - x^2 - 1]}{(x^2 - 1)^2}$$

$$= \frac{2x(x^2 + 1)^2 (x^2 - 3)}{(x^2 - 1)^2}$$

Ans

$$2(a) \quad y = (1+2\sqrt{x})^3 \cdot x^{2/3}$$

$$\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$$

$$\text{let } u = x$$

$$\Rightarrow y = (1+2\sqrt{u})^3 u^{2/3}$$

$$\frac{dy}{du} = (1+2\sqrt{u})^2 \cdot \frac{2}{3} u^{-1/3} + u^{2/3} \cdot 3(1+2\sqrt{u})^2 \cdot \frac{1}{2\sqrt{u}}$$

$$\frac{dy}{du} = (1+2\sqrt{u})^2 \left[\frac{2}{3} u^{-1/3} + 3 u^{1/6} (1+2\sqrt{u})^2 \right]$$

$$\frac{du}{dx} = 1$$

use chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= (1+2\sqrt{u})^2 \left[\frac{2}{3} u^{-1/3} + 3 u^{1/6} \right] \times (1)$$

$$\Rightarrow \frac{dy}{dx} = (1+2\sqrt{u})^2 \left[\frac{2}{3} (1+2\sqrt{u}) u^{-1/3} + 3 u^{1/6} \right]$$

$$\text{use } u = x$$

$$\Rightarrow \frac{dy}{dx} = (1+2\sqrt{x})^2 \left[\frac{2}{3} (1+2\sqrt{x}) x^{-1/3} + 3 x^{1/6} \right]$$

2(b)

$$y = \sqrt{\frac{1-x}{1+x}}$$

let $u = \frac{1-x}{1+x}$ — (1)

$$y = \sqrt{u}$$
 (2)

$$1) \Rightarrow \frac{dy}{dx} = \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{-1-x-1+x}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{-2}{(1+x)^2}$$

$$2) \Rightarrow \frac{dy}{du} = \frac{1}{2\sqrt{u}}$$

use chain rule.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{u}} \times \frac{(-2)}{(1+x)^2}$$

$$\text{use } u = \frac{1-x}{1+x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \sqrt{\frac{1+x}{1-x}} \times \frac{(-2)}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} (1+x)^{3/2}$$

$$(1+x)^{2-1/2}$$

$$3) a \quad \int \frac{1}{\sqrt{x^3}} dx$$

$$= \int \frac{1}{(x^3)^{1/2}} dx$$

$$= \int \frac{1}{x^{3/2}} dx$$

$$= \int x^{-3/2} dx$$

use formula $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$$= \frac{x^{-3/2+1}}{-3/2+1} + C$$

$$= \frac{x^{-1/2}}{-1/2} + C$$

$$\int \frac{1}{\sqrt{x^3}} dx = \frac{-2}{\sqrt{x}} + C$$

$$3) b) \int \frac{1}{(6x+7)^6} dx$$

$$= \int (6x+7)^{-6} dx$$

use again above mention formula

$$= \frac{1}{6} \int (6x+7)^{-6} (6) dx \rightarrow \text{check } \left(\frac{1}{6} \times 6 = 1\right)$$

$$= \frac{1}{6} \frac{(6x+7)^{-6+1}}{-6+1} + C$$

$$\int \frac{1}{(6x+7)^6} dx = \frac{-1}{30(6x+7)^5} + C$$