Mid Exam Summer

Course Name: Linear Algebra

Submitted By:

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BS (SE-8) Section: A

Submitted To:

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Department of Computer Science,
IQRA National University, Peshawar Pakistan **Linear Algebra Mid Assignment**

Note:

- *If your student ID is e.g. 14589 then ID1 = 1 , ID2 = 4, ID3 = 5 etc*
- *Submission time 21-08-2020 before 6:00 pm (4 Hrs)*
- *Copied papers will both be marked zero*

Question No: 1

Solve the system of equations that corresponds to this augmented Matrix

$$\begin{bmatrix} \mathbf{1} & \mathbf{-3} & \mathbf{4} & \mathbf{-ID2} \\ \mathbf{3} & \mathbf{-7} & \mathbf{7} & \mathbf{-ID4} \\ \mathbf{-4} & \mathbf{6} & \mathbf{-1} & \mathbf{ID3} \end{bmatrix}$$

Question No: 2

a) Find Inverse of a Matrix

$$\begin{bmatrix} \mathbf{ID3} & \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{ID3} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

Question # 2 (a)

Find Inverse of a Matrix

$$\begin{bmatrix} ID3 & -1 & 0 \\ 0 & 1 & ID3 \\ 1 & 1 & 0 \end{bmatrix}$$

Solution:

As my ID is 12280

So, ID3 = 2

putting value in the matrix.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

∴ Taking determi-

$$|A| = \begin{vmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2 \times \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} + 1 \times \begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} + 0 \times \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= 2 \times (1 \times 0 - 2 \times 1) + 1(0 \times 0 - 2 \times 1) + 0(0 \times 1 - 1 \times 1)$$

$$= 2(0-2) + 1(0-2) + 0(0-1)$$

$= 2(-2) + 1(-2) + 0(-1)$

$= -4 - 2 + 0$

$= -6$

Now Taking adj...

$$\text{Adj}(A) = \text{Adj}\begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} +\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix} & +\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} & +\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} \\ +\begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} +(1\times 0 - 2\times 1) & -(0\times 0 - 2\times 1) & +(0\times 1 - 1\times 1 \\ -(-1\times 0 - 0\times 1) & +(2\times 0 - 0\times 1) & -(2\times 1 - (-1)\times \\ +(-1\times 2 - 0\times 1) & -(2\times 2 - 0\times 0) & +(2\times 1 - (-1 \end{bmatrix}$$

$$= \begin{bmatrix} +(0-2) & -(0-2) & +(0-1) \\ -(0+0) & +(0+0) & -(2+1) \\ +(-2+0) & -(4+0) & +(2+0) \end{bmatrix}^t$$

$$= \begin{bmatrix} -2 & 2 & -1 \\ 0 & 0 & -3 \\ -2 & -4 & 2 \end{bmatrix}^{T}$$

$$= \begin{bmatrix} -2 & 0 & -2 \\ 2 & 0 & -4 \\ -1 & -3 & 2 \end{bmatrix}$$

Now taking, $A^{-1} = \frac{1}{|A|} \times Adj(A)$.

$$\left(\frac{1}{-6}\right) \begin{bmatrix} -2 & 0 & -2 \\ 2 & 0 & -4 \\ -1 & -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3333 & 0 & 0.3333 \\ -0.3333 & 0 & 0.6667 \\ 0.1667 & 0.5 & -0.3333 \end{bmatrix}$$

b) Find an echelon form for the below matrix using row operations

$$\begin{bmatrix} \mathbf{1} & \mathbf{ID3} & \mathbf{8} \\ \mathbf{2} & \mathbf{ID4} & \mathbf{-1} \\ \mathbf{-3} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{-ID3} & \mathbf{16} \end{bmatrix}$$

Question # 2 (b)

Find an echelon form for the below matrix using row operations

$$\begin{bmatrix} 1 & ID3 & 8 \\ 2 & ID4 & -1 \\ -3 & 0 & 0 \\ 1 & -ID3 & 16 \end{bmatrix}$$

Solution :-

My ID = 12280

So, ID3 = 2 and ID4 = 8

putting values in the given matrix.

$$= \begin{bmatrix} 1 & 2 & 8 \\ 2 & 8 & -1 \\ -3 & 0 & 0 \\ 1 & -2 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 2 & 8 & -1 \\ 1 & 2 & 8 \\ 1 & -2 & 16 \end{bmatrix}$$ ∴ Interchanging rows. $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 1 & 2 & 8 \\ 1 & -2 & 16 \end{bmatrix}$$ ∴ $R_2 \leftarrow R_2 + 0.6667 \times R_1$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 0 & 2 & 8 \\ 1 & -2 & 16 \end{bmatrix} \quad \therefore R_3 \leftarrow R_3 + 0.3333 \times R_1$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 0 & 2 & 8 \\ 0 & -2 & 16 \end{bmatrix} \quad \therefore R_4 \leftarrow R_4 + 0.3333 \times R_1$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 0 & 0 & 8.25 \\ 0 & -2 & 16 \end{bmatrix} \quad \therefore R_3 \leftarrow R_3 - 0.25 \times R_2$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 0 & 0 & 8.25 \\ 0 & 0 & 15.75 \end{bmatrix} \quad \therefore R_4 \leftarrow R_4 + 0.25 \times R_2$$

$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & -1 \\ 0 & 0 & 15.75 \\ 0 & 0 & 8.25 \end{bmatrix}$$

$\therefore$ Interchanging rows $R_3 \leftrightarrow R_4$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 15.75 \\ 0 & 0 & 8.25 \end{bmatrix} \quad \therefore R_2 \leftarrow R_2 + 0.0635 \times R_3$$

$$= \begin{bmatrix} -3 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 15.75 \\ 0 & 0 & 0 \end{bmatrix} \quad \therefore R_4 \leftarrow R_4 - 0.5238 \times R_3$$

Question No: 3

Find the Eigen values and Eigen vectors of the below Matrix

$$\begin{bmatrix} \mathbf{ID3} & -6 & 2 \\ -6 & \mathbf{ID2} & -4 \\ 2 & -4 & \mathbf{ID4} \end{bmatrix}$$

Question # 3:-

Find the Eigen values and Eigen vectors of the below matrix.

$$\begin{bmatrix} ID3 & -6 & 2 \\ -6 & ID2 & -4 \\ 2 & -4 & ID4 \end{bmatrix}$$

Solution:-

My ID = 12280

So, ID2 = 2 , ID3 = 2 , ID4 = 8

putting values in the matrix.

$$= \begin{bmatrix} 2 & -6 & 2 \\ -6 & 2 & -4 \\ 2 & -4 & 8 \end{bmatrix}$$

$$\therefore \; |A - \lambda I| = 0$$

$$\begin{vmatrix} (2-\lambda) & -6 & 2 \\ -6 & (2-\lambda) & -4 \\ 2 & -4 & (8-\lambda) \end{vmatrix} = 0$$

$\Rightarrow (2-\lambda)\big((2-\lambda)(8-\lambda)-(-4)(-4)\big)-(-6)\big((-6)(8-\lambda)$
$-(-4)\times 2\big)+2\big((-6)(-4)-(2-\lambda)2\big)=0$

$\Rightarrow (2-\lambda)\big((16-10\lambda+\lambda^2)-16\big)+6\big((-48+6\lambda)$
$-(-8)\big)+2\big(24-(4-2\lambda)\big)=0$

$\Rightarrow (2-\lambda)(-10\lambda+\lambda^2)+6(-40+6\lambda)+2(20+2\lambda)=0$

$\Rightarrow (-20\lambda+12\lambda^2-\lambda^3)+(-240+36\lambda)+(40+4\lambda)=0$

$\Rightarrow (-\lambda^3+12\lambda^2+20\lambda-200)=0$

$\Rightarrow -(\lambda^3-12\lambda^2-20\lambda+200)=0$

$\Rightarrow (\lambda^3-12\lambda^2-20\lambda+200)=0$

Roots can be find using newton raphson method

$\Rightarrow$ Newton Raphson method for $x^3-12x^2-20x+200=0$

Here $x^3-12x^2-20x+200=0$.

let $f(x)=x^3-12x^2-20x+200$

$\therefore f'(x)=3x^2-24x-20$

$x_0 = 3$

1st Iteration :-

$f(x_0) = f(3) = 3^3 - 12 \times 3^2 - 20 \times 3 + 200 = 59$

$f'(x_0) = f'(3) = 3(3^2) - 24(3) - 20 = -65$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$x_1 = 3 - 59 / -65$

$\boxed{x_1 = 3.9077}$

2nd Iteration :-

$f(x_1) = f(3.9077) = (3.9077)^3 - 12(3.9077)^2 - 20(3.9077) + 200$
$= 1.7239$

$f'(x_1) = f'(3.9077) = 3(3.9077)^2 - 24(3.9077) - 20 = -67.9744$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$x_2 = 3.9077 - \dfrac{-1.7239}{-67.9744}$

$\boxed{x_2 = -3.8823}$

3rd Iteration :-

$$f(x_2) = f(3.8823) = (3.8823)^3 - 12(3.8823)^2 - 20(3.8823) + 200 = -0.0002$$

$$f(x_2) = f(3.8823) = 3(3.8823)^2 - 24(3.8823) - 20 = -67.9585$$

$$x_3 = x_2 - f(x_2) / f(x_2)$$

$$x_3 = 3.8823 - \frac{-0.0002}{-67.9585}$$

$$\boxed{x_3 = 3.8823}$$

The system associated with the eigenvector value $\lambda = -4.1867$

$$(A + 4.1867I)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -5.1087 \\ 0 & 1 & -5.601 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 - 5.1087x_3 = 0, \; x_2 - 5.601x_3 = 0$

$\Rightarrow x_1 = 5.1087x_3, \; x_2 = 5.601x_3$

$\therefore$ eigenvectors corresponding to the eigenvalue $\lambda = -4.1867$ is

$$v = \begin{bmatrix} 5.1087x_3 \\ 5.601x_3 \\ x_3 \end{bmatrix}$$

let $x_3 = 1$

$$v_1 = \begin{bmatrix} 5.1087 \\ 5.601 \\ 1 \end{bmatrix}$$

2 → Eigenvectors for $\lambda = 3.8823$

$$A - \lambda I = \begin{bmatrix} 2 & -6 & 2 \\ -6 & 2 & -4 \\ 2 & -4 & 8 \end{bmatrix} - 3.8823 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -6 & 2 \\ -6 & 2 & -4 \\ 2 & -4 & 8 \end{bmatrix} - \begin{bmatrix} 3.8823 & 0 & 0 \\ 0 & 3.8823 & 0 \\ 0 & 0 & 3.8823 \end{bmatrix}$$

$$= \begin{bmatrix} -1.8823 & -6 & 2 \\ -6 & -1.8823 & -4 \\ 2 & -4 & 4.1177 \end{bmatrix}$$

Now reducing the matrix.

interchanging rows $R_1 \leftrightarrow R_2$

$$= \begin{bmatrix} -6 & -1.8823 & -4 \\ -1.8823 & -6 & 2 \\ 2 & -4 & 4.1177 \end{bmatrix}$$

$R_1 \leftarrow R_1 \div -6$

$$= \begin{bmatrix} 1 & 0.3137 & 0.6667 \\ -1.8823 & -6 & 2 \\ 2 & -4 & 4.1177 \end{bmatrix}$$

$$\sim R_2 \leftarrow R_2 + 1.8823 \times R_1$$

$$= \begin{bmatrix} 1 & 0.3137 & 0.6667 \\ 0 & -5.4095 & 3.2549 \\ 2 & -4 & 4.1177 \end{bmatrix}$$

$$\sim R_3 \leftarrow R_3 - 2 \times R_1$$

$$= \begin{bmatrix} 1 & 0.3137 & 0.6667 \\ 0 & -5.4095 & 3.2549 \\ 0 & -4.6274 & 2.7843 \end{bmatrix}$$

$$\sim R_2 \leftarrow R_2 \div -5.4095$$

$$= \begin{bmatrix} 1 & 0.3137 & 0.6667 \\ 0 & 1 & -0.6017 \\ 0 & -4.6274 & 2.7843 \end{bmatrix}$$

$$\sim R_1 \leftarrow R_1 - 0.3137 \times R_2$$

$$= \begin{bmatrix} 1 & 0 & 0.8554 \\ 0 & 1 & -0.6017 \\ 0 & -4.6274 & 2.7843 \end{bmatrix}$$

$\therefore R_3 \leftarrow R_3 \div 4.6274 \times R_2$

$$= \begin{bmatrix} 1 & 0 & 0.8554 \\ 0 & 1 & -0.6017 \\ 0 & 0 & 0 \end{bmatrix}$$

The system is associated with eigenvalue $\lambda = 3.8823$

$$(A - 3.8823I)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.8554 \\ 1 & 1 & -0.6017 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\cancel{x_1 + 0.8554x_3 = 0,\ x_2 - 0.6017x_3 = 0}$

$\Rightarrow x_1 + 0.8554x_3 = 0,\ x_2 - 0.6017x_3 = 0$

$\Rightarrow x_1 = -0.8554x_3,\ x_2 = 0.6017x_3$

$\therefore$ eigenvectors correspondings to the eigenvalue $\lambda = 3.8823$ is

$$V = \begin{bmatrix} -0.8554x_3 \\ 0.6017x_3 \\ x_3 \end{bmatrix}$$

let. $x_3 = 1$

$$V_2 = \begin{bmatrix} -0.8554 \\ 0.6017 \\ 1 \end{bmatrix}$$