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Final Semester Assignment

Q no 1 => A man throws two fair dice. what is the Conditional probability that the Sum of two dice will be 7, given that

(1) The Sum is even

(2) The Sum is greater than 8

(3) The two dice had the Same outcome

Solution

Sample Space for this experiment is

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1)$
$(2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2) (3,3)$
$(3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5)$
$(4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1)$
$(6,2), (6,3), (6,4), (6,5), (6,6)\}$

Let $A = \{$ The Sum is $7\}$

↳ $A = \{(1,6), (2,5), (3,4), (4,3), (5,2) (6,1)\}$

$B = \{$ The Sum is even $\}$

↳ $B = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6)$
$(3,1), (3,3), (3,5), (4,2), (4,4), (4,6)$
$(5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

C = The Sum is greater than 8

↳ C = {(3,6), (4,5), (4,6), (5,4), (5,5), (5,6)
(6,3), (6,4), (6,5), (6,6)}

D = The two dice had the Same Outcome

↳ D = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

Now

$A \cap B = \emptyset$

$A \cap C = \emptyset$

$A \cap D = \emptyset$

$P(A) = \frac{6}{36}$, $P(B) = \frac{18}{36}$, $P(C) = \frac{10}{36}$

$P(D) = \left(\frac{6}{36}\right)$

Hence $P(A/B) = \frac{P(AB)}{P(B)} = 0 \times \frac{36}{18} = 0$

$P(A/C) = \frac{P(A \cap C)}{P(C)} = 0 \times \frac{36}{10} = 0$

$P(A/D) = \frac{P(A \cap D)}{P(D)} = 0 \times \frac{36}{6} = 0$

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Q no 2 Solution

Sample Space for this experiment is

S = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1)
(2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3)
(3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4)
(4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5)
(5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

Throwing more than 7 is equal to less than 7

more than 7

A = (2,6), (3,5), (3,6), (4,4), (4,5), (4,6)
(5,3), (5,4), (5,5), (5,6), (6,2), (6,3)
(6,4), (6,5), (6,6)

A = 15/36

Less than 7

B = [(1,1), (1,2), (1,3), (1,4), (1,5)
(2,1), (2,2), (2,3), (2,4), (3,1)
(3,2), (3,3), (4,1), (4,2), (5,1)]

Less than 7

$$B = \frac{15}{36}$$

$A = B \quad = \frac{15}{36} \quad = \frac{15}{36}$

Exact 7

$C = [(1,6), (2,5), (3,4), (4,3)$
$(5,2), (6,1)]$

$\boxed{C = \frac{6}{36}}$ 6 is the probability of exact 7

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Q3 :- A and B play a game in which A's probability of winning is 2/3 in a Series of '8'. games what is the probability of A will win

(1) Exactly 4 Games

(2) Atleast 4 Games

(3) from 3 to 6 Games.

Sol

Given that $p = 2/3$ $\quad n = 8$

$q = 1 - p$

$q = 1 - 2/3$

$q = 1/3$

'X' denotes the number of Games won by A

(i) $p(X=4) = \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4$

$= \frac{1120}{6561} = \boxed{0.1707}$

(ii) $p(x \geq 4)$

$1 - p(x < 4)$

$1 - \sum_{x=0}^{3}\binom{8}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{8-x}$

$$1 - \left[\left(\frac{1}{3}\right)^8 + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^7 + 28\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6 + 56\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5\right]$$

$$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561} = \frac{5984}{6561} = \boxed{0.912}$$

(iii) $p(3 \leq x \leq 6)$

$$\sum_{x=3}^{6} \binom{8}{x}\left(\frac{2}{3}\right)^{x}\left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{4} +$$

$$\binom{8}{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{3} + \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{3^8}\left[56 + 140 + 224 + 224\right.$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = 0.7852$$

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Q no 5

Solution $\Rightarrow$ $P(X=x) = (n/x)\, p^{(x)} (1-p)^{n-x}$

$x = 0, 1, 2, 3 \text{-----} n$

$\mu p = np, \quad \sigma^2 - np(1-p)$

$$E(x) = \sum_{n=0}^{n} x \binom{x}{x} p^{x} (1-p)^{n-n}$$

$$\sum_{x=0}^{n} x \frac{1}{x!(n-x)!} px (1-p)^{n-x}$$

$$\sum_{x=0}^{n} = \frac{n!}{(x-1)(n-x)!} p^{x} (1-p)^{n-x}$$

$x = 0$

$y = x-1, \quad m = n-1$

$x = y+1, \quad n = m+1$

$$E(x) = \sum_{y=0}^{m} \frac{m+1!}{y!(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)\, p \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y} (1-p)^{m-}$$

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y} (1-p)^{m-y}$$

Binomial theorm Say that

$$(a+b)^m = \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

Setting $a = p$. $b = 1-p$

$$= \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^y b^{m-y} = (a+b)^m = (p+1-p)^m = 1$$

So that $E(X) = np$ derived

Now Using $y = x-2$ and $m = n-2$

$$E(X(X-1)) = \sum_{x=0}^{n} x(x-1) \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} = n(n-1) \frac{n!}{x!(n-x)} p^x (1-p)^{n-x}$$

$$\sum_{x=2}^{n} \frac{n!}{(x-2)(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1) p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)(n-x)!} p^{(x-2)} (1-p)^{n-x}$$

$$= n(n-1) p^2 \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= n(n-1) p^2 (p + (1-p))^m$$

$$= n(n-1) p^2$$

So the variance of X is

$$E(x^2) - E(X)^2 = E(X(X-1)) + E(X) - E(X)^2 =$$

$$n(n-1)p^2 + np - (np)^2$$

$$= np(1-p)$$

Hence derived

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Q6 :- Differentiate b/w Bi-nomial frequency distribution and Bi-nomial distribution and Bi-nomial distribution with the help of formulas?

Solution

If the binomial probability distribution is multiplied by N, The number of experiments or Sets, the resulting distribution is known as the bi-nomial frequency distribution

formulas :- $N \binom{n}{x} p^x q^{n-x}$

Bi-nomial Distribution

Many experiments Consists of repeated Independent trak. each trials having two possible outcome for example the two possible outcome of a trial may be head and Tail Success and failure. if the probability of each trial remain the Same through out the trials. are Called Binomial trials and the experiment is Called Binomial experiment.

Formula

$$P(X=x)\ f(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$x = 0, 1, 2 \text{ ---- } n$$

n = Number of Sample Size
x = The number of Success
p = The probability of Success
q = The probability of failure

Note

$$p+q=1, \quad q=1-p, \quad p=1-q$$

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Qno 7

Measure	Data Set A	B	C	D
Mean	45	60	50	25
SD	3	11	5	15
Sample Size	1500	3200	500	2700

find Co-efficient of variation

$$C.V = \frac{3}{X} \times 100$$

for A $= \frac{3}{45} \times 100 = 6.66$

For B $= \frac{11}{60} \times 100 = 18.33$

For C $= \frac{5}{50} \times 100 = 10$ nearest tauth

For D $= \frac{15}{25} \times 100 = 60$

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