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Sec :- B

Subject :- Advanced Fluid Mechanics

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(1)

Question No 1 Part (A)

Define Drag with its components. write down the equations for friction Drag coefficient both in laminar and turbulant boundary layer.

Answer

Forced on Immersed Bodies

A body which is whole immersed in a homogenous fluid may be subject to two kind of forces arising from relative motion between body and fluid. These force are termed as drag and lift depending on whether force is Parallel or at right angle to motion.

Drag forces on submerged body can have two components.

(1) Pressure Drag F_p => It is equal to the integration of component in the direction of motion of all Pressure force exerted on the surface of body

$$F_p = C_p \rho \frac{V^2}{2} A$$

c_p - depend on shape

(2) Friction Drag :-

It is equal to integration of components of shear stress along the surface of the body in direction of motion.

$$F_f = C_f \rho \frac{V^2}{2} BL$$

$C_f \rightarrow$ depends on viscosity

Friction Drag of Boundary Layer

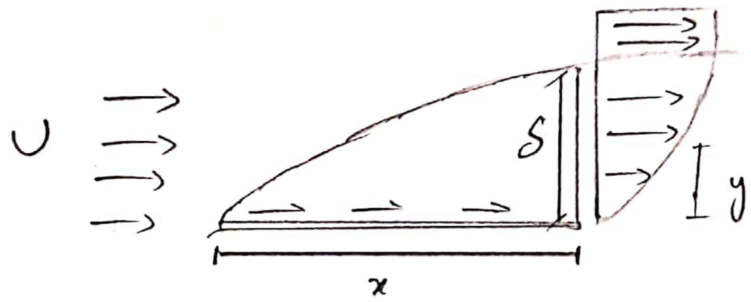
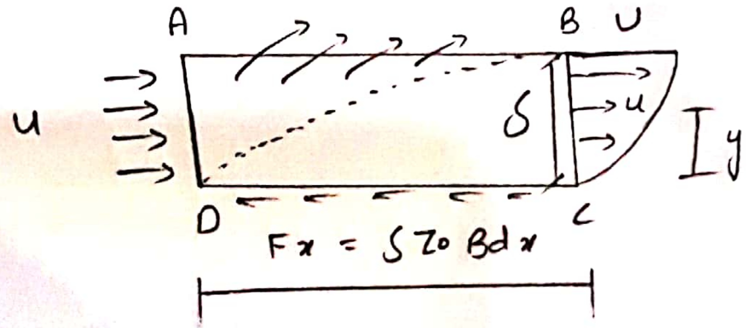


Fig shows growth of boundary layer along one side of smooth plate in steady flow of incompressible fluid. consider is control volume.



where δ is distance from boundary layer to plate - U is undistrubed velocity.

③

Now $-F_x = -\text{drag} =$ rate of momentum in x - direction leaving through BC

+ rate of momentum in x - direction leaving through AB - rate of momentum in x direction entering through DA.

According to Impulse momentum Principle :-

$$\Sigma F = \frac{d(mv)}{dt} = \frac{(\rho \times Vd) \times V}{dt} = \rho Q V$$

$$\Sigma F_x = \rho Q_2 V_2 - \rho Q_1 V_1$$

DA $\rightarrow$ $\rho U (U \times B \times \delta)$

CB $\rightarrow$ $\rho B \int_0^\delta u^2 dy$

AB $\rightarrow$ $\rho \left(U \times B \times \delta - B \int_0^\delta u\, dy \right) U$

Putting value

$$F_x = \rho B \int_0^\delta u(U - u)\, dy$$

Solving this $F_x = \rho B U^2 \delta \alpha$ where α the function of boundary layer velocity condition distribution.

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Now to find local wall shear stress

$$\tau = \frac{F_x}{Area} \Rightarrow \tau_0 = \frac{dF_x}{B \cdot dx}$$

$$F_x = \rho B U^2 \delta \alpha$$

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$

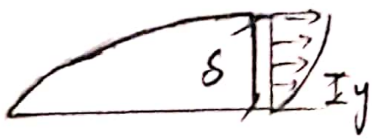
Laminar Boundary layer :-

In case of laminar flow,

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = \frac{\mu}{\delta}\left(\frac{du}{d\eta}\right) = \frac{\mu U}{\delta}\left[\frac{df(\eta)}{d\eta}\right]$$

By solving

$$\tau_0 = \frac{\mu U \beta}{\delta} \rightarrow (1)$$



$$\frac{u}{U} = f\left(\frac{y}{\delta}\right)$$

$$\eta = \frac{y}{\delta}$$

$$\therefore \frac{u}{U} = f(\eta)$$

$$u = U f(\eta)$$

Equation $\Rightarrow \tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

$$\delta \, d\delta = \frac{\mu \beta}{\rho U \alpha} dx$$

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Solving it

$$\frac{\delta^2}{2} = \frac{\mu \beta}{\rho U \alpha} x + c$$

At $x = 0, \ \delta = 0 \ . \ c = 0$

$$\delta = \sqrt{\frac{2 \mu \beta x}{\rho U \alpha}} = \sqrt{\frac{2\beta}{\alpha}} \cdot \frac{x}{\sqrt{R_x}} \qquad R_x = \frac{x U \rho}{\mu}$$

Experimentally.

$\beta = 1.63, \ \alpha = 0.135$ putting values in (1) $\rightarrow$

$$\frac{\delta}{x} = \sqrt{\frac{2 \times 1.63}{0.135}} \times \frac{1}{\sqrt{R_x}} = \frac{4.91}{\sqrt{R_x}}$$

where R_x may be called the local reynolds number. It should be noted that R_x increases linearly in down stream direction.

Now

$$F_f = B \int_0^L Z_0 \, dx \Rightarrow$$

$$Z_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

$$R_x = \frac{x U \rho}{\mu}$$

Thus

$$F_f = 0.664 B \sqrt{\rho \mu L U^3}$$

where

$$F_f = C_F \rho \frac{V^2}{2} BL$$

$$C_F = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \frac{1.328}{\sqrt{R}}$$

where R is based on characteristics length of whole plate. The laminar boundary layer will remain laminar if R_x is of about 500,000.

Question No 1 Part (B)

Derive equation for critical depth. critical velocity of rectangular section of a channel.

Answer :-

Specific energy :- It is defined as the energy head referred to channel bed to datum.

Thus $$E = y + \frac{v^2}{2g}$$

If channel is uniform depth and relatively wide, flow near center of channel will be unaffected by side boundaries. Thus.

Flow q, Per unit width b can be expressed as $q = Q/b$

Now average velocity will be

$$V = \frac{Q}{A} = \frac{qb}{by} = q/y$$

⑧

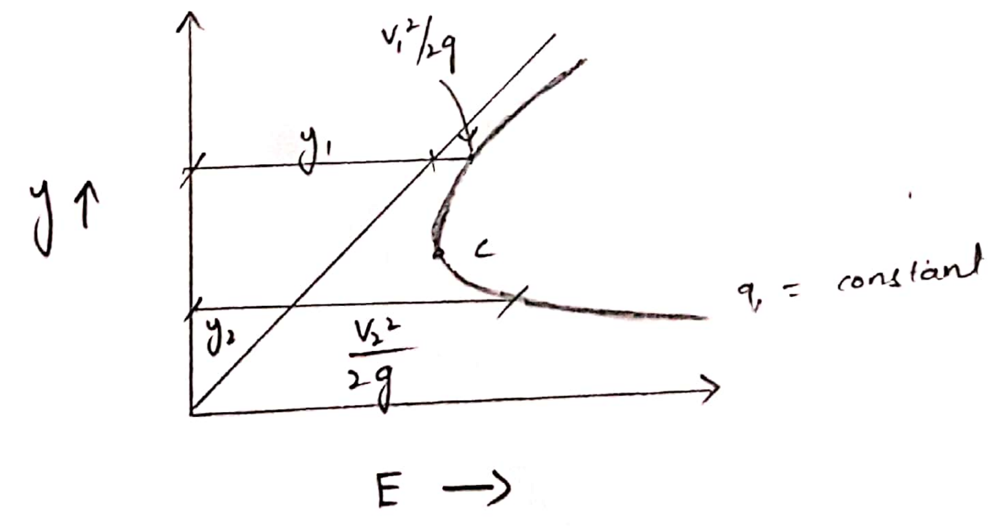
Thus $$E = y + \frac{1}{g}\left(\frac{q^2}{y^2}\right)$$

Lets consider how E will vary with y if q remains constant

Thus $$(E-y) = \frac{q^2}{2g(y^2)}$$

$$(E = y)\, y^2 = \frac{q^2}{2g} = \text{constant}$$

Plot of E Vs y is Parabola



This is specific energy diagram.

For particular q, There will be two kind of possible values of y. for given E

The equation is cubic with three roots with third root 3 being negative, giving no value (Physical meaning). Thus

(9)

Thus two alternate depth represent two totally different flow regimes - slow and deep on upper portion and fast & shallow on lower portion.

Point represent dividing point between two regimes of flow - Thus for given q, value of E is minimum and flow at this point is critical flow. Depth of flow at this point is critical depth y_c and velocity at this point is critical velocity.

Thus relation of critical depth can be found as.

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

for minimum spec. energy $\frac{dE}{dy} = 0$

$$\frac{dE}{dy} = 1 + \frac{\cancel{2}}{\cancel{2}g}\left(\frac{q^2}{y^3}\right)$$

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$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3} \Rightarrow 0$$

$$1 = \frac{q^2}{gy^3} \Rightarrow q^2 = gy^3 \quad \text{or} \quad \frac{q^2}{g} = y^3_{cr}$$

$$q^2 = gy^3 \Rightarrow y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

As $q = V_c y \quad \therefore \quad V_c^2 = g y_3$

or

$$V_c = \sqrt{g y_c} \quad \text{or}$$

$$\boxed{y_c = \frac{V_c^2}{g}}$$

Now

$$\frac{y_2}{2} = \frac{V_c^2}{2g}$$

$$E_{min} = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2}$$

$$= \frac{3}{2} y_c \quad \text{or} \quad \boxed{y_{cr} = \frac{2}{3} E_{min}}$$

(11)

Question No 02 :-

Find depth of a rectangular channel if water flows at the rate of $3.5 m^3/s$ with bed slope of 0.0008 and n= 0.0219. width of bed is your student ID number in mm.

Also find the critical depth, critical velocity:

Is flow sub-critical or super critical?

Given data

$Q = 3.5 \ m^2/s$

$S_o = 0.0008$

$n = 0.0219$

$b = 7847 \ mm = 7.847 m$

Required data:

$y = ?$

$y_{cr} = ?$

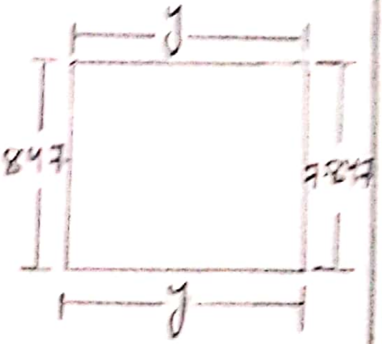
$V_{cr} = ?$

Sol :- (12)

$$Q = \frac{1}{n} \cdot A \cdot R_h^{2/3} \cdot S_o^{1/2}$$

$A = y \times b$

$= y \times 7.847$



$$R_h = \frac{A}{P} = \frac{7.847y}{2y + 15.694}$$

Putting values in eq, (1)

$$3.5 = \frac{1}{0.0219} \times 7.847y \times \left(\frac{7.847y}{2y+15.694}\right)^{2/3} (0.0008)^{1/2}$$

$y = 0.7222 \, m$

$y = 722.8 \, mm$

Now

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$q = Q/b = \frac{3.5}{7.847} = 0.446 \, m^2/s$$

$$y_{cr} = \left(\frac{0.446^2}{9.81}\right)^{1/3}$$

$y_{cr} = 0.2726 \, m$

$y_{cr} = 272.6 \, mm$

(13)

$\rightarrow \quad V_{cr} = ?$

$$V_{cr} = \sqrt{g y_{cr}}$$

$$V_{cr} = \sqrt{9.81 \times 0.272}$$

$$\boxed{V_{cr} = 1.63 \ m/s}$$

Since $\quad y > y_{cr}$

Thus the flow is subcritical

Question No 03:-

Find the friction drag on one side of a smooth plate with 200 mm wide and 800 mm length Placed longitudinally in stream of oil with specific gravity of 0.89. The undisturbed velocity is 5 m/s and kinetic viscosity is $0.93 \times 10^{-4} m^2/s$.

Given data:-

wide. $B = 200\ mm$

length. $L = 800\ mm$

Specific gravity $= 0.89$

undisturbed velocity, $U = 5$

kinetic viscosity. $\nu = 0.93 \times 10^{-4} m^2/s$.

solution:-

As we know that

$$R = \frac{LU}{\nu}$$

(15) $\nu = 0.93 \times 10^{-4} \, m^2/s$

$L = 0.80$

$U = 5$

Putting these value

$$R = \frac{0.80 \times 5}{0.93 \times 10^{-4}} = 43010 < 500,000$$

Thus

$$C_f = \frac{1.328}{\sqrt{R}}$$

$$= \frac{1.328}{\sqrt{43010}} = 0.0064$$

Now

$$F_f = C_f \, \rho \frac{V^2}{2} \times BL$$

$$F_f = 0.006 \times 0.89 \times 1000 \times \frac{(5)^2}{2} \times 0.20 \times 0.80$$

$$= 53.4$$

To find the thickness of boundray layer.

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$$\frac{\delta}{x} = \frac{4.91}{\sqrt{Rx}} \quad \text{at} \quad x = L$$

$$\delta = \frac{4.91}{\sqrt{43010}} \times 80 \text{ cm}$$

$$\delta = 1.89 \text{ cm}$$

$$F_f = 0.664 \times B \sqrt{\rho U . L U^3}$$

$$= 0.664 \times 0.20 \sqrt{0.89 \times 1000 \times 1000 \times 0.89 \times 0.93 \times 10^{-4} \times 0.80 \times (5)^3}$$

$$F_f = 11.39 \text{ N}$$