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Section : B

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Program : B.Sc Civil Engineering

Assignment : Plain and Reinforced  
Concrete Design - I

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# QUESTION - 01

①

Explain in detail types of stirrups with figures and Also explain ACI codes for shear design.

Ans:

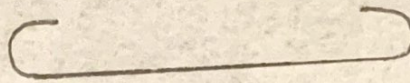
STIRRUP:

Stirrups are closed-loop bars tied at regular intervals in beam reinforcement to hold the bars in position.

Types of Stirrups :

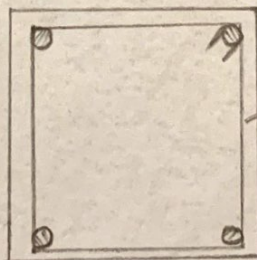
① Single legged Stirrup :

The single leg stirrups have rarely been used because they are mostly used when binding only two rods.



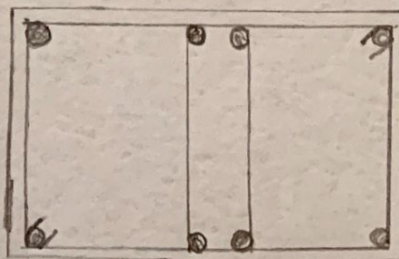
② Two legged Stirrup :

It is commonly and widely used stirrup. Minimum 4 bars are required for providing this stirrup.



2 legged stirrup

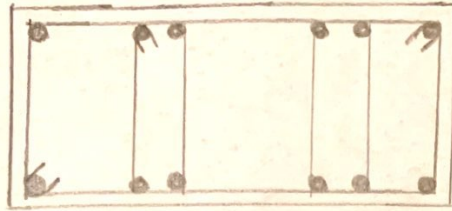
③ Four legged Stirrup :  
These stirrups are used in case of web reinforcement



4 legged stirrup

④ Sin legged Stirrup :

②



ACI CODES FOR SHEAR DESIGN OF A BEAM:

According to ACI-318, following are the formulas used for the shear design of a beam.

① Critical Section: It occurs at  $45^\circ$  and is at distance  $(d)$  from the face of support which is equal to effective depth.

② Shear Strength Capacity of concrete is

$$V_c = 2 \times \sqrt{f'_c} \times b_w \times d$$

③ Minimum web Reinforcement:

If  $V_u \leq \phi V_c$ , then theoretically no web reinforcement is required. However ACI code require provision of atleast a minimum area of web reinforcement equal to,

$$\phi = 0.75 \rightarrow \text{For shear design.}$$

( $\therefore V_u =$  Total factored shear applied at a given section)

$\Rightarrow$  For minimum reinforcement area:

$$A_{u \min} = \frac{0.75 \times \sqrt{f'_c} \times b_w \times S}{f_y} \quad \text{or} \quad \frac{S_0 \times b_w \times S}{f_y} \rightarrow \left[ \begin{array}{l} \text{Higher} \\ \text{value is} \\ \text{selected} \end{array} \right]$$

By interchanging the above formulas, we can obtain the formula for maximum spacing.

$$S_{\max} = \frac{A_u \times f_y}{0.75 \times \sqrt{f'_c} \times b_w} \quad \text{or} \quad \frac{A_u \times f_y}{S_0 \times b_w} \rightarrow \left[ \begin{array}{l} \text{lesser value} \\ \text{is selected} \end{array} \right]$$

④ No web - reinforcement is required if

③

$$V_u < \frac{1}{2} \phi V_c$$

→ between critical section " $V_u$ " and " $\phi V_c$ ", spacing b/w web requirement can be find by,

$$S = \frac{\phi \times A_u \times f_y \times d}{V_u - \phi V_c}$$

⑤ If  $V_s \leq 4 \times \sqrt{f'_c} \times b_w \times d$ , then max spacing for stirrups will be the smallest of the following.

1- 24'

2-  $d/2$

3-  $S_{max} = \frac{A_u \times f_y}{0.75 \times \sqrt{f'_c} \times b_w}$

4-  $S_{max} = \frac{A_u \times f_y}{50 \times b_w}$

⇒ If  $V_s > 4 \times \sqrt{f'_c} \times b_w \times d$   
↓  
max. spacing will be halved

⇒ If  $V_s > 8 \times \sqrt{f'_c} \times b_w \times d$   
↓  
Then either increase cross-sectional dimensions or increase  $f'_c$

## QUESTION - 02

(4)

A simply supported rectangular beam 14" wide having an effective depth of 22" to carry a lateral load of 6.5 k/ft on a 18' simple span. It is reinforced with 7 in<sup>2</sup> of tensile steel area, if  $f'_c = 4$  ksi and  $f_y = 60$  ksi, then design the beam for shear.

Given :

Breadth of web of beam ( $b_w$ ) = 14"

Effective depth ( $d$ ) = 22"

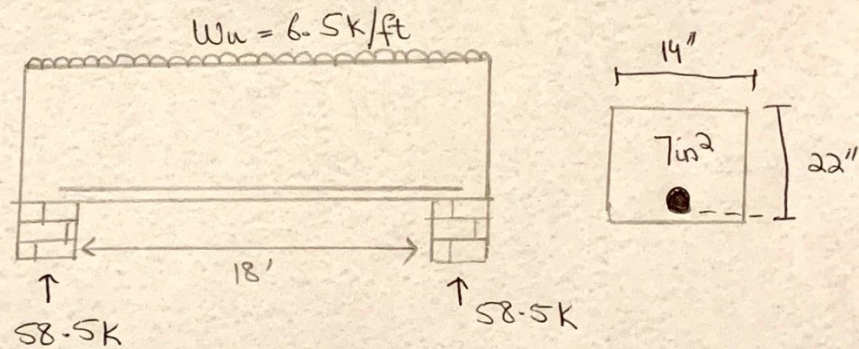
Given load = 6.5 k/ft

Steel Area = 7 in<sup>2</sup>

$f'_c = 4$  ksi

$f_y = 60$  ksi

Sol:



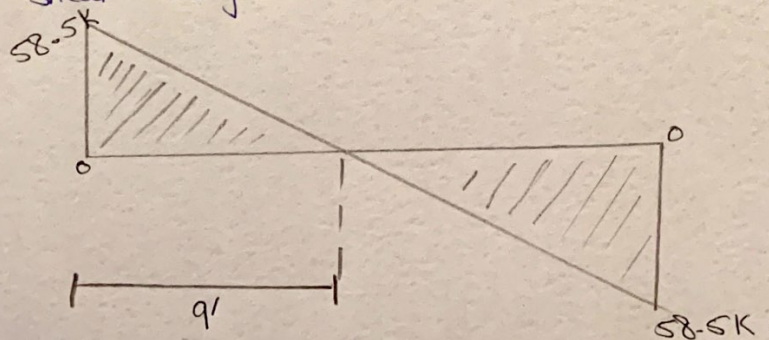
Step # 1 : (Reactions on Supports)

Finding the reactions due to applied load.

$$\text{Total load} = \frac{6.5 \times 18}{2} = 58.5 \text{ Kips}$$

Step # 2 : (Shear Force Diagram)

The required Shear diagram will be

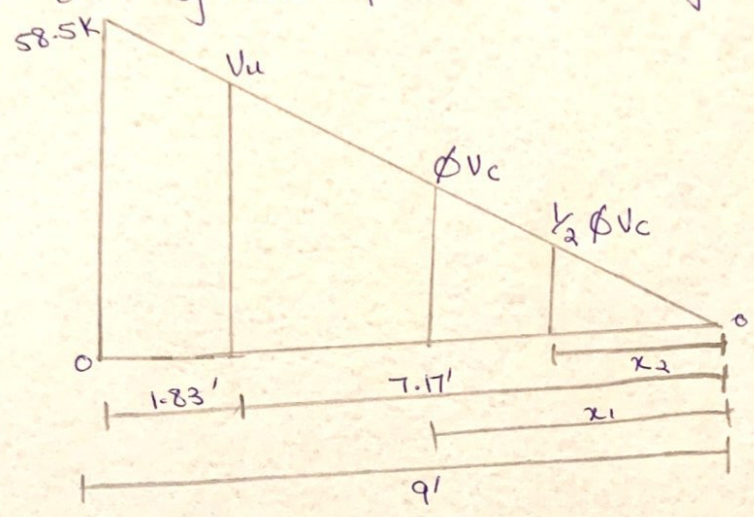


Step # 3:

Finding the value of critical shear "Vc" and its location.

As, we know that critical shear is located at distance "d" from face of support (d) = 2' = 1.83'

=> we will find the values of critical shear at distance "d" by use of similar triangles.



From similar triangles,

$$\frac{58.5}{9} = \frac{Vu}{8.17}$$

$$Vu = \frac{58.5 \times 7.17}{9}$$

$$Vu = 46.61 \text{ Kips}$$

Step # 4

Finding the value of 'phi Vc' and '1/2 phi Vc' and also the distances from zero shear to right side.

By formulas,

$$\Rightarrow \phi Vc = \phi \times 2 \times \sqrt{f'c} \times bw \times d$$

$$= 0.75 \times 2 \times \sqrt{4000} \times 14 \times 22 = 29219 \text{ lbs}$$

$$= 29.2 \text{ Kips}$$

=> location of phi Vc by similar triangles,

$$\frac{58.5}{9} = \frac{\phi Vc}{x_1} \Rightarrow \frac{58.5}{9} = \frac{29.21}{x_1}$$

$$= x_1 = 4.491'$$

=> Similarly,

$$\frac{1}{2} \phi Vc = \phi Vc / 2 \Rightarrow 29.21 / 2 = 14.60 \text{ Kips}$$

=> location of 1/2 phi Vc will be,

$$\frac{58.5}{9} = \frac{14.60}{x_2} \Rightarrow x_2 = 2.24'$$

STEP #5:

Finding the value of  $\phi V_c$   
 By formula,  $V_u = \phi V_s + \phi V_c$   
 $\Rightarrow \phi V_s = V_u - \phi V_c$   
 $= 46.61 - 29.2$

$$\phi V_s = 17.4 \text{ kips}$$

Step # 6:

check on section adequacy,

By formula,  
 $= \phi \times 8 \times \sqrt{f'_c} \times b_w \times d$   
 $= 0.75 \times 8 \times \sqrt{4000} \times 14 \times 22 = 116877 \text{ lbs}$   
 $= 116.87 \text{ kips}$

As  $\phi \times 8 \times \sqrt{f'_c} \times b_w \times d > \phi V_s$   
 So section is Adequate!

Step # 7:

check on maximum spacing for stirrups,

By formula,  
 $= \phi \times 4 \times \sqrt{f'_c} \times b_w \times d$   
 $= 0.75 \times 4 \times \sqrt{4000} \times 14 \times 22 = 58438 \text{ lbs}$   
 $= 58.43 \text{ kips}$

As  $\phi \times 4 \times \sqrt{f'_c} \times b_w \times d > \phi V_s$

so maximum will be selected from the following 4 conditions

- 1-  $S_{max} = 24"$
- 2-  $d/2 = 22/2 = 11"$
- 3-  $S_{max} = \frac{A_v \times f_y}{0.75 \times \sqrt{f'_c} \times b_w}$

Here we are using # 3  
 stirrup, dia =  $(\frac{3}{8})" = 0.375"$   
 so area =  $\frac{\pi}{4} (0.375)^2 = 0.11 \text{ in}^2$   
 For 2-legged stirrup  
 $\Rightarrow \text{area} \times 2$   
 $\Rightarrow 0.11 \times 2 = 0.22 \text{ in}^2$

$$3 - S_{max} = \frac{0.22 \times 60,000}{0.75 \times \sqrt{4000} \times 14} = 19.87''$$

$$4 - S_{max} = \frac{A_v \times f_y}{50 \times b_w} = \frac{0.22 \times 60,000}{50 \times 14} = 18.85''$$

from above 4 conditions, best value of spacing for #3 2 legged stirrup will be selected as,  
 $S_{max} = 11''$

Step 8:

Stirrups spacing from/at critical section will be, by formula,

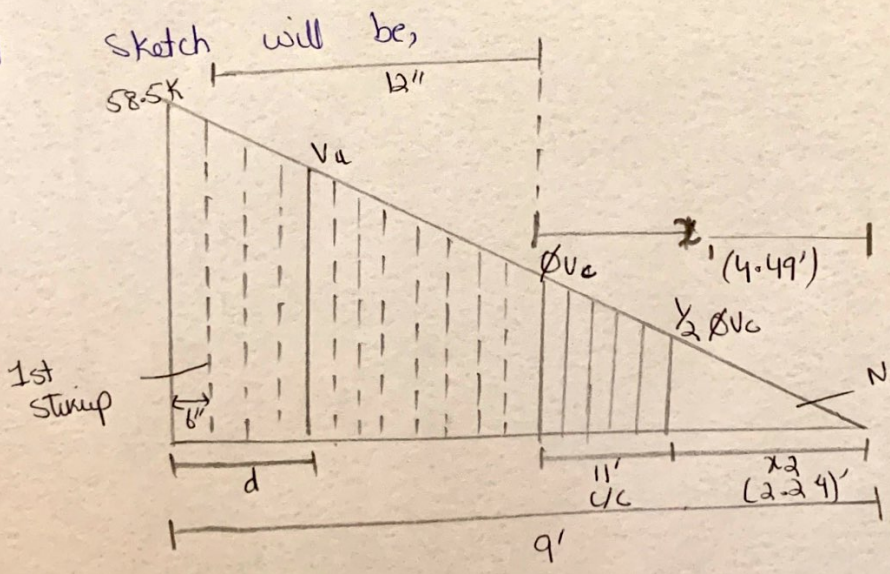
$$S = \frac{\phi \times A_v \times f_y \times d}{V_u - \phi V_c} = \frac{0.75 \times 0.22 \times 60 \times 22}{46.61 - 29.21}$$

$$S = 12.5'' \approx 12''$$

so 12" c/c

Step 9:

final



As first stirrup from face of support,  
 $\frac{S}{2} = \frac{12}{2} = 6''$



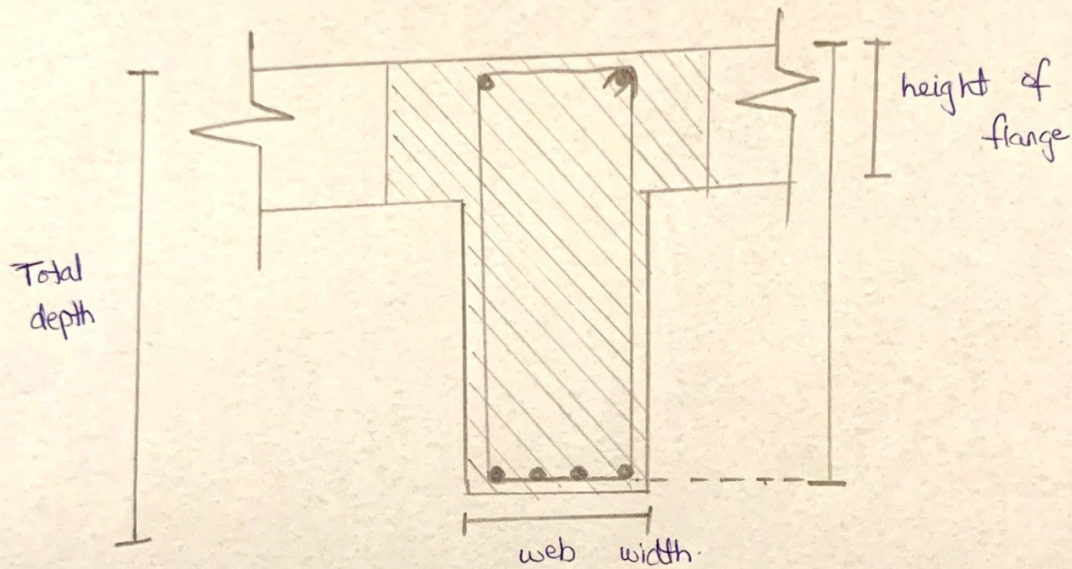
## QUESTION - 03

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Define both the T-beam & L-beam with the help of diagram. Also explain flexural analysis of T-beam.

T-Beam :

⇒ In most of the required concrete structures, concrete slabs are cast monolithically with slab so, in this case the beam that act as an intermediate beam are called T-beams.



⇒ Because of their T-shape, these beams are called T-Beams.

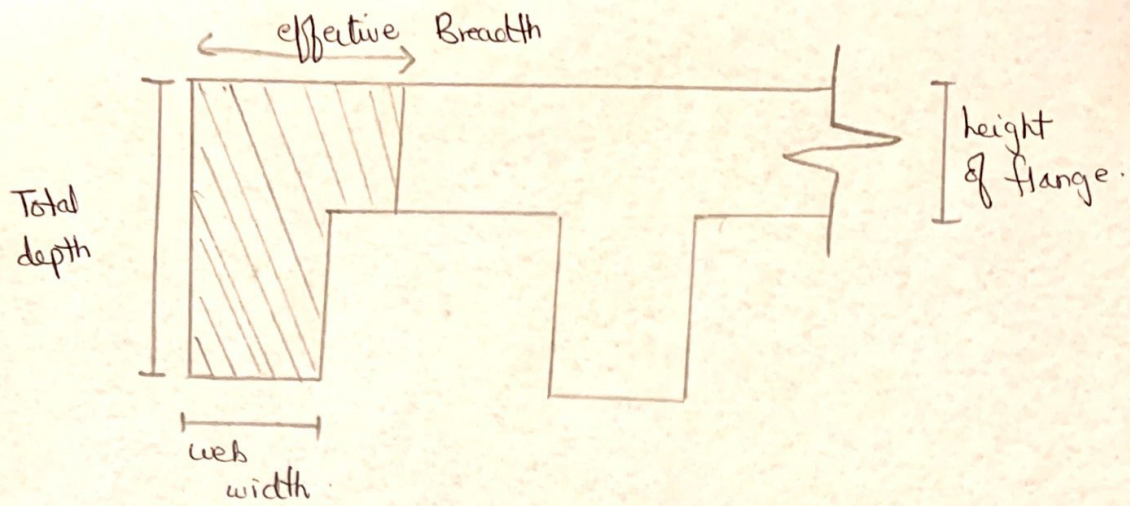
⇒ It is provided at the center of the slab to resist the loads.

⇒ The upper most area of the beam attached to the slab is called flange.

⇒ The bottom rectangular portion of the beam is called web of the beam.

L-Beam :

⇒ L-shaped structure that is in contact with the slab and present at the corner of the floor is called L-Beam.



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- => L-Beams are also called Edge Beams.
- => J's always provided at the corners of the slab.
- => L-Beams are typical floor beams because of their reduced overall structural depth, the beams are in prestressed or reinforced concrete.

### FLEXURAL ANALYSIS OF T-BEAM:

Flexural analysis of T-Beam consists of the following steps:

- 1- For finding the ultimate factored moment, we use the following formula,
 
$$M_u = \frac{W_u \times L^2}{8}$$
 (  $W_u$  = total factored load  
 $L$  = Total span of the beam )

2- Effective width ( $b_e$ ) for T-beam is calculated as:

- 1-  $16(h_f) + b_w$
- 2- c/c distance
- 3-  $\text{span}/4$
- 4-  $\frac{CTS + b_w}{2}$

$\therefore$  (  $h_f$  = height of flange  
 $CTS$  = clear transverse span )

- we've to select the least value from above formulas

- If c/c distance is given, then there's no need of "  $\frac{CTS}{2} + b_w$  "

3- Checking whether Rectangular or T-Beam Analysis is required. (10)

i- If  $a > hf \rightarrow$  special Analysis is required.

ii- If  $a < hf \rightarrow$  Rectangular beam Analysis is required.

where

( $a$  = depth of compression block)  
 ( $hf$  = height of flange)

4- For finding area of steel, we've to use.

$$A_{st} = \frac{M_u}{\phi \times f_y \times (d - a/2)}$$

where

$$a = \frac{A_{st} \times f_y}{0.85 \times f_c \times b_w}$$

$\phi$  = strength reduction factor  
 $d$  = effective depth  
 $a$  = compression block depth.  
 $b_w$  = web width of beam

5- For checking the range of reinforcement Ratio

$$S_{max} = 0.85 \times B \times \frac{f_c}{f_y} \times \left( \frac{E_s}{E_c + E_s} \right)$$

$$S_{min} = \frac{200}{f_y}$$

$$S = \frac{A_{st}}{b \times d}$$

6- Formula for finding no. of bars required is:

$$\text{no. of bars} = \frac{\text{area of steel}}{\text{area of single bar}}$$

7- For checking minimum width for bars accommodation,

$$b_{min} = 2(\text{clear cover}) + 2(\text{dia of stirrup}) + \text{no. of bars} \times (\text{dia of bar}) + \text{spacing of bars} \times (\text{dia of bar})$$

8- Design moment is given by.

$$M_d = \phi \times f_y \times A_{st} \times (d - a/2) \rightarrow \text{if } a < hf$$

$$M_d = \phi \times [A_s \times f_y \times (d - hf/2) + (A_s - A_{st}) \times f_y \times (d - a/2)]$$

$\downarrow$   
 if  $a > hf$ .

## QUESTION - 4

(ii)

What is the difference b/w case-1 & case-2 in the design of T-beam?

CASE - I :

from the figure

$$a < hf$$

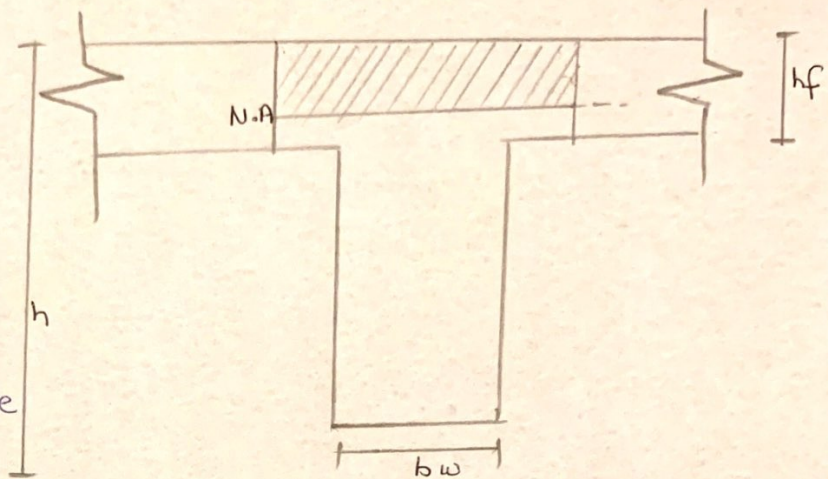
So in this case,

Rectangular Beam

Analysis is Required

So, The Design

Moment formula will be



$$M_d = \phi \times f_y \times A_{st} \times (d - a/2)$$

CASE - II :

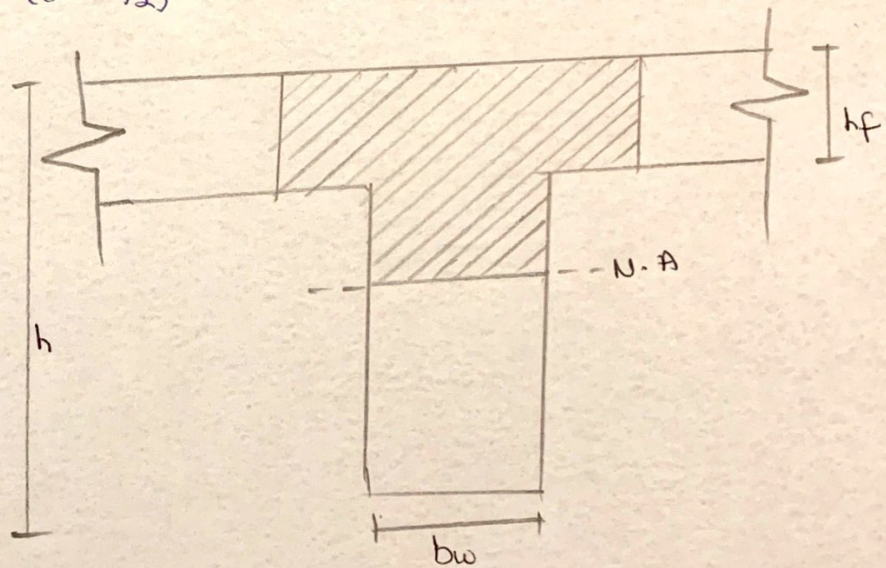
from the figure,

$$a > hf$$

So in this, special  
beam analysis i.e.,

T-Beam Analysis

is required.



So the required Design Moment  
will be,

$$M_d = \phi \times \left[ A_s \times f_y \times (d - \frac{hf}{2}) + (A_s - A_{st}) \times f_y \times (d - \frac{a}{2}) \right]$$

## QUESTION - 05

(12)

A floor system consists of 3.5" concrete slab supported by 16' simple span spaced at 9' c/c, the beam having a web width of 10" & effective depth of 18" and total height is 23". Calculate the necessary flexural reinforcement if the factored applied moment is 5800 kip-inch. Use  $f'_c = 3 \text{ ksi}$  &  $f_y = 60 \text{ ksi}$

Given :

$$\text{height of flange (} h_f) = 3.5''$$

$$\text{c/c distance} = 9'$$

$$\text{length / span of beam} = 16'$$

$$\text{web width (} b_w) = 10''$$

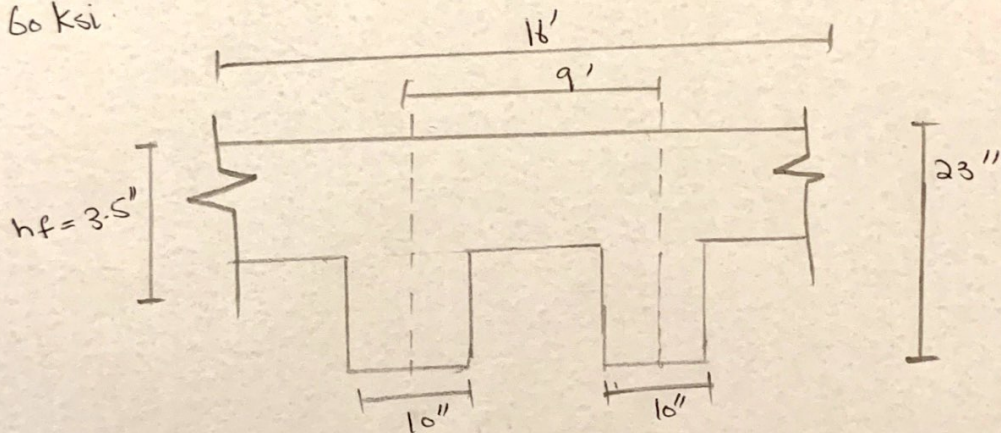
$$\text{effective depth (} d) = 18''$$

$$\text{height (} h) = 23''$$

$$\text{Total factored moment (} M_u) = 5800 \text{ kip inch.}$$

$$f'_c = 3 \text{ ksi}$$

$$f_y = 60 \text{ ksi.}$$



Step # 1

Calculate the effective width ( $b_e$ ) for T-beam.

$$1 - b_f(h_f) + b_w = 16(3.5) + 10 = 66''$$

$$2 - \text{c/c distance} = 9 \times 12 = 108''$$

$$3 - \text{Span}/4 = \frac{16}{4} \times 12 = 48''$$

Selecting the least value of  $b_e$  as,

$$\boxed{b_e = 48''}$$

Step # 2:

Check whether rectangular or T-Beam Analysis is required

Trial # 01 : let  $a = hf = 3.5''$

$$A_{st} = \frac{M_u}{\phi \times f_y \times (d - a/2)} = \frac{5800}{0.90 \times 60 \times (18 - 8.5/2)} = 6.61 \text{ in}^2$$

Trial # 02 :

$$a = \frac{A_{st} \times f_y}{0.85 \times f_c \times b_e}$$

$$a = \frac{6.61 \times 60}{0.85 \times 3 \times 48} = 3.2''$$

$= 3.2'' < 3.5''$

$\epsilon_t$   $A_{st} = 6.55 \text{ in}^2$

So Rectangular Beam Design is Required!

Trial # 03:  $a = 3.21''$

$$\epsilon_t A_{st} = \frac{5800}{0.90 \times 60 \left(18 - \frac{3.21}{2}\right)} = 6.55 \text{ in}^2$$

So Area of steel is  $6.55 \text{ in}^2$

Step # 3:

check  $S_{max}$  &  $S_{min}$

$$\Rightarrow S_{max} = 0.85 \times \beta \times \frac{f_c}{f_y} \left( \frac{\epsilon_u}{\epsilon_u + \epsilon_t} \right)$$
$$= 0.85 \times 0.85 \times \frac{3}{60} \left( \frac{0.003}{0.003 + 0.005} \right) = 0.013$$

$$\Rightarrow S_{min} = \frac{200}{f_y} = \frac{200}{60,000} = 0.0033$$

$$= S = \frac{A_{st}}{b \times d} = \frac{6.55}{10 \times 18} = 0.036$$

$$\rho_{min} < \rho < \rho_{max}$$

$$0.003 < 0.036 < 0.013$$

As the value of  $\rho_{max}$  is less than  $\rho$ , so we've to design it as "doubly reinforced Beam"  
=> first we've to find the area of steel against  $\rho_{max}$ .

$$\rho_{max} = \frac{A_{st}}{b \times d} \Rightarrow A_{st} = \rho_{max} \times (b \times d)$$

$$A_{st} = 0.013 \times (10 \times 18) = \boxed{2.34 \text{ in}^2}$$

Step # 4:

Finding the value of  $M_{u2}$ :

By formula,

$$M_{u2} = \phi \times A_{st} \times f_y \times (d - \frac{a}{2})$$

first finding the value of "a"

$$\Rightarrow a = \frac{A_{st} \times f_y}{0.85 \times f_c \times b} = \frac{2.43 \times 60}{0.85 \times 3 \times 10}$$

$$a = 5.72''$$

$$\Rightarrow M_{u2} = 0.90 \times 2.43 \times 60 \times (18 - \frac{5.72}{2})$$

$$M_{u2} = 1986.67 \text{ kip-inch}$$

$$\text{As } M_{u2} < M_u \\ 1986.67 < 5800$$

So we've to design the beam in such way that it can resist more bending moment than the applied external moment.

STEP # 5:

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Finding difference in moments & Area of steel.

$$M_{u1} = M_u - M_{u2} \\ = 5800 - 1986.67$$

$$M_{u1} = 3813.33 \text{ kip-inch}$$

By formula,

$$A_{st} = \frac{M_u}{\phi \times f_y \times (d-d')} = \frac{3813.33}{0.9 \times 60 \times (18.25)} = 4.56 \text{ in}^2$$

STEP # 6:

Finding total steel Area:

$$A_s = A_{st} + A_{st}' \\ = 2.43 + 4.56 = 6.99 \text{ in}^2$$

STEP # 7:

Selection of Bar:

In Tension Zone:

let we use # 8 bar

$$\text{dia} = (8/8) = 1'' , \text{ Area} = \frac{\pi}{4} (1)^2 = 0.785 \text{ in}^2$$

By formula:

$$\text{no. of bars} = \frac{\text{area of steel}}{\text{area of single bar}} = \frac{6.99}{0.785} = 8.9 \approx 9$$

In compression zone:

let we use # 7 bar

$$\text{dia} = (7/8)'' , \text{ Area} = \frac{\pi}{4} (7/8)^2 = 0.601 \text{ in}^2$$



By formula,  

$$\text{no. of bars} = \frac{\text{area of steel}}{\text{area of single bar}} = \frac{4.56}{0.601} = 7.5 \approx 8$$

so 8 # 7 bars.

STEP # 8:

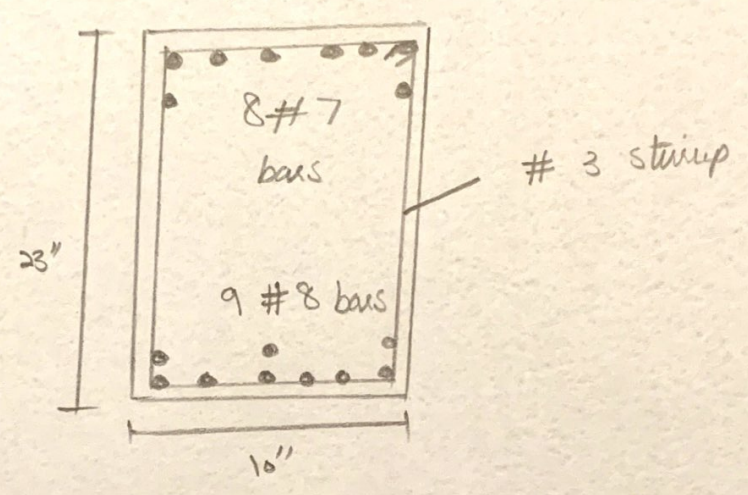
minimum width for accommodation of bars.

$$b_{min} = (2 \times 1.5) + (2 \times 3/8) + 9(8/8) + 8(8/8)$$

$$= 20.75''$$

As 20.75'' > 16''

so, the bars will be placed in multiple layers:



Effective depth (d) = 23 - 1.5 + 3/8 + 8/8 + 1/2 (8/8) = 19.6''

Effective cover (d') = 1.5 + 3/8 + 7/8 + 1/2 (7/8) = 3.18''

STEP # 9:

Finding the Design moment:

$$M_d = \phi [A_s \times f_y \times (d - d') + (A_s - A_{st}) \times f_y \times (d - a/2)]$$

Just a = 
$$\frac{(A_s - A_{st}) \times f_y}{0.85 \times f_c \times b} = \frac{(9 \times 0.785 - 8 \times 0.601) \times 60}{0.85 \times 3 \times 10} = 5.31''$$

$$\Rightarrow M_d = 0.90 [(8 \times 0.601) + 60 \times (19.6 - 3.18) + (9 \times 0.785 - 8 \times 0.601) \times 60 \times (19.6 - \frac{5.31}{2})]$$

As 6328.38 > 5800 → so design is ok!

## QUESTION-06:

(17)

A beam is required to develop ultimate moment of 6000 kip-inches limited to 14x26 inch size, use  $f'_c = 4 \text{ ksi}$  &  $f_y = 60 \text{ ksi}$ . Determine flexural reinforcement assume two rows of tensile reinforcement & effective depth of beam is 22 inches.

Breadth (b) = 14"

height = 26"

$f'_c = 4 \text{ ksi}$

$f_y = 60 \text{ ksi}$

$M_u = 6000 \text{ kip-inches}$

$d = 22"$

$d' = 2.5$  — assume.

Step # 1 : Reinforcement Ratio.

By formula,

$$S_{max} = 0.85 \times B \times \frac{f'_c}{f_y} \times \left( \frac{E_u}{E_u + E_y} \right)$$

$$= 0.85 \times 0.85 \times \frac{4}{60} \times \left( \frac{0.003}{0.003 + 0.005} \right) = 0.0180.$$

Step # 2 : Area of Steel.

As we know that.

$$S_{max} = \frac{A_{st}}{b \times d} \Rightarrow A_{st} = S_{max} \times (b \times d)$$

$$A_{st} = 0.0180 \times (14 \times 22) = 5.54 \text{ in}^2.$$

### STEP # 3 : Design Moment

$$M_{u2} = \phi \times A_{st} \times f_y \times (d - a/2)$$

$$\Rightarrow a = \frac{A_{st} \times f_y}{0.85 \times f_c \times b} = \frac{5.54 \times 60}{0.85 \times 4 \times 14} = 6.98''$$

$$\text{So } M_{u2} = 0.90 \times 5.54 \times 60 \left( 22 - \frac{6.98}{2} \right) = 5537.4 \text{ kip-inch}$$

$$\text{As } 5537.4 < 6000$$

so we've to design a section as doubly reinforced.

### Step #4 Difference in moments.

$$M_{u1} = M_u - M_{u2} = 6000 - 5537.4$$

$$M_{u1} = 462.6 \text{ kip-inches}$$

### Step # 5. Area of steel

$$M_{u1} = \phi \times A'_{st} \times f_y \times (d - d')$$

So area of steel in compression zone will be

$$\Rightarrow A'_{st} = \frac{M_{u1}}{\phi \times f_y \times (d - d')} = \frac{462.6}{0.90 \times 60 \times (22 - 2.5)}$$

$$= 0.44 \text{ in}^2.$$

Step # 6: Total Steel Area

$$A_s = A_{st} + A'_{st} \\ = 5.54 + 0.44 = 5.98 \text{ in}^2$$

Step # 7: Selection & no. of bars used.

1- Steel in Tension zone:

we use # 7 bar,

$$\text{dia} = (7/8)'' = 0.875'', \text{ area} = \frac{\pi}{4} (0.875)^2 \\ = 0.601 \text{ in}^2$$

$$\text{so, no. of bars} = \frac{A_s}{\text{area of single bar}} = \frac{5.98}{0.601} = 9.9 \approx 10 \text{ bars.}$$

so 10 # 7 bars.

2- Steel in compression zone:

we use # 5 bars,

$$\text{dia} = (5/8)'' = 0.625'', \text{ area} = \frac{\pi}{4} (0.625)^2 \\ = 0.306 \text{ in}^2$$

$$\text{so, no. of bars} = \frac{A_{st}'}{\text{area of single bar}} \\ = \frac{0.44}{0.306} = 1.43 \approx 2 \text{ bars}$$

so 2 # 5 bars.

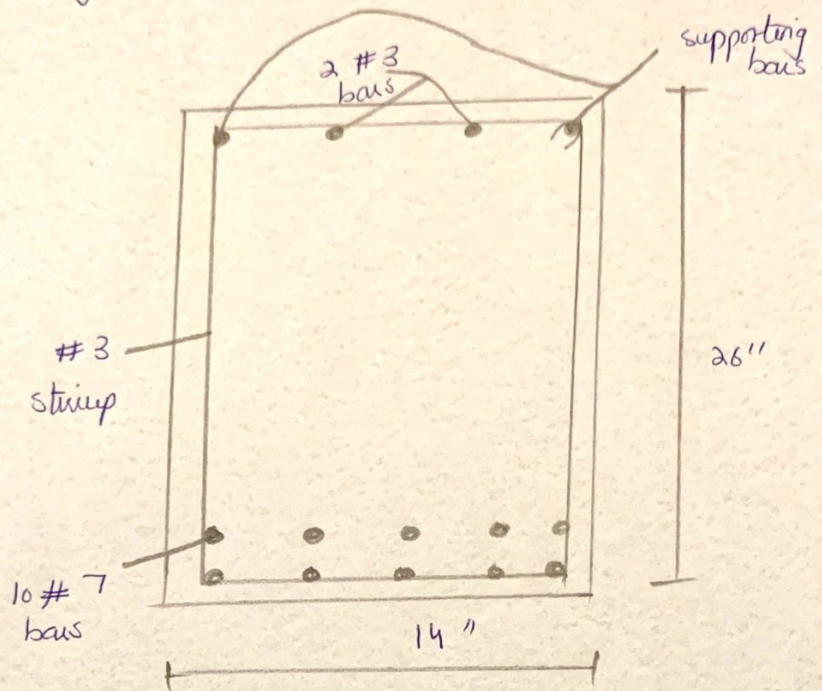
Step # 8 Minimum width of Beam.



$$b_{min} = 2(1.5) + 2(3/8) + 10(7/8) + 9(7/8)$$

$$b_{min} = 20.37 > 14''$$

so not good in one layer.



Now,

$$\Rightarrow \text{effective depth } (d) = 26 - 1.5 - 3/8 - 7/8 - \frac{1}{2}(7/8)$$

$$= 22.82''$$

$$\Rightarrow \text{effective cover } (d') = 1.5 + 3/8 + \frac{1}{2}(7/8)$$

$$= 2.18''$$

Step # 9 Design moment.

$$M_d = \phi \times [A_{st}' \times f_y \times (d - d') + (A_{st} - A_{st}') \times f_y \times (d - a/2)]$$

$$a = \frac{(A_{st} - A_{st}') \times f_y}{0.85 \times f_c' \times b}$$

$$= \frac{(10 \times 0.601 - 2 \times 0.306) \times 60}{0.85 \times 4 \times 14} = 6.80''$$

$$M_d = 0.90 [(2 \times 0.306) \times 60 (22.82 - 2.18) + (10 \times 0.601 - 2 \times 0.306) \times 60 \times (22.82 - \frac{6.80}{2})]$$

$$M_d = 7047.6 \text{ kip-inches}$$

$$A_s \quad 7047.6 > 6000$$

Design is ok!