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Final Term Exam

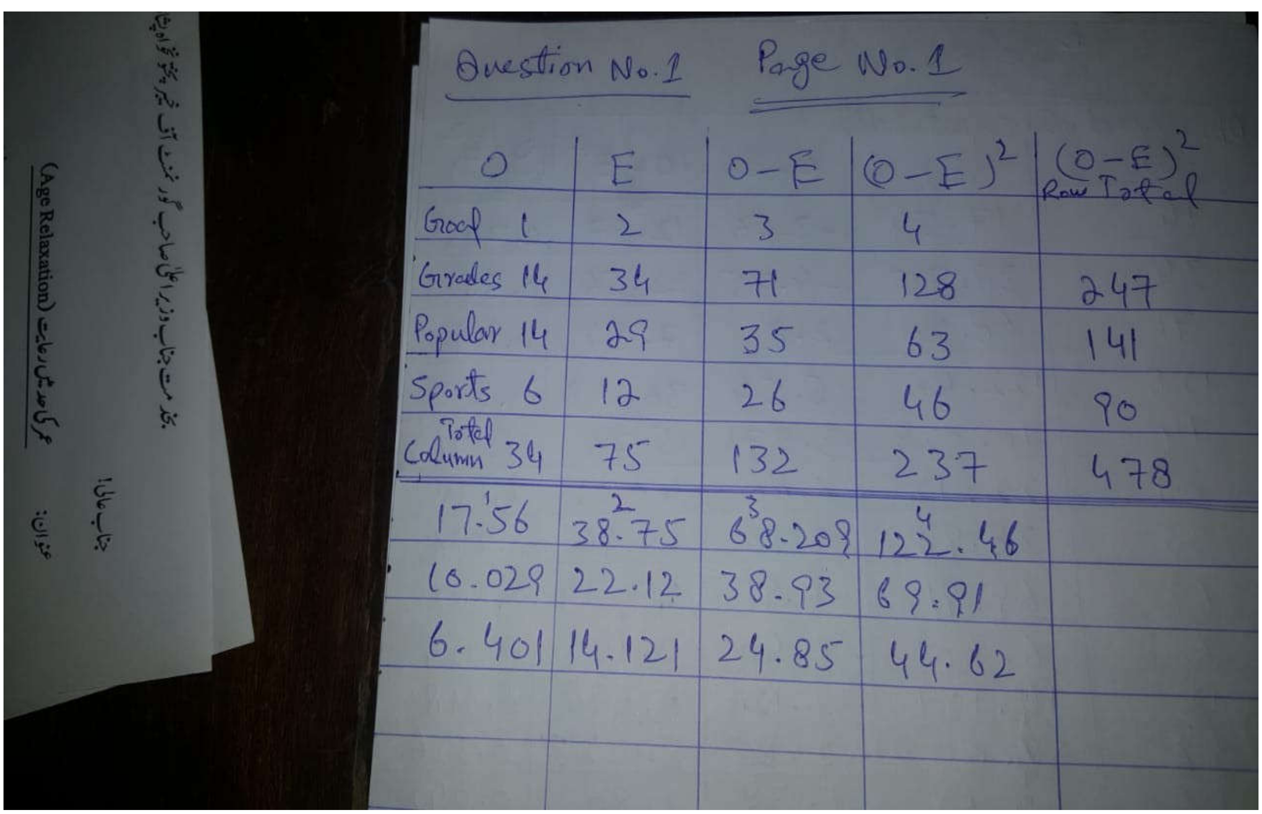
Question No: 01

Students at multiple grade schools were asked what their personal goal (get good grades, be popular, be good at sports) was and how important having money were to them (1 very important and 4 least important). Do the data provide enough evidence to show that goal attainment and importance of money are independent in following given table? Test at the 5% level.

Goal	Money Importance Rating				Row Total
	1	2	3	4	
Grades	14	34	71	128	247
Popular	14	29	35	63	141
Sports	6	12	26	46	90
Column Total	34	75	132	237	478

Table: Personal Goal and Importance of Money

<u>Solution</u>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O	E	O-E	$(O-E)^2$	$\frac{(O-E)^2}{\text{Row Total}}$
Goal 1	2	3	4	
Grades 14	34	71	128	247
Popular 14	29	35	63	141
Sports 6	12	26	46	90
Total Column 34	75	132	237	478
17.56	38.75	68.209	122.46	
10.029	22.12	38.93	69.91	
6.401	14.121	24.85	44.62	

Page No 2

Observed value (O)	Expected value (E)	O - E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
14	17.54	-3.56	12.67	0.705
34	38.75	-4.75	22.56	0.582
71	68.20	2.8	7.84	0.114
128	122.46	5.54	30.69	0.250
14	10.029	3.971	15.76	1.571
29	22.12	6.88	47.33	2.139
35	38.93	-3.93	15.44	0.396
63	69.91	-6.91	47.74	0.682
6	6.401	-0.401	0.160	0.024
12	14.121	-2.121	4.498	0.318
26	28.85	-2.85	8.1225	0.326
46	44.62	-1.38	1.90	0.042

$= \frac{(O-E)^2}{E}$

$= 6.828$

So degree of freedom = (Column - 1)(row - 1)

$(4-1)(3-1) = 3 \times 2 = 6$

$= 0.05$

Tabular value is 12.59

χ^2 tabular = 12.59

χ^2 calculated = 6.828

χ^2 calculated < χ^2 tabular

Question No:
02

a) Write down the basic assumptions of Binomial Distribution.

Solution

Binomial distribution is a probability distribution that summarizes the likelihood that a value will take one of two independent values under a given set of parameters or assumptions.

The **assumptions** of the binomial distribution are that there is only one outcome for each trial, that each trial has the same probability of success, and that each trial is mutually exclusive, or independent of each other.

Binomial distribution is a common discrete distribution used in statistics, as opposed to a continuous distribution, such as the normal distribution. This is because the binomial distribution only counts two states, typically represented as 1 (for a success) or 0 (for a failure) given a number of trials in the data. The binomial distribution, therefore, represents the probability for x successes in n trials, given a success probability p for each trial.

b) If X is binomially distributed with 8 trails and probability of success equal to ¾ at each attempt, what is the probability of:
   i. Exactly 5 successes   ii. At least one success

Solution

i. Exactly 5 successes

Question 2 (b) (i)

Solution Exactly 5 Success

$n = 8$ $\quad x = 5$

formula

$n = 8$, $P = \frac{3}{4}$, $q = \frac{1-3}{4} \Rightarrow \frac{1}{4}$

$x = 5$ $\quad P(5\ \text{success}) = C = \left(\frac{3}{4}\right)^5 \left(\frac{1}{4}\right)^{8-5}$

$$\Rightarrow \frac{8!}{(8-5)!\,5!} \left(\frac{3}{4}\right)^5 \left(\frac{1}{4}\right)^3$$

$$\Rightarrow \frac{40320}{120} (0.75)^5 (0.2)^3$$

$\cancel{336}$

$$336\,(0.237)(0.008)$$

$\boxed{P = 0.63}$ Ans

ii. At least one success

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Question No. 2 → (b) → (ii)

Solution   At least one Success

$n = 8,\qquad x = 1$

$\Rightarrow {}_{8}C_{1}\left(\frac{3}{4}\right)^{1}\left(\frac{1}{4}\right)^{8-1}$

$\Rightarrow \frac{8!}{(8-1)!\,1!}\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{7}$

$\Rightarrow \frac{8\times7\times6\times5\times4\times3\times2}{7\times6\times5\times4\times3\times2}\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{7}$

$\Rightarrow \frac{\overset{8}{\cancel{40320}}}{\cancel{5040}}(0.75)(0.2)^{7}$

$8\,(0.75)(0.0000128)$

$\boxed{P = 0.0000768}$ Ans.

Question No: 03

a. Differentiate between Z-test, t-test & ANOVA test

Solution

Z-Test

A **z-test** is used for testing the mean of a population versus a standard, or comparing the means of two populations, with large ($n \geq 30$) samples whether you know the population standard deviation or not. It is also used for testing the proportion of some characteristic versus a standard proportion, or comparing the proportions of two populations.
Example: Comparing the average engineering salaries of men versus women.
Example: Comparing the fraction defectives from 2 production lines.

T-Test

A **t-test** is used for testing the mean of one population against a standard or comparing the means of two populations if you do not know the populations' standard deviation and when you have a limited sample ($n < 30$). If you know the populations' standard deviation, you may use a z-test.
Example: Measuring the average diameter of shafts from a certain machine when you have a small sample.

ANOVA test

Analysis of variance (**ANOVA**) is a collection of statistical models and their associated estimation procedures (such as the "variation" among and between groups) used to analyze the differences among group means in a sample. ANOVA was developed by the statistician Ronald Fisher. The ANOVA is based on the law of total variance, where the observed variance in a particular variable is partitioned into components attributable to different sources of variation. In its simplest form, ANOVA provides a statistical test of whether two or more population means are equal, and therefore generalizes the t-test beyond two means.

Example: As a crop researcher, you want to test the effect of three different fertilizer mixtures on crop yield.

b. Write down the basic assumptions for Chi-square test.

Solution

Basic assumptions for Chi-square test.

The **chi-square test** for independence, also called Pearson's chi-square test or the chi-square test of association, is used to discover if there is a relationship between two categorical variables.

Assumptions

When we choose to analyses we data using a chi-square test for independence, we need to make sure that the data we want to analyses "passes" two assumptions. We need to do this because it is only appropriate to use a chi-square test for independence if our data passes these two assumptions. If it does not, you cannot use a chi-square test for independence. These two assumptions are:

- **Assumption #1:** Our two variables should be measured at an ordinal or nominal level (i.e., categorical data). You can learn more about ordinal and nominal variables in our article: Types of Variable.
- **Assumption #2:** Our two variable should consist of two or more categorical, independent groups. Example independent variables that meet this criterion include gender (2 groups: Males and Females), ethnicity (e.g., 3 groups: Caucasian, African American and Hispanic), physical activity level (e.g., 4 groups: sedentary, low, moderate and high), profession (e.g., 5 groups: surgeon, doctor, nurse, dentist, therapist), and so forth.

Question No: 04

The p.d.f of the age of babies, x years, being brought to a post-natal clinic is given by

$$f(x) = \begin{cases} \frac{3}{4}\, x\,(3 - x) & 0 < x < 2 \\ 0 & ot\ \ erwise \end{cases}$$

If 45 babies are brought in on a particular day, how many are expected to be under 8 months old?

Question No. 4

Eight months $= \frac{2}{3}$ years so

~~$P(x = \frac{3}{4}$~~

$P(x = \frac{2}{3} = \int^{\frac{2}{3}} \frac{2}{\cancel{3}4} x(2-x)\,dx$

$= \frac{3}{4} \int^{\frac{2}{3}} (2x - x^3)\,dx$

$= \frac{3}{4} \left(x^2 - \frac{x^3}{3}\right)^{\frac{2}{3}}$

$= \frac{3}{4} \left[\frac{4}{9} - \frac{8}{81}\right] - [0]$

$= \frac{3}{4} \left[\frac{4}{9} - \frac{8}{81}\right] - [0]$

$= \frac{3}{4} \left[\frac{28}{81}\right] = \frac{7}{27} \Rightarrow 0.259$

Hence the expected number of babies $\leq$ 8 months

$= 45 \times \frac{7}{27} = \boxed{11.6}$

So the expected number of babies under 8 months