Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: **Total Marks:** 50

Student Details

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Q1.	(a)	Determine the response y(n), n ≥ 0, of the system described by the second order difference equation ( )−4 (−1)+4 (−2)= ( )− (−1) To the input ( ) = (−1) ( ). And the initial conditions are y (-1) = y (-2) = 0.	**Marks** **7** **CLO** **2**
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. ( ) − 0.7 (− 1) + 0.1 (− 2) = 2 ( ) − (− 2)	**Marks** **7** **CLO** **2**
Q2.	(a)	Determine the causal signal x(n) having the z-transform ( ) = 1 / (1 − 2 −1)(1 − −1)2 (Hint: Take inverse z-transform using partial fraction method)	**Marks** **6** **CLO** **2**
	(b)	Evaluate the inverse z- transform using the complex inversion integral ( ) = 1 / 1− −1 ||>||	**Marks** **6** **CLO** **2**
Q.3	(a)	A two- pole low pass filter has the system response ( ) = (1 − −1)2 Determine the values of b_o and p such that the frequency response H(ω) satisfies the condition H(0) = 1 and | (4)| 2 = 1 2 .	**Marks** **6** **CLO** **3**

	(b)	Design a two-pole bandpass filter that has the center of its passband at ω = π/2, zero in its frequency response characteristics at ω = 0 and ω = π and its magnitude response in $\frac{1}{\sqrt{2}}$ at ω = 4π/9.	**Marks 6** **CLO 3**
Q 4	(a)	A finite duration sequence of Length L is given as 1, 0≤≤−1 ( )={0, h Determine the N- point DFT of this sequence for N ≥ L	**Marks 6** **CLO 2**
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $_1(\ ) = \{^2,1,2,1\}$ ↑ $_2(\ ) = \{^1,2,3,4\}$ ↑	**Marks 6** **CLO 2**

Date: ________ (1)

Q1 Part (a)

Solution:

$$y(n)-4y(n-1)+4y(n-2)=x(n)-x(n-1)$$

The Characteristic equation is

$$\lambda^2-4\lambda+4=0$$

$$\lambda=2,2 \text{ Hence}$$

$$y_h(n)=c_1 2^n+c_2 2^n$$

The Particular Solution

$$y_p(n)=k(-1)^n u(n)$$

Substituting this Solution into the difference equation we obtain

$$k(-1)^n u(n)-4k(-1)^{n-1}u(n-1)+4k(-1)^{n-4}u(n-2)=(-1)^n u(n)-(-1)^{n-1}u(n-1)$$

For $n=2$, $k(1+4+4)=2 \Rightarrow k=\frac{2}{9}$

The total Solution is

$$y(n)=\left[c_1 2^n+c_2 n 2^n+\frac{2}{9}(-1)^n\right]u(n)$$

From the initial Condition we obtain

$y(0)=1$, $y(1)=2$ Then

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(2)

$$C_1 + \frac{2}{9} = 1$$

$$\Rightarrow C_1 = \frac{7}{9}$$

$$2C_1 + 2C_2 - \frac{2}{9} = 2$$

$$C_2 = \frac{1}{3}$$

So Put in eq (i)

$$y(n) = \left[\frac{7}{9}2^n + \frac{1}{3}n2^n + \frac{2}{9}(-1)^n\right]u(n)$$

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Q1 Part (B)

Solution:

The Characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2}, \frac{1}{5}$ Hence

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$$

$$y(0) = 2$$

$$y(1) = 0.7y(0) = 0 \rightarrow y(1) = 1.4$$

Hence $C_1 + C_2 = 2$ and

$$\frac{1}{2}C_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$\rightarrow C_1 + \frac{2}{5}C_2 = \frac{14}{5}$$

These equation Yield

$$C_1 = \frac{10}{3}, \quad C_2 = \frac{-4}{3}$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$$

The Step Response is

$$S(n) = \sum_{K=0}^{n} h(n-K)$$

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(4)

$$= \frac{10}{3} \sum_{K=0}^{n} \left(\frac{1}{2}\right)^{n-K} - \frac{4}{3} \sum_{K=0}^{n} \left(\frac{1}{5}\right)^{n-K}$$

$$\frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{K=0}^{n} 2^{K} - \frac{4}{3} \left(\frac{1}{5}\right)^{n} \sum_{K=0}^{n} 5^{K}$$

$$\frac{10}{3} \left(\frac{1}{2}^{n} (\cdot 2^{n+1} - 1\right) u(n) - \frac{1}{3} \left(\frac{1}{5}^{n} (5^{n+1} - 1\right) u(n)$$

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Q 2 Part (a)

$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$

$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$

Multiply by L.C.M

$1 = A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + C(1-2z^{-1})$

Put $z = 0$

$1 = A(1-\frac{1}{2})^2 + 0 + 0$

$A = 4$

Put $z = 1$

$1 = 0 + 0 + C(1-2)$

$1 = -C$

$C = -1$

Comparing z^{-1}

$0 = -2A - 3B\ 2C$

$0 = -2(4) - 3B\ 2(-1)$

$= 0 - 8 - 3B + 2$

$6 = -3B$

$B = -2$

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then

$$X(z) = \frac{4}{1-2z^{-1}} - \frac{2}{1-z^{-1}} - \frac{z^{-1}}{(1-z^{-1})^2}$$

Apply inverse laplace

$= 4(+2)^n u(n) - 2(+1)^n u(n) - n\, u(n)$

$= [4(2)^n - 2 - n]\, u(n)$

For casual System

Q 2 Part (B)

$$x(z) = \frac{1}{(1-az^{-1})} \quad |z| > |a|$$

In this

$1 - az^{-1} = 0$

$1 = az^{-1}$

$z = a$

Pole is at $z = a$

For convergence of $x(z)$

$|az^{-1}| < 1$ and $|z| > |a|$

$\sum_{n=0}^{\alpha} |az^{-1}| < \alpha$

$= \sum_{n=0}^{\alpha} (az^{-1})^n$

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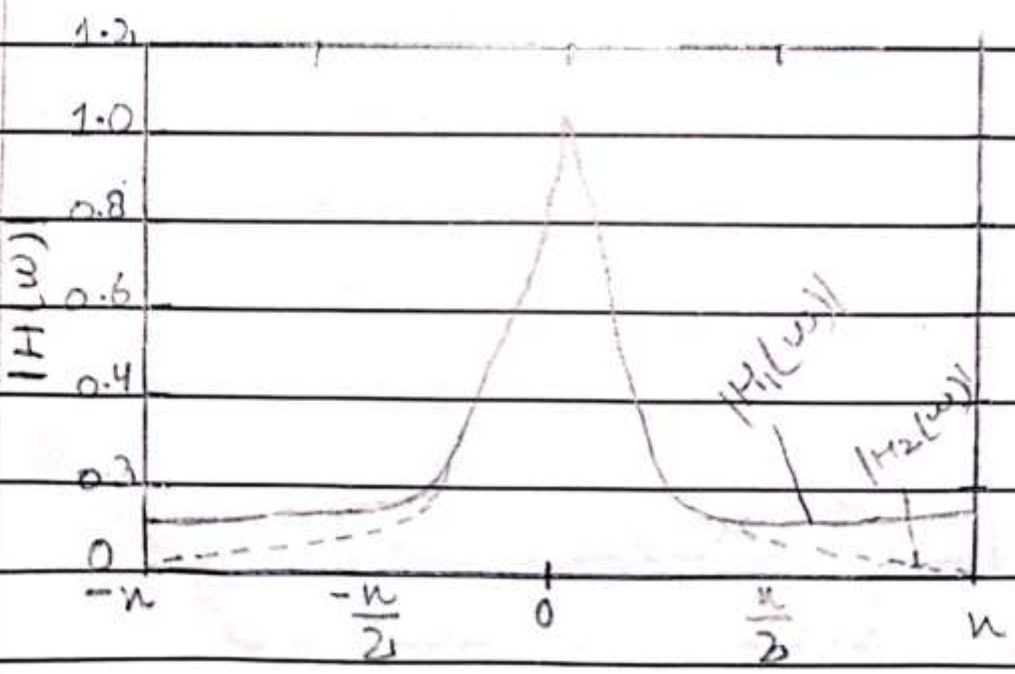
$$= \sum_{n \geq 0}^{\infty} a^n z^{-n}$$

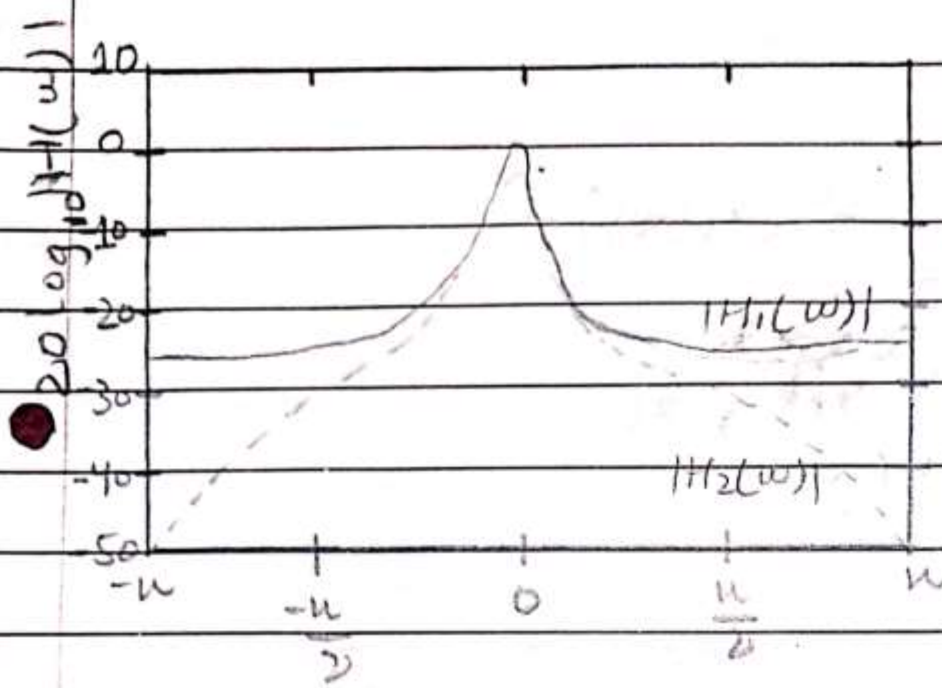
$$X(z) = \sum_{n=-\infty}^{\infty} a^n (u(n)) z^{-n}$$

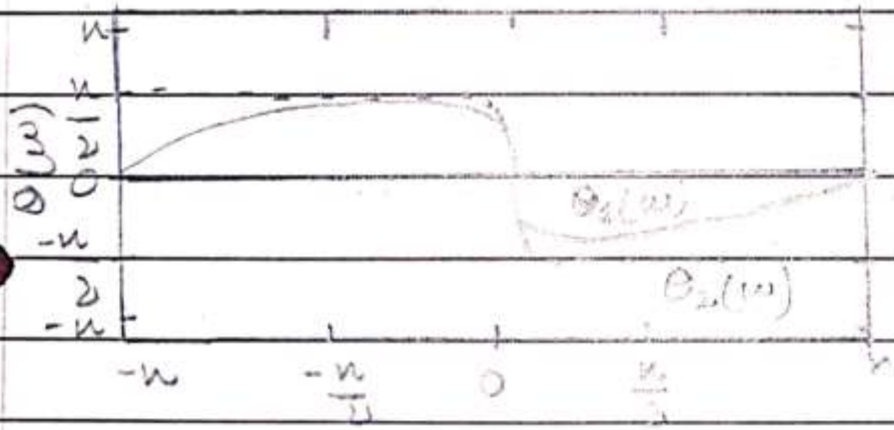
This gives u

$$X(n) = a^n (u(n))$$

Q 3 Part (a)







Magnitude and Phase response of (1) a Single Pole filter and (2) a one Pole one Zero filter

$H_1(z) = (1-a)/(1-az^{-1})$,

$H_2(z) = [(1-a)/2]\,[(1+z^{-1})/(1-az^{-1})]$

Determine the value of b_0 and P such that the frequency response $H(\omega)$

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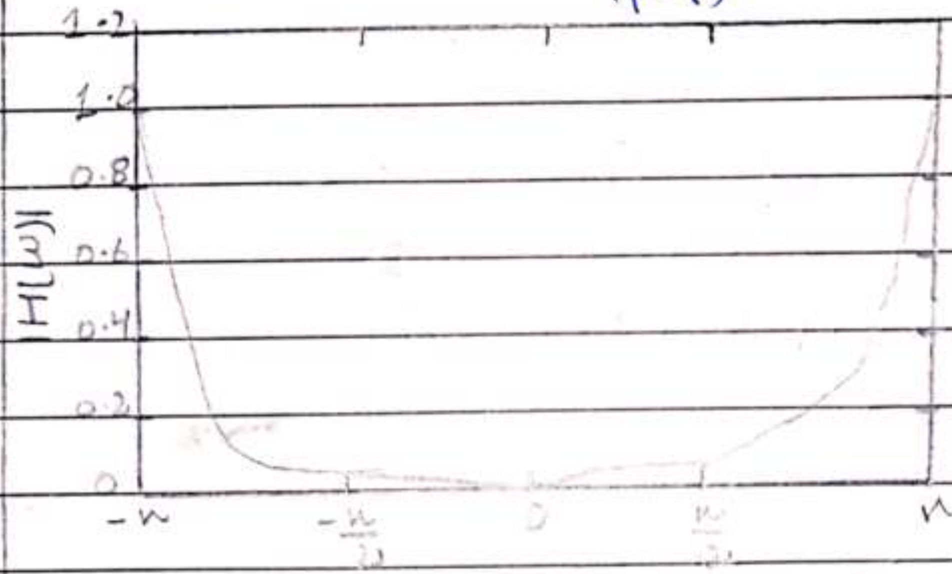
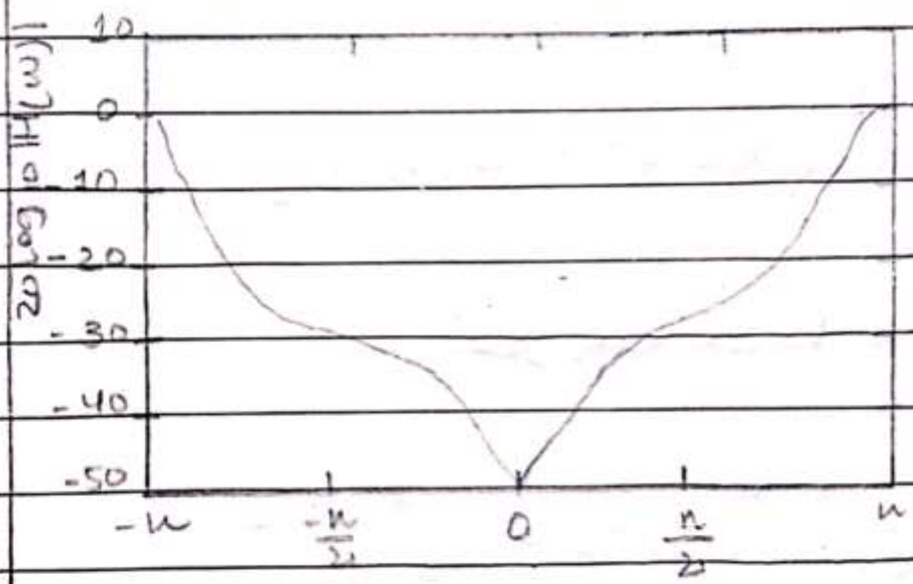
Satisfies the condition, $H = 0$

$|H(\frac{\pi}{4})| = \frac{1}{2}$

As $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1 \Rightarrow b_0 = (1-p)^2$$





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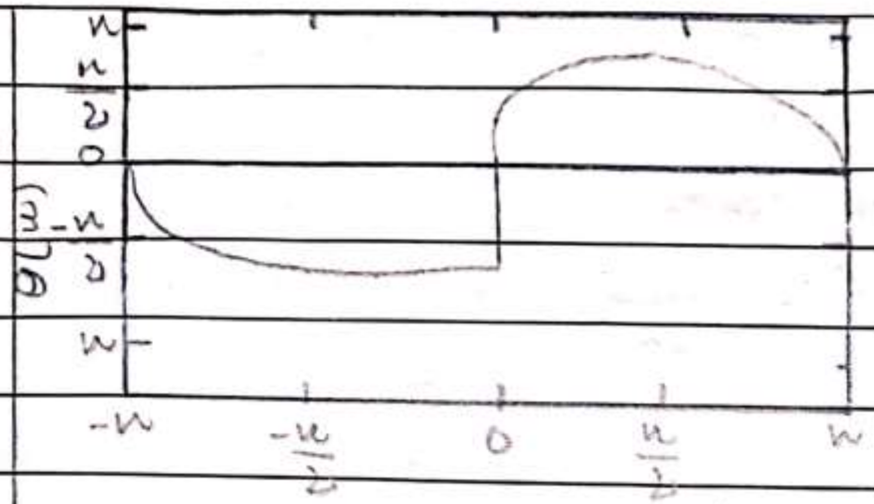

Magnitude and Phase response of a simple high pass filter

$H z = [(1-a)/2][(1-z^{-1})/(1+az^{-1})]$

with $a = 0.9$

At $\omega = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4)+jP\sin(\pi/4))^2}$$

Hence $\frac{(1-P)^2}{(1-P/\sqrt{2}+jP/\sqrt{2})^2}$

$$\frac{(1-P)^4}{[(1-P/\sqrt{2})^2+P^2/2]^2} = \frac{1}{2}$$

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Q3 Part B

Solution:

Clearly the filter must have poles at

$p_{1,2} = \pm jr$

and zero at $z=1$ and $z=-1$

Consequently the system function is

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{(z^2-1)}{z^2+r^2}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \frac{\pi}{2}$

Thus we have

$$H(\pi/2) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

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The value of r is determined by the evaluating $H(\omega)$ at $\omega = \frac{4\pi}{9}$. Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

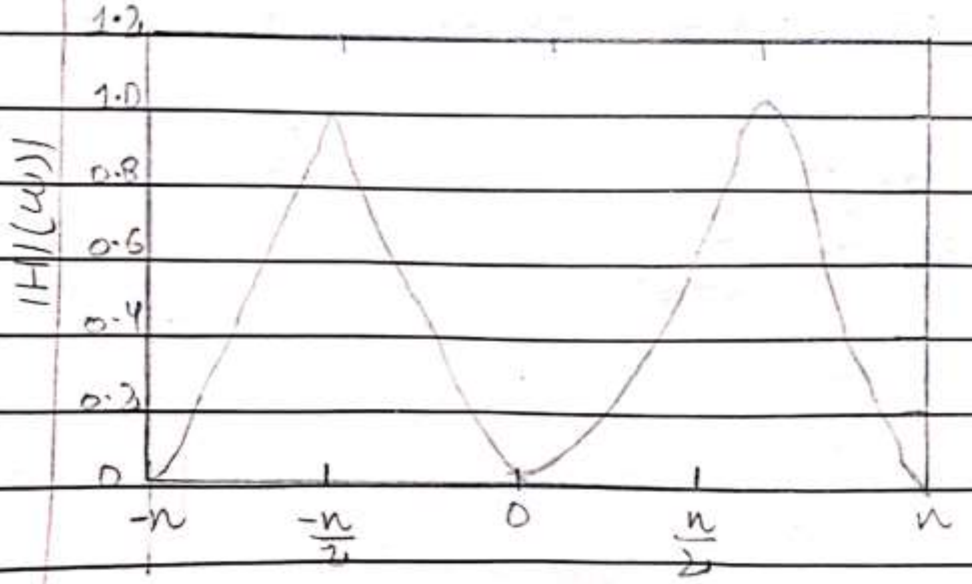
or equivalently

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of $r^2 = 0.7$ satisfies this equation. Therefore the system function for the desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

Its frequency response is illustrated in Fig



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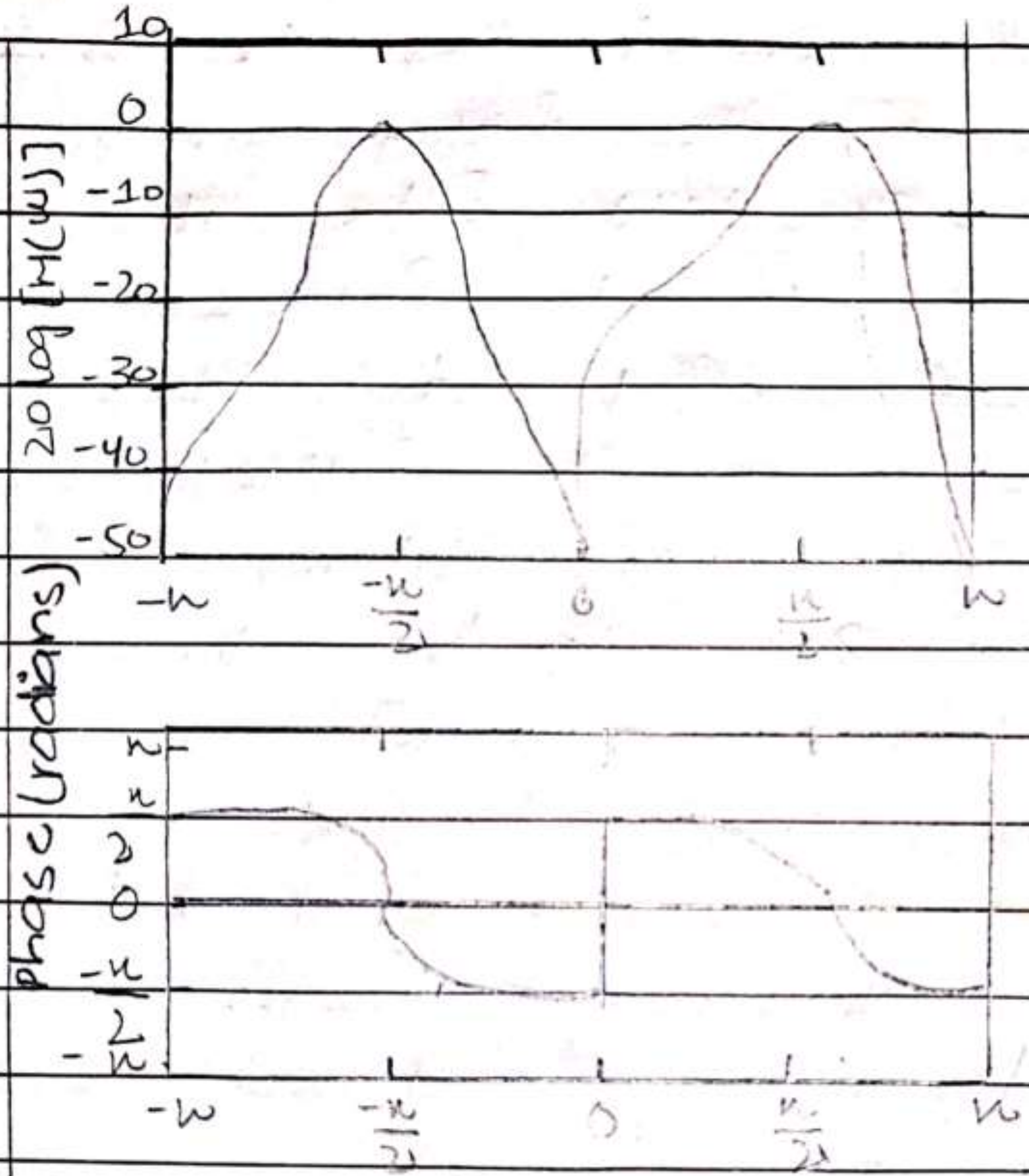

It should be emphasized that the main purpose of the foregoing methodology for designing simple digital filter by pole-zero placement is to provide insight into the effect that poles and zero have on the frequency response characteristic of system.
A simple band pass filter transformation.
The methodology is not intended as a good method for designing digital filter with well-specified passband and stopband characteristic. Systematic method for design of sophisticated digital filter for practical application are discuss.

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Q 4 (a)

A finite duration sequence of length L is given a

$$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the N-Point DFT of this sequence ofor N>L

Solution:

The Fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude of and Phase of $X(\omega)$ are illustrated in Fig fo L=10. The N Point DFT of $x(n)$ is simply $X(\omega)$ evaulated at the set of N equally spaced frequencies $\omega_k = 2\pi k/N$, k=0,0 ...... ,N-1 Hence

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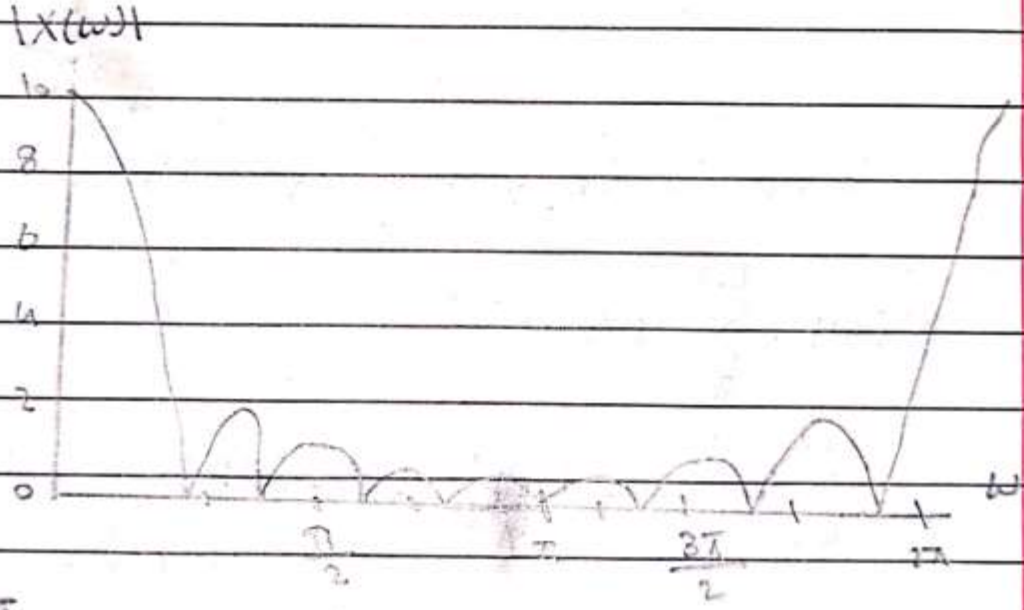
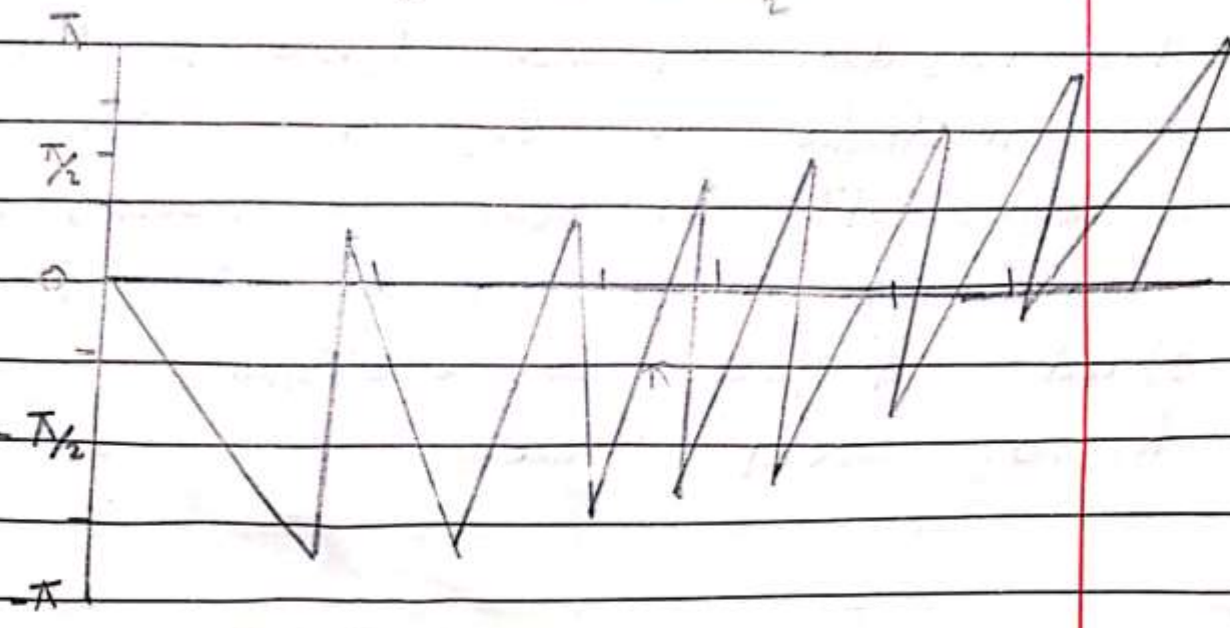
$$X(k) = \frac{1 - e^{-j2\pi kL/N}}{1 - e^{-j2\pi k/N}}, \quad k = 0, 1, \ldots N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$





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The Discrete Fourier Transform: Its Properties and Application

If N is is selected such that N = L, then the DFT becomes

$$X(K) = \begin{cases} L, & K = 0 \\ 0, & K = 1, 2, \ldots L-1 \end{cases}$$

Thus there is only one nonzero value in the DFT. This is apparent from observation of $X(\omega)$, since $X(\omega) = 0$ at the frequencies $\omega_k = 2\pi K/L$, $K \neq 0$. The reader should be verify that $x(n)$ can be recovered from $X(K)$ by performing an L-point IDFT.

Although the L-point DFT is sufficient to uniquely represent the sequence $x(n)$ in the frequency domain, it is apparent that it does not provide sufficient detail to yield a good picture of the spectral characteristic of $x(n)$.

If we wish to have a better

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batter Picture we must evaluate X(w) at more closely spaced frequencies, say $w_k = 2k\pi/N$ where $N > L$. In effect, we can view this computation as expending the size of the sequence from L point to N point by appending N-L zeros to the sequence x(n) that is zero padding. Then the N-point DFT provides finer interpolation than the L-point DFT.

Figure provides a plot of the N point DFT, in magnitude and phase for L=10, N=50 and N=10. Now the spectral characteristic of the sequence are more clearly evident, as one will conclude by comparing these spectra with the continuous spectrum X(w)

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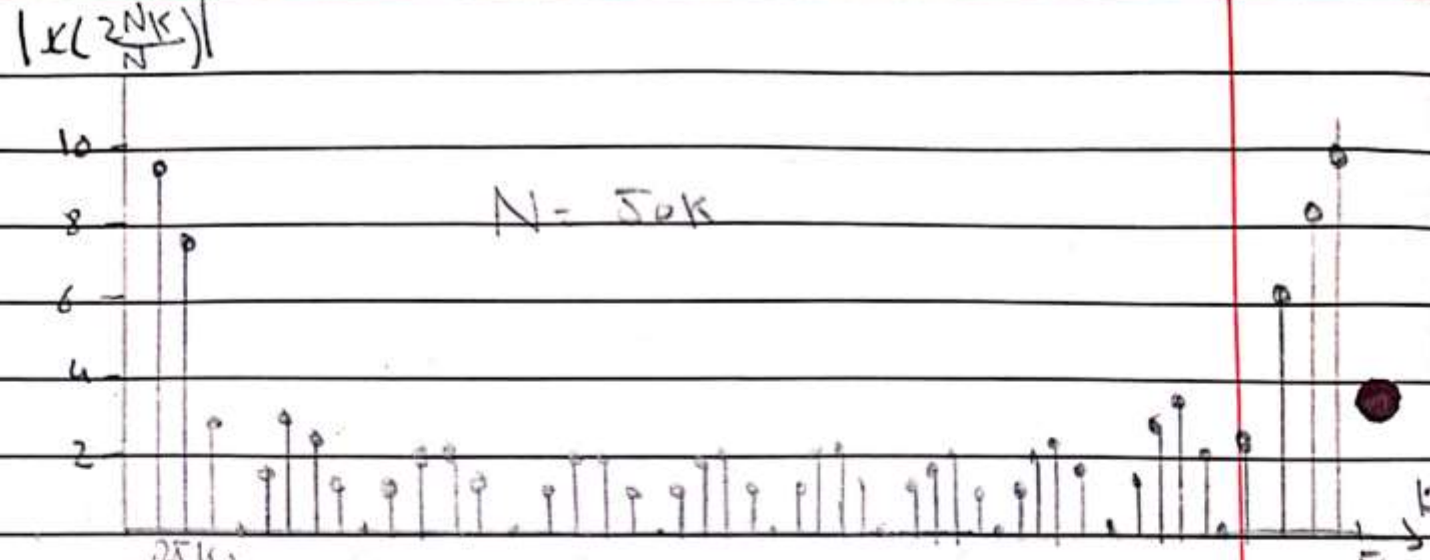

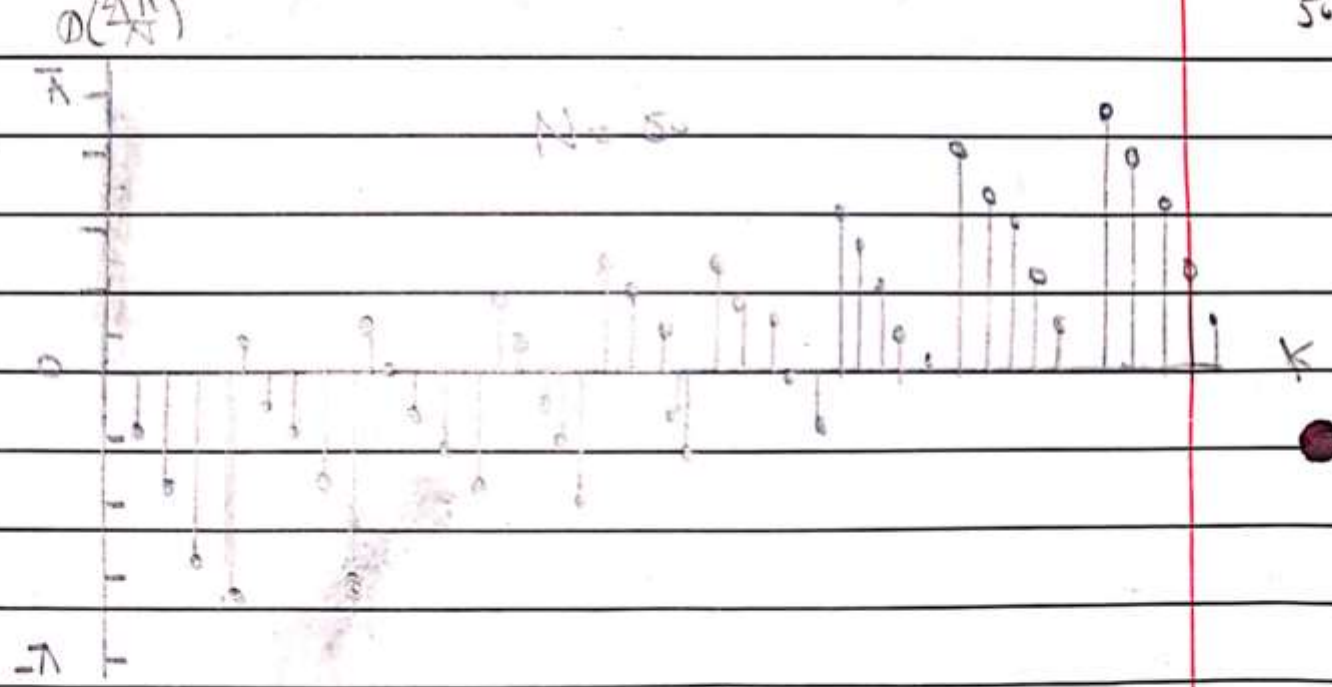

Magnitude and Phase of an N-Point DFT in

$L = 10$ $N = 50$; $L = 10$ $N = 16$

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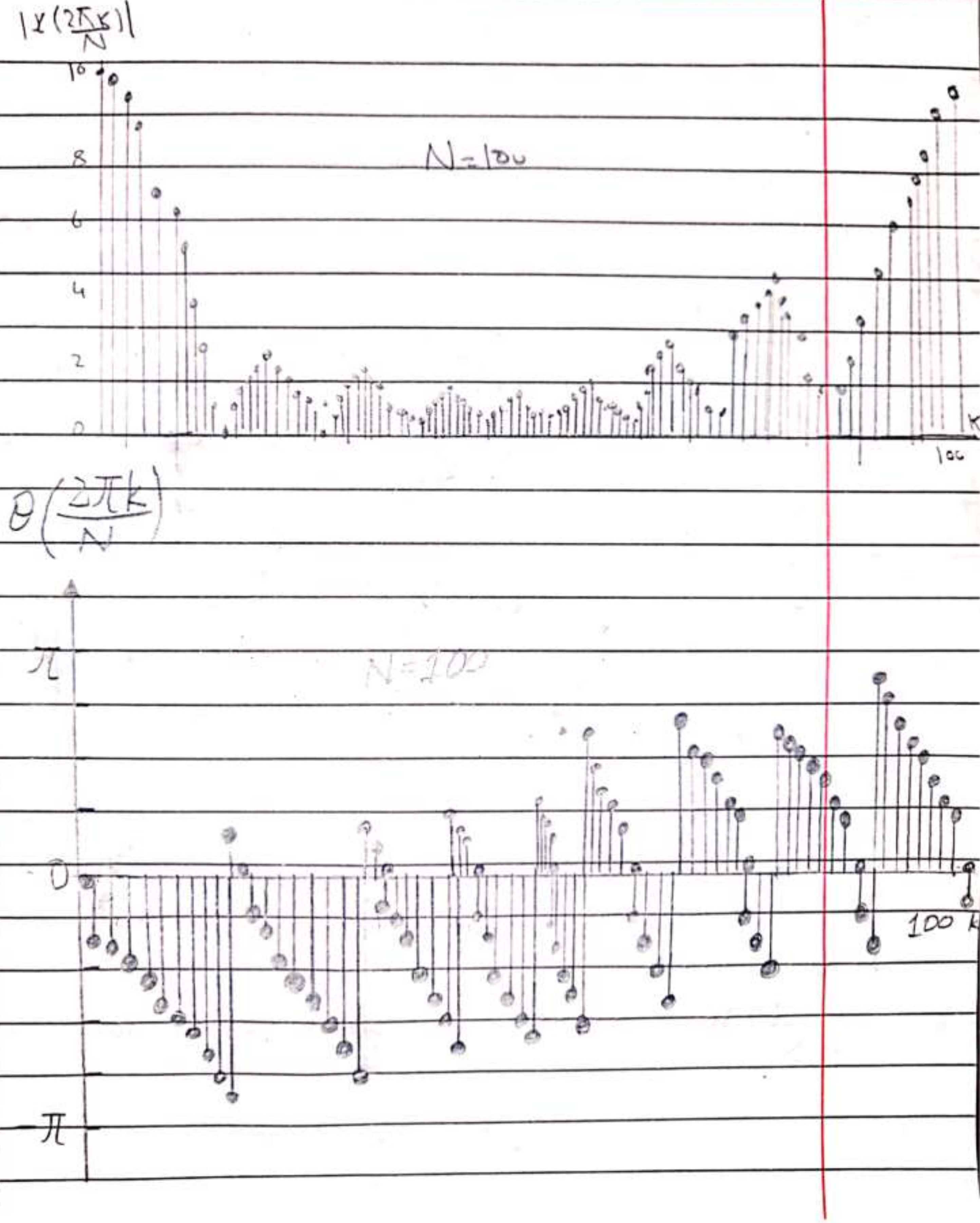

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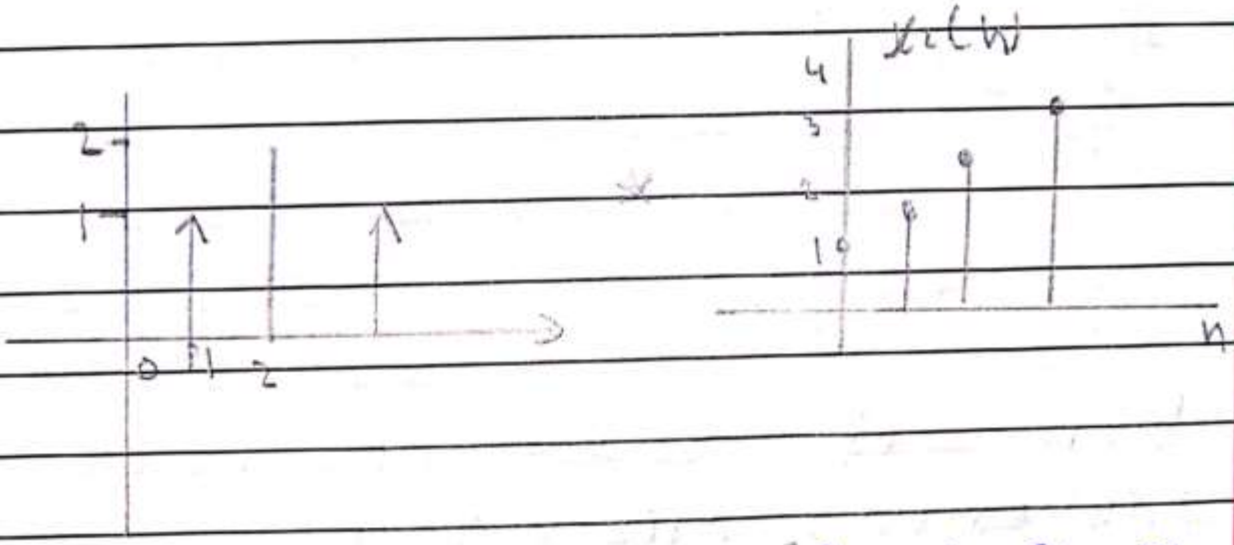
Q 4 Part (B)

Solution:

$x_1(n) = \{2, 1, 2, 1\}$ (arrow under 2)

$x_2(n) = \{1, 2, 3, 4\}$ (arrow under 1)

$x_1(n) * x_2(n) =$



$x_1(n) = 2\delta(n) + \delta(n-1) + 2\delta(n-2) + \delta(n-3)$

$x_2 = \delta(n) + 2\delta(n-1) + 3\delta(n-2) + 4\delta(n-3)$

$x_1(n) * x_2(n)$

$= [\delta(n) + 2\delta(n-1) + 3\delta(n-2) + 4\delta(n-3)] \times$

$[2\delta(n) + \delta(n-1) + 2\delta(n-2) + \delta(n-3)]$

$= 2\delta(n) + \delta(n-1) + 2\delta(n-2) + \delta(n-3) + 2\delta(n-1)$

$+ 2\delta(n-2) + 4\delta(n-3) + 2\delta(n-4)$