Q.No 1

$P(4,1,3) = 4\hat{i} + \hat{j} + 3\hat{k}$

$Q(1,2,4) = \hat{i} + 2\hat{j} + 4\hat{k}$

Now distance b/w PQ

so $|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

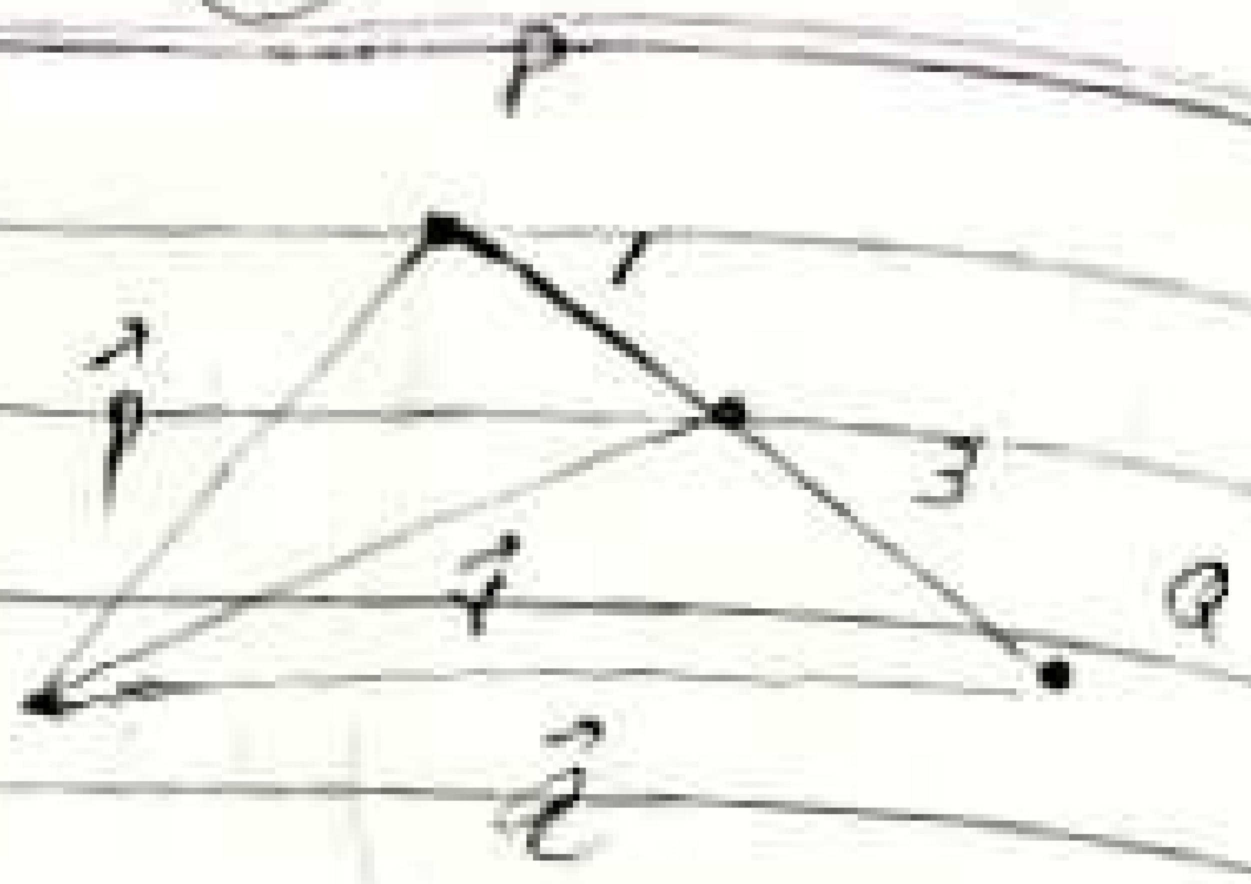
$= \sqrt{(4-1)^2+(1-2)^2+(3-4)^2}$

$= \sqrt{3^2+(-1)^2+(-1)^2}$

$= \sqrt{9+1+1} = \sqrt{11}$

Now find the position vector of the point dividing PQ in the ratio of 1:3

9



$a : b = 1 : 3$

$$\vec{r} = \frac{b\vec{p} + a\vec{q}}{b+a}$$

$$= \frac{3(4\hat{i}+\hat{j}+3\hat{k}) + 1(\hat{i}+2\hat{j}+4\hat{k})}{3+1}$$

$$= \frac{12\hat{i}+3\hat{j}+9\hat{k}+\hat{i}+2\hat{j}+4\hat{k}}{4}$$

$$= \frac{13\hat{i}+5\hat{j}+13\hat{k}}{4}$$

$$\boxed{\vec{r} = \frac{13}{4}\hat{i} + 5/4\,\hat{j} + 13/4\,\hat{k}}$$

Ans

Q No 2:- $\int \frac{4x^2+10x+4}{2x^2+x}\,dx$

Sol:→ $\int \frac{4x^3+10x+4}{2x^2+x}\,dx$

* Add and sub with $-2x^2-9x-4$

$$= \int\left(\frac{4x^3+10x+4-2x^2-9x-4+2x^2+9x+4}{2x^2+x}\right)dx$$

Saparete the fraction.

$$= \int\left(\frac{4x^2+10x+4-2x^2-9x-4}{2x^2+x} + \frac{2x^2+9x+4}{2x^2+x}\right)dx$$

$$= \int\left(\frac{\cancel{2x^2+x}}{\cancel{2x^2+x}} + \frac{2x^2+9x+4}{2x^2+x}\right)dx$$

$$= \int 1\,dx + \int \frac{2x^2+9x+4}{2x^2+x}\,dx$$

$$= x + \int \frac{2x^2+9x+4}{2x^2+x}\,dx$$

* By using partial decompsition

$$= x + \int\left(1+\frac{4}{x}\right)dx$$

2

$$= x + \int 1\,dx + \int \frac{4}{x}\,dx$$

$$= x + x + 4\ln|x| + c$$

$$= 2x + 4\ln x + c$$

Ans

QNo3 (b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Sol:→ $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

let $t = (x)^{\frac{1}{2}}$

$dt = \frac{1}{2\sqrt{x}}\,dx$

$2dt = \frac{dx}{\sqrt{x}}$ put value

$= \int_1^2 2\sin(t)\,dt$

$= 2\int_1^2 \sin(t)\,dt$

$= 2\left(-\cos t\right|_1^2$ put $t=\sqrt{x}$

$= -2\left(\cos\sqrt{x}\right|_1^2$

$= -2\left(\cos\sqrt{2} - \cos\sqrt{1}\right)$ Ans

$= -2\,(0.15 - 0.54)$ ∴ more simplyfy.

$= -2\,(-.39)$

$= 0.78$ Ans

Q.No.3 (a):-

$$\int_0^2 x^2 e^x \, dx$$

Sol:-

$$\int_0^2 x^2 e^x \, dx$$

By using integration by par

$$= x^2 \int_0^2 e^x dx - \int_0^2 \frac{d}{dx} x^2 \int e^x dx \, dx$$

$$= x^2 e^x \Big|_0^2 - \int_0^2 2x e^x \, dx$$

$$= (2^2 e^2 - 0^2 e^0) - 2\left[x \int_0^2 e^x dx - \int_0^2 \frac{d}{dx} x \int e^x$$

$$= 4e^2 - 2\left[x e^x \Big|_0^2 - \int_0^2 e^x dx \right]$$

$$= 4e^2 - 2\left[(2e^2 - 0e^0) - e^x \Big|_0^2 \right]$$

$$= 4e^2 - 2[2e^2 + (e^2 - e^0)$$

$$= \cancel{4e^2} + \cancel{4e^2} + 2e^2$$

$$= \boxed{-2e^2}$$

$$= \cancel{4e^2} - 2\cancel{4e^2} + 2e^2 + 2$$

$$= -2e^2 + 2$$

Q4

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Laplace for $u(x,y,z)$ is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial u}(x^2+y^2+z^2)^{-\frac{1}{2}}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-\frac{3}{2}}(\cancel{2}x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left[-x(x^2+y^2+z^2)^{-\frac{3}{2}}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = \Big\{-\Big[x\left(\frac{-3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-\frac{5}{2}}\cancel{2}x + (x^2+y^2+z^2)^{-\frac{3}{2}}$$

Scanned by TapScanner

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial z}\left(-z(x^2+y^2+z^2)\right)$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-\frac{5}{2}} - (x^2+y^2+z^2)^{-\frac{3}{2}}$$

Now $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

$$3x^2(x^2+y^2+z^2)^{-\frac{5}{2}} - (x^2+y^2+z^2)^{-\frac{3}{2}} + 3y^2(x^2+y^2+z^2)^{-\frac{5}{2}}$$

$$- (x^2+y^2+z^2)^{-\frac{3}{2}} + 3z^2(x^2+y^2+z^2)^{-\frac{5}{2}} - (x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$(x^2+y^2+z^2)^{-\frac{5}{2}}\left[\cancel{3x^2} - \cancel{x^2} - \cancel{y^2} - \cancel{z^2} + \cancel{3y^2} - \cancel{x^2} - \cancel{y^2} - \cancel{z^2} + \cancel{3z^2} - \cancel{x^2} - \cancel{y^2} - \cancel{z^2}\right]$$

$$(x^2+y^2+z^2)(0) = 0$$

Hence $u(x, y, z)$ satisfies solution of laplace solution

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial y} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left[-y(x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y\left(\frac{-3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = \left[3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}(x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial z} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$