

**Sessional Assignment**

**Advanced mechanics of Materials**

**Total Marks: 20**

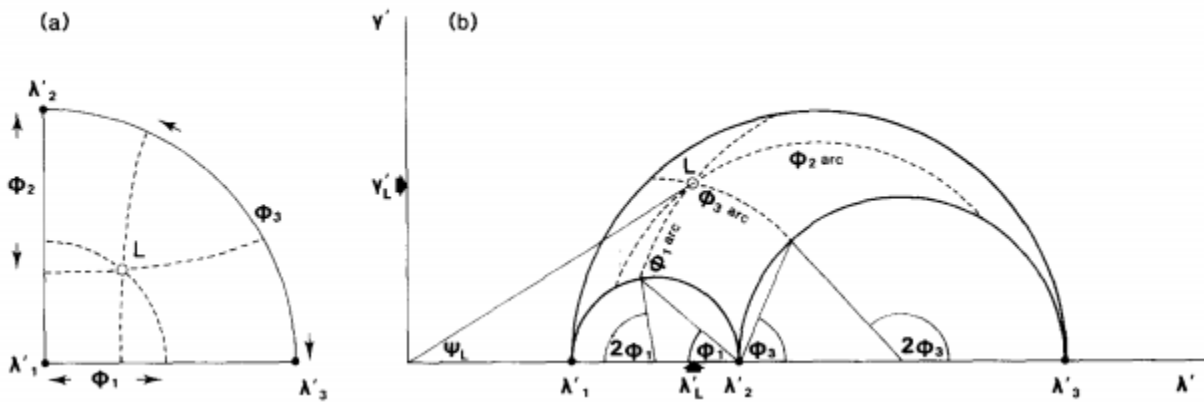
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# 1. Application of Mohr's Circle to the Three

THE TERM Mohr diagram is used to encompass a family of graphical representations, either of stress or strain, which are characterized by circles, known as Mohr circles. The Mohr diagram for stress is most commonly used in the study of fracture, in which a Mohr envelope is constructed from successive Mohr circles at points of failure. In strain studies, a single Mohr circle is sometimes labeled a Mohr diagram: such circles may be used to illustrate two-dimensional strain tensors or as a practical method of strain determination. In modern structural geology, the Mohr diagram is restricted almost exclusively to two-dimensional analyses, either of stress or strain. This was not the case for the original Mohr diagram.

The Mohr diagram for strain is rarely used in its full form, as a representation of three-dimensional strain. Recent attention has focused on various uses of the Mohr circle to express two-dimensional strain tensors. This contribution red scribes the Mohr diagram for three-dimensional strain and illustrates some new applications. The Mohr diagram for any strain ellipsoid provides an immediate method for ellipsoid shape classification. However, its greatest new potential is considered to be in the representation of strain ellipses as sections of ellipsoids.

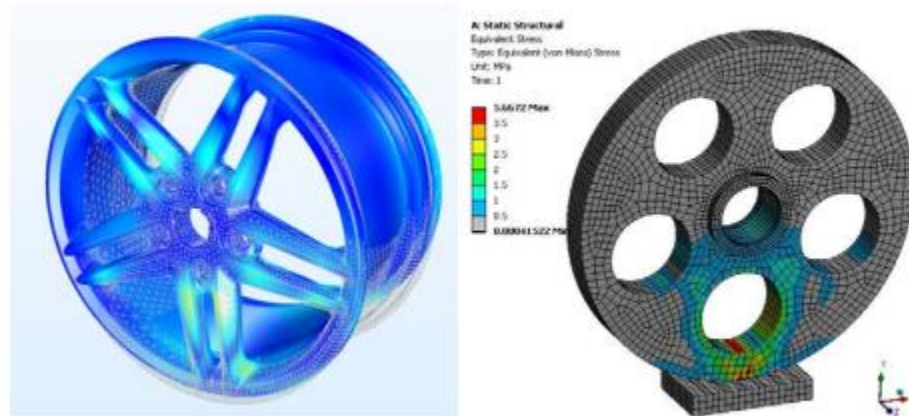


#### 4. Dimensional Analysis of Stress.

This element is useful for the stress analysis of general three-dimensional bodies that require more precise analysis than is possible through two-dimensional and/or axisymmetric analysis. Examples of three dimensional problems are arch dams, thick-walled pressure vessels, and solid forging parts as used, for instance, in the heavy equipment and automotive industries.

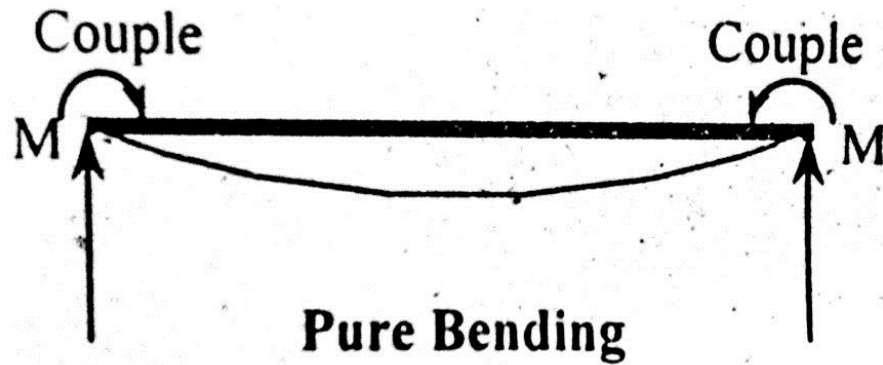
Finite elements can be used for analyzing linear and nonlinear stress characteristics of structures and mechanical components.

Finite element discretization, stresses, and deformations of a wheel rim in a structural analysis.

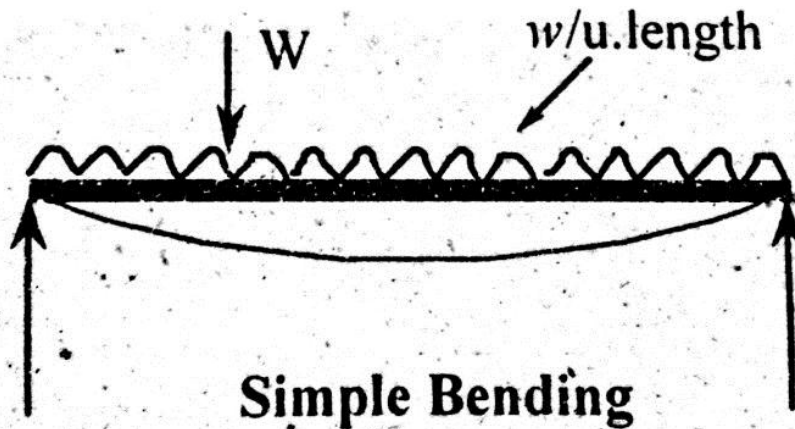


#### 3. Simple bending and Pure Bending

Bending will be called as pure bending when it occurs solely because of coupling on its end. In that case there is no chance of shear stress in the beam. But, the stress that will propagate in the beam as a result will be known as normal stress. Normal stress because it not causing any damages to beam. As shown below in the picture.



Bending will be called as simple bending when it occurs because of beam self-load and external load. This type of bending is also known as ordinary bending and in this type of bending results both shear stress and normal stress in the beam. As shown below in the figure.



#### 4. Assumption made in theory of pure bending.

Following are the assumptions made in theory of simply bending

- The material of the beam is perfectly homogeneous.
- The beam material is stressed within its elastic limit and thus obeys Hooke's law.
- The transverse sections which are plane before bending remain plane after bending also.
- Each layer of the beam is free to expand or contract independently of the layer, above or below it.
- The value of Young's modulus for the material of beam is same in tension and compression.

#### 5. Classic Flexure Equation.

Flexure Formula. Stresses caused by the bending moment are known as flexural or bending stresses. Consider a beam to be loaded as shown. Consider a fiber at a distance  $y$  from the neutral axis, because of the beam's curvature, as the effect of bending moment; the fiber is stretched by an amount of  $cd$ .

$$\epsilon = cd/av = y/\rho$$

## 6. Section Modulus.

Section modulus is a geometric property for a given cross-section used in the design of beams or flexural members. Other geometric properties used in design include area for tension and shear, radius of gyration for compression, and moment of inertia and polar moment of inertia for stiffness. Any relationship between these properties is highly dependent on the shape in question. Equations for the section moduli of common shapes are given below. There are two types of section moduli, the elastic section modulus and the plastic section modulus. The section moduli of different profiles can also be found as numerical values for common profiles in tables listing properties of such.

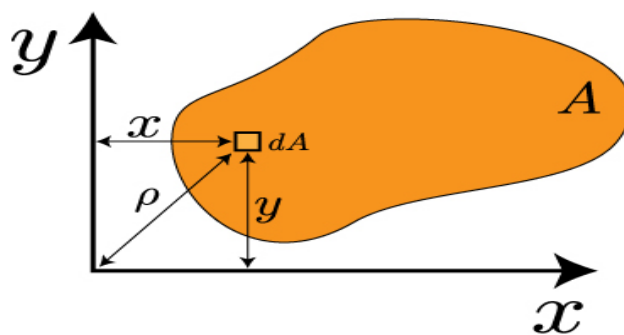
## 7. Application of Bending Equation in any object.

### Moments of Area

In order to calculate stress (and therefore, strain) caused by bending, we need to understand where the neutral axis of the beam is, and how to calculate the second moment of area for a given cross section.

Let's start by imagining an arbitrary cross section – something not circular, not rectangular, etc.

### Arbitrary Cross Section



In the above image, the arbitrary shape has an area denoted by  $A$ . We can look at a small, differential area  $dA$  that exists some distance  $x$  and  $y$  from the origin. We can look at the first moment of area in each direction from the following formulas:

$$Q_x = \int_A y \, dA$$

$$Q_y = \int_A x \, dA$$

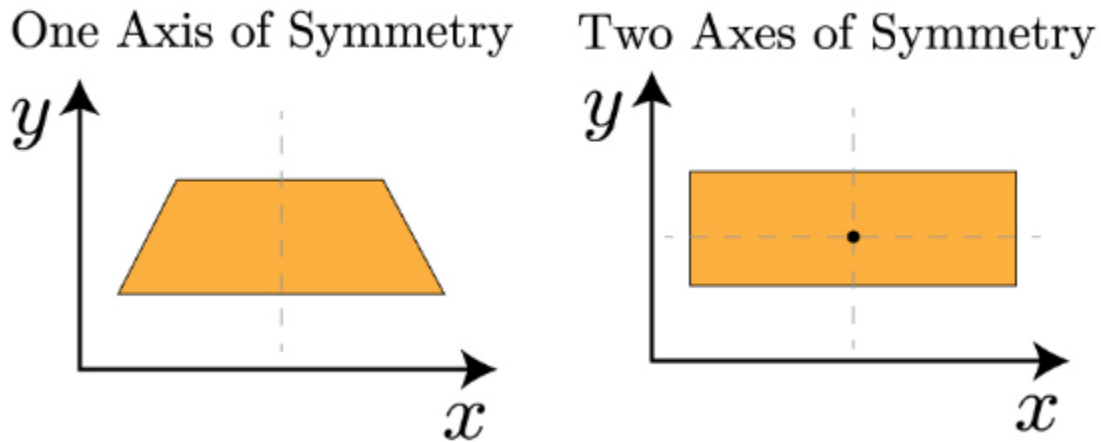
The first moment of area is the integral of a length over an area – that means it will have the units of length cubed [L<sup>3</sup>]. It is important because it helps us locate the centroid of an object. The centroid is defined as the "average x (or y) position of the area". Mathematically, this statement looks like this:

$$\bar{x} = \frac{Q_y}{A} = \frac{\int_A y \, dA}{\int_A dA} = \frac{\sum_i \bar{x}_i \bar{A}_i}{\sum_i \bar{A}_i}$$

$$\bar{y} = \frac{Q_x}{A} = \frac{\int_A x \, dA}{\int_A dA} = \frac{\sum_i \bar{y}_i \bar{A}_i}{\sum_i \bar{A}_i}$$

The far right side of the above equations will be very useful in this course – it allows us to break up a complex shape into simple shapes with known areas and known centroid locations. In most engineering structures there is at least

one axis of symmetry – and this allows us to greatly simplify finding the centroid. **The centroid has to be located on the axis of symmetry.** For example:



For the cross section on the left, we know the centroid has to lie on the axis of symmetry, so we only need to find the centroid along the  $y$ -axis. The cross section on the right is even easier – since the centroid has to line on the axes of symmetry, it has to be at the center of the object.

Now that we know how to locate the centroid, we can turn our attention to the second moment of area. As you might recall from the previous section on torsion, this is defined as:

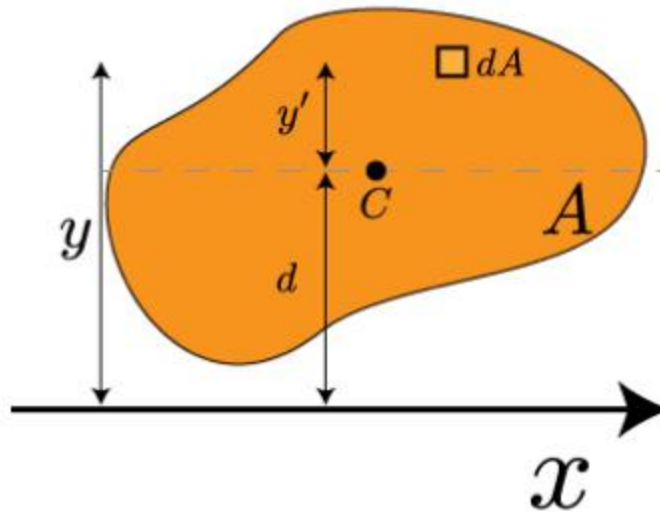
$$I_x = \int_A y^2 dA$$

$$I_y = \int_A x^2 dA$$

$$J = \int_A \rho^2 dA = I_x + I_y$$

And, finally sometimes we will need to determine the second moment of area about an arbitrary  $x$  or  $y$  axis – one that does not correspond to the centroid. In this case, we can utilize the parallel axis theorem to calculate it. In this case, we utilize the second moment of area with respect to the centroid, plus a term that includes the distances between the two axes.

# Arbitrary Cross Section



$$I_x = \bar{I}_{x'} + Ad^2$$

This equation is referred to as the **Parallel Axis Theorem**. It will be very useful throughout this course. As described in the introductory video to this section, it can be straightforward to calculate the second moment of area for a simple shape. For more complex shapes, we'll need to calculate  $I$  by calculating the individual  $I$ 's for each simple shape and combining them together using the parallel axis theorem.

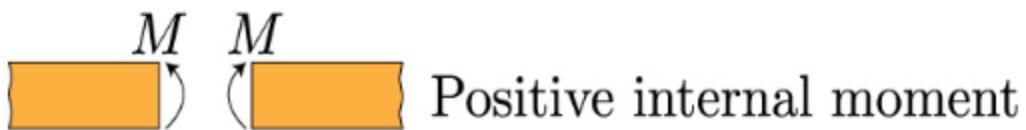
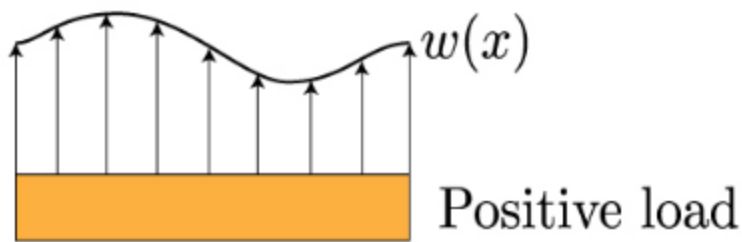
## Shear and Moment Diagrams

Transverse loading refers to forces that are perpendicular to a structure's long axis. These **transverse loads** will cause a bending moment  $M$  that induces a **normal stress**, and a shear force  $V$  that induces a **shear stress**. These forces can and will vary along the length of the beam, and we will use **shear & moment diagrams (V-M Diagram)** to extract the most relevant values. Constructing these diagrams should be familiar to you from **statics**, but we will review them here. There are two important considerations when examining a transversely loaded beam:

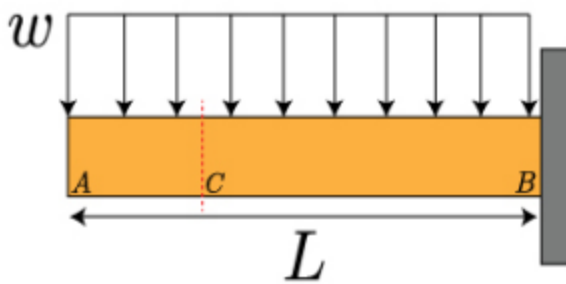
1. How is the beam loaded?
  - point load, distributed load (uniform or varying), a combination of loads...
1. How is the beam supported?
  - simply supported, cantilevered, overhanging, statically indeterminate...

Knowing about the loads and supports will enable you to sketch a *qualitative* V-M diagram, and then a statics analysis of the free body will help you determine the *quantitative* description of the curves. Let's start by recalling our **sign conventions**.

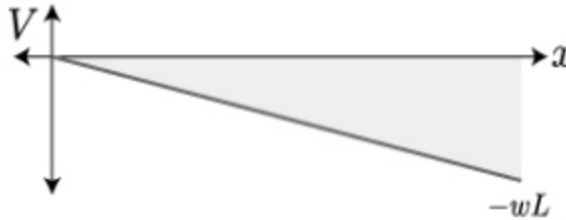
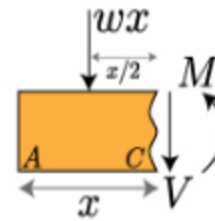




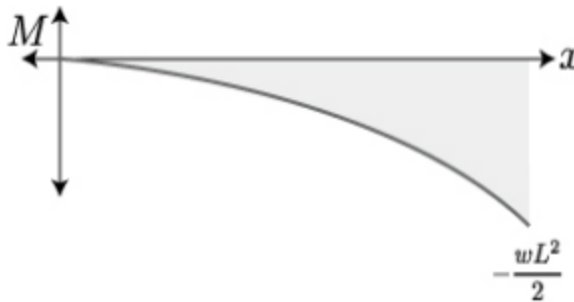
These sign conventions should be familiar. If the shear causes a counterclockwise rotation, it is positive. If the moment bends the beam in a manner that makes the beam bend into a "smile" or a U-shape, it is positive. The best way to recall these diagrams is to work through an example. Begin with this cantilevered beam – from here you can progress through more complicated loadings.



Free Body Diagram



$$\begin{aligned}
 +\uparrow \sum F_y &= 0 \\
 -wx - V &= 0 \\
 \boxed{V} &= -wx
 \end{aligned}$$



$$\begin{aligned}
 +\curvearrowright \sum M_C &= 0 \\
 wx \left(\frac{x}{2}\right) + M &= 0 \\
 \boxed{M} &= -\frac{wx^2}{2}
 \end{aligned}$$

### Normal Stress in Bending

In many ways, bending and torsion are pretty similar. Bending results from a couple, or a **bending moment**  $M$ , that is applied. Just like torsion, in pure bending there is an axis within the material where the stress and strain are zero. This is referred to as the **neutral axis**. And, just like torsion, the stress is no longer uniform over the cross section of the structure – it varies. Let's start by looking at how a moment about the z-axis bends a structure. In this case, we won't limit ourselves to circular cross sections – in the figure below, we'll consider a prismatic cross section.

axis. To restate this in the language of this class, we can say that the bottom surface is under tension, while the top surface is under compression. Something that is a little more subtle, but can still be observed from the above overlaid image, is that the displacement of the beam varies linearly from the top to the bottom – passing through zero at the neutral axis. Remember, this is exactly what we saw with torsion as well – the stress varied linearly from the center to the center. We can look at this stress distribution through the beam's cross section a bit more explicitly:

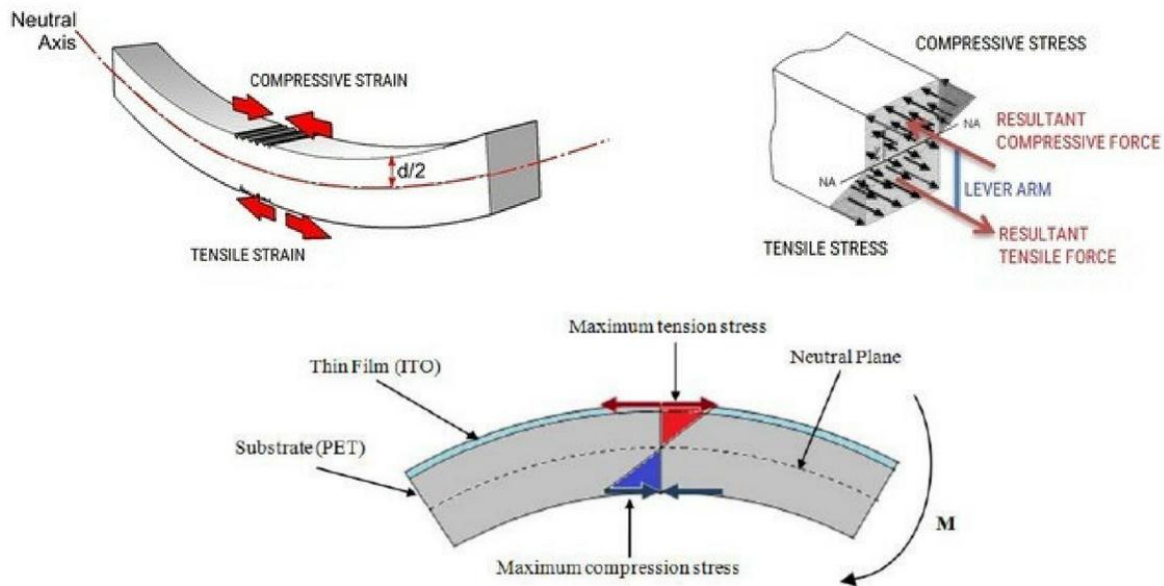
$$M_z = \int_A y dF = \int_A y(\sigma_x dA) = \int_A y \left( \frac{y}{c} \sigma_{\max} \right) dA$$
$$M_z = \frac{\sigma_{\max}}{c} \int_A y^2 dA$$

The final term in the last equation – the integral over  $y$  squared – represents the [second moment of area](#) about the  $z$ -axis (because of how we have defined our coordinates). In Cartesian coordinates, this second moment of area is denoted by  $I$  (in cylindrical coordinates, remember, it was denoted by  $J$ ). Now we can finally write out our equation for the maximum stress, and therefore the stress at any point along the  $y$ -axis, as:

$$\sigma_{\max} = \frac{cM_z}{I_z}$$
$$\sigma_x = -\frac{yM_z}{I_z}$$

## 8. Moment of resistance.

When a beam bends under load, the horizontal fibers will change in length. In technical terms it is referred to as the internal moment of resistance. ... The tensile and compressive stresses result in a turning effect about the neutral axis. These are called moment  $M_T$  and  $M_C$  respectively.



## 9. Design of Columns Under Centric Load

Eccentric load was defined as a displacement of the point of application of the compressive load in the end sections of the column relative to its centreline, in two directions:  $e_1$ :  $0 < e_1 < 10$  mm and  $e_2$ :  $-10 < e_2 < 10$  mm.

## 10. Deflections of beam by Castiglione's Theorem

Castiglione's first theorem the first partial derivative of the total internal energy (strain energy) in a structure with respect to any particular deflection component at a point is equal to the force applied at that point and in the direction corresponding to that deflection component.