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Section

'C'

Department

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Subject

Differential
Equation.

Submitted To

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QNO # 1



Solution:-

$$f(t) = 1 + t \quad -\pi \leq t \leq \pi$$

Here we use formula :-

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt \rightarrow (i)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t) dt$$

$$a_0 = \frac{1}{2\pi} \left[t + \frac{t^2}{2} \right]_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2} \right) \right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2} \right)$$

$$a_0 = \frac{1}{2\pi} (2\pi + \pi^2)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t) (\cos nt) dt$$

$$a_n = \frac{1}{\pi} \left((1+t) \frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{\sin nt}{n} \frac{d}{dt} (1+t) \right)$$

$$a_n = \frac{1}{\pi} \left((1+t) \frac{\sin nt}{n} - \frac{\cos nt}{n^2} \Big|_{-\pi}^{\pi} \right)$$

$$a_n = \frac{1}{n^2\pi} (\cos n\pi - \cos n(-n))$$

$$a_n = \frac{-1}{n^2\pi} (-1 - (-1))$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t) \sin t \, dt$$

$$b_n = \frac{1}{\pi} \left((1+t) \int_{-\pi}^{\pi} \sin nt - \int_{-\pi}^{\pi} \left(\int \sin nt \frac{d}{dt} (1+t) dt \right) \right)$$

$$b_n = \frac{1}{\pi} \left(\frac{(1+t)(-\cos nt)}{n} \right) \Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \left(\frac{-\cos nt}{n} (1) \right)$$

$$b_n = \frac{1}{\pi} \left(\frac{-(1+t)(\cos nt)}{n} \right) \Big|_{-\pi}^{\pi} + \left(\frac{\sin nt}{n^2} \right) \Big|_{-\pi}^{\pi}$$

$$b_n = \frac{-1}{n\pi} \left((1+\pi)(\cos n\pi) - (1+(-\pi))(\cos n\pi) \right)$$

$$b_n = \frac{-1}{n\pi} \left(\cos n\pi + \pi \cos n\pi - \cos n\pi + \pi \cos n\pi \right)$$

$$b_n = \frac{-1}{n\pi} \left(2\pi \cos n\pi \right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$.

$$b_n = \frac{2}{n} (-1)^{n+1}$$

So putting the value of a_0, a_n
and b_n in equation (i).

$$f(x) = \frac{1}{2\pi} (2\pi + \pi) + 0 + \sum_{n=1}^{\infty} \frac{2}{n} (-1)^{n+1} \sin t.$$



QNO # 02*

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen value = ?

Solution:-

Step = 01

We have ;

$$(A - \lambda I) = 0$$

A = Given Matrix

Step = 02

We have ; The characteristics equation is given by.

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix}$$

Step # 03

$$\lambda^3 - \left| \begin{smallmatrix} \text{sum of} \\ \text{Diagonal element} \end{smallmatrix} \right| \lambda^2 + \left| \begin{smallmatrix} \text{sum of} \\ \text{Diagonal} \\ \text{minors} \end{smallmatrix} \right| \lambda - |A| = 0$$

(B)

$$\text{Sum of Diagonal element} = 1 + 1 + 2 = 4$$

$$\text{Sum of Diagonal minors} = \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$$

$$= (-6) + (2) + (1)$$

$$= -6 + 2 + 1$$

$$= -3.$$

Putting the values in equation (B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \rightarrow (C)$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0 \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 1(2-8) - 0 + 1(6-0)$$

$$= -6 + 6$$

$$= 0$$

Now putting the values in equation (C):-

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0$$

Now using Quadratic Equation

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} a &= 1 \\ b &= -4 \\ c &= -3 \end{aligned}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \lambda = \frac{4 - \sqrt{28}}{2}$$

We have Eigen value :-

$$\lambda = \left(0, \frac{4 + \sqrt{28}}{2}, \frac{4 - \sqrt{28}}{2} \right) \text{ Required Answers.}$$

QNO # 03



Given Equation :-

$$5x + 4z + 2m = 3$$

$$x - y + 2x + m = 1$$

$$4x + y + 2z = 1$$

$$x + y + z + m = 1$$

Writing in Augmented form.

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right] R_2 = 5R_1$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right]$$

$$R_2 - 5R_1$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right]$$

$$R_3 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 0 & -12 & -7 & -5 \\ 1 & 1 & 1 & 1 & 0 \end{array} \right]$$

$$R_4 - R_1$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 0 & 0 & -12 & -7 & -5 \\ 0 & 2 & -1 & 0 & -1 \end{array} \right]$$

$$R_4 + \frac{2}{5}R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 1 \\ 0 & -5 & -6 & -8 & -2 \\ 0 & 0 & -2 & -7 & -5 \\ 0 & 0 & -\frac{17}{5} & \frac{6}{5} & -\frac{7}{5} \end{array} \right] R_3 \leftrightarrow R_4$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ 0 & -5 & -6 & -3 & -2 \\ 0 & 0 & -\frac{17}{5} & \frac{6}{5} & -\frac{17}{5} \\ 0 & 0 & -12 & -7 & -5 \end{array} \right] R_4 - 2R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 1 \\ 0 & -5 & -6 & -3 & 3 \\ 0 & 0 & -\frac{17}{5} & \frac{6}{5} & -\frac{17}{5} \\ 0 & 0 & 0 & -1 & 1 \end{array} \right]$$

$$\Rightarrow -m = 1$$

$$\Rightarrow \boxed{m = 1}$$

$$-\frac{17}{5}z + \frac{6}{5}m = -\frac{17}{5}$$

$$-\frac{17}{5}z = \frac{17}{5} - \frac{6}{5}$$

$$z = \frac{-23}{8} \times \frac{8}{-17}$$

$$\Rightarrow z = \frac{23}{17}$$

$$-5y - 6z - 3m = 3$$

$$-5y - 6\left(\frac{23}{17}\right) - 3(1) = 3$$

$$-5y - \frac{138}{17} = 6$$

$$= \frac{102 + 138}{17}$$

$$y = \frac{240}{17x-5}$$

$$\Rightarrow \boxed{y = -\frac{48}{17}}$$

$$x + y - 2z + m = 1$$

$$x - \frac{48}{17} - 2 \frac{(23)}{17} + 1 = 1$$

$$x - \left(\frac{48 - 46}{17} \right) = 0$$

$$x = \frac{2}{17} = 0$$

$$\Rightarrow \boxed{x = \frac{2}{17}}$$

Required
Solution.

Question #4.

Solution: -

$$U(x, t) = \sin(x + vt)$$

Solution: -

$$\frac{\partial^2 u}{\partial t^2} = \frac{c^2 \partial^2 u}{\partial x^2}$$

$U(x, t) = \sin(x + vt)$ is

Solution of $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

It will satisfy the above equation.

$$\frac{\partial U}{\partial t} = \cos(x + vt) \cdot \frac{d}{dt}$$

$(x + vt)$

$$\frac{du}{dx} = \cos(x+2t) (1+0)$$

$$\frac{du}{dx} = \cos(x+2t)$$

$$\frac{d^2u}{dx^2} = \frac{d}{dx} \cos(x+2t)$$

$$\frac{d^2u}{dx^2} = \frac{d}{dx} \cos(x+2t)$$

$$\frac{d^2u}{dx^2} = -\sin(x+2t) \cdot \frac{d}{dx}(x+2t)$$

$$\frac{d^2u}{dx^2} = -\sin(x+2t) (1+0)$$

$$\Rightarrow \boxed{\frac{d^2u}{dx^2} = -\sin(x+2t)}$$

$$\xi \quad u(x, t) = \sin(x+2t)$$

Differentiate w.r.t "t".

$$\frac{du}{dt} = \frac{d}{dt} \sin(x+2t)$$

$$\frac{du}{dt} = \cos(x+2t) (0+2)$$

$$\frac{du}{dt} = 2 \cos(x+2t)$$

$$\frac{d^2u}{dt^2} = (2) - \sin(x+2t) (0+2)$$

$$\Rightarrow \frac{d^2u}{dt^2} = -4 \sin(x+2t)$$

We know that one dimensional equation is,

$$\frac{d^2 u}{dt^2} = c^2 \frac{d^2 u}{dx^2}$$

$$-4 \sin(u+2t) = c^2 [-\sin(u+2t)]$$

$$-4 \sin(u+2t) = -c^2 \sin(u+2t)$$

$$-4 \sin(u+2t) + c^2 \sin(u+2t) = 0$$

for the arbitrary constant $c = \pm 2$.

$$-4 \sin(u+2t) + (\pm 2)^2 \sin(u+2t) = 0$$

$$-4 \sin(u+2t) + 4 \sin(u+2t) = 0$$

$$0 = 0.$$

Then it be verified for the arbitrary constant $c = 2$

Required Answer.

