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Section ; B

Subject ; "Earth-Quake ENGG"

Submitted To ; "Engg; Yaseen M"

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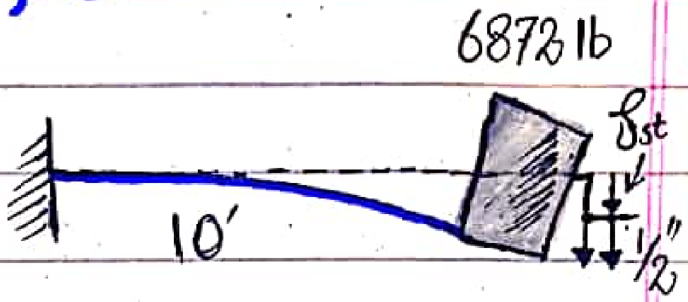
"Problem ; 01"



"Given Data";

$E = 29000$ ksi

$I = 150$ in^4

$\delta_{st} = 6872$ lb

"Solution";

The general E.O.M for "SDOF" system is;

$$Ku + c\dot{u} + m\ddot{u} = p(t)$$

In our case system is undamped ($c=0$) undergoing free vibration ($p(t)=0$). Hence general "E.O.M" become;

$$Ku + m\ddot{u} = 0 \quad \text{——— "A"}$$

Now;

$$K = \frac{3EI}{L^3} = \frac{3 \times 29000 \frac{k}{in^2} \times 150 in^4}{(10 \times 12 in)^3}$$

$$K = 7.55 \; k/in \times 26.455$$

OR $K =$ ~~90625 lb~~/ft $= \boxed{199.735 \; lb/ft}$

OR $K = 90625$ lb/ft

- Now;

$$m = \frac{6872 \text{ lb}\cdot\text{sec}^2}{32.2 \text{ ft}} = 213.41 \text{ slug}$$

- As;

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{199.735}{213.41}} \text{ OR } \sqrt{\frac{90625}{213.41}}$$

$$\boxed{\omega_n = 0.967 \text{ rad/sec}} \text{ OR } \boxed{20.60}$$

- As;

$$T_n = \frac{2\pi}{\omega_n} = \frac{2(3.14)}{0.967} \text{ OR; } \frac{2(3.14)}{20.60}$$

$$\boxed{T_n = 6.494 \text{ sec}} \quad \boxed{0.304 \text{ sec}}$$

- Now by putting values in eq "A".

$$199.735u + 213.41\ddot{u} = 0$$

~~where~~; or; $90625 + 213.41\ddot{u} = 0$

Where; $K = \text{lb/ft}$; $m = \text{lb sec/ft}^2$

- General solution to the "E.O.M" for undamped free vibration is;

$$u(t) = u(0)\cdot\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\cdot\sin(\omega_n t)$$

- But;

$$u(0) = 1/2'' = \frac{1}{2} \times \frac{1}{12} = \frac{1}{24}\, ft$$

and; $u(0) = 0$

$$\rightarrow u(t) = \left(\frac{1}{24}\right) \cdot \cos(0.967) + 0$$

$$= \left(\frac{1}{24}\right)(\cos(0.967)) \text{ OR } \left(\frac{1}{24}\right)\cos(20.60)$$

- Equivalent static force at any time "t" is;

$$f_{(s)}(t) = K \cdot u(t) = \frac{199.735 \cdot \cos(0.967)}{24}$$

$$f_{(s)}(t) = 8.322 \cdot \cos(0.967) \text{ or; } 3776.041 \cos(20.6)$$

- Amplitude of dynamic displacement, u_0 for undamped free vibration is;

$$u_0 = \sqrt{\left[(u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2\right]} = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

$$\boxed{u_0 = \frac{1}{24}\, ft}$$

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- Amplitude of equivalent static force; f_{so}:

$$Ku_o = 199.735 \times \frac{1}{24} \text{ or; } 90625 \times 1/24$$

$$\boxed{Ku_o = 8.322} \qquad \boxed{= 3776.041}$$

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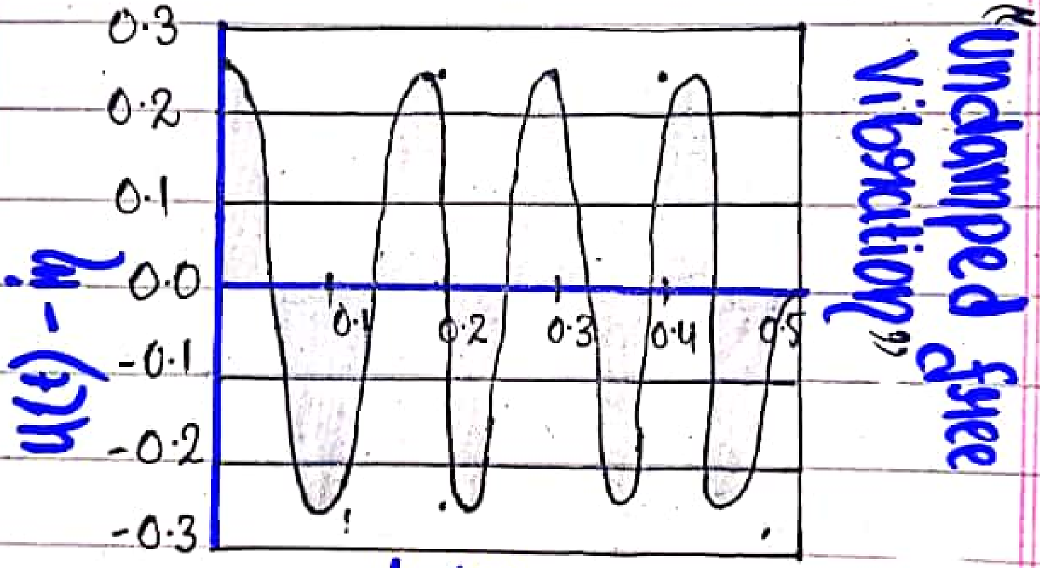
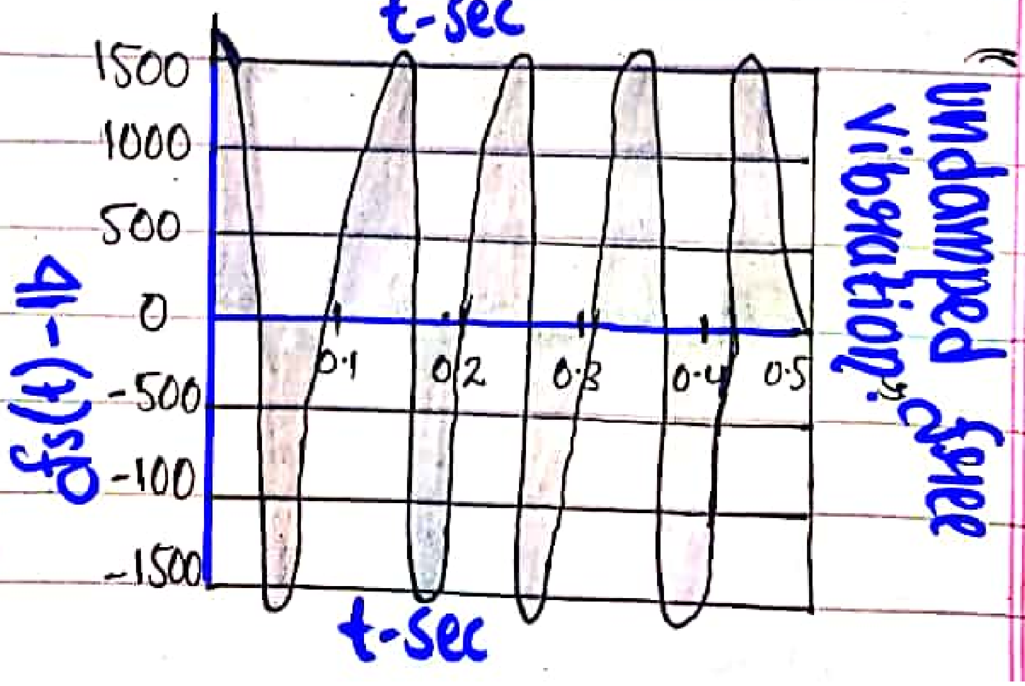




"Problem ; 02"

"Given Data";

$E = 2900\ ksi$

$I = 150\ in^4$

$\delta_{st} = 6872\ lb$

Take; $\zeta = 2.5\%$

"Solution";

E.O.M for Damped free vibration;

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{—— "A"}$$

From problem "no ; 01"

$K = 90625\ lb/ft$

$m = 213.41\ sec^2/ft$

- Now;

$$C = \zeta \times 2m\omega_n$$

$$C = 0.025 \times 2 \times 213.41 \times 20.60$$

$$\boxed{C = 219.81\ lb \cdot sec/ft}$$

- By putting values in eq "A".

$$Ku + C\dot{u} + m\ddot{u} = 0$$

$$90625u + 219.81\dot{u} + 213.41\ddot{u} = 0$$

- Solution to the E.O.M for dammed free vibration is;

$$u(t) = e^{-\zeta\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[u(0)\zeta\omega_n\right] \times \sin(\omega_D t)\right]$$

$$\Rightarrow \omega_D = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{213.41}}$$

$$\omega_D = 20.60 \text{ rad/sec}$$

$$\rightarrow u(t) = e^{-0.025(20.60)t}\left[\tfrac{1}{24}\cos(20.60t) + \frac{1}{20.60}\left[\tfrac{1}{24}(20.60)(0.025)\right]\sin(20.60t)\right]$$

$$u(t) = e^{-0.515t}\left[0.041667 \cdot \cos(20.60t) + 0.0485 \times \right.$$

$$\left. [0.0416(0.025)(20.60) \cdot \sin(20.60t)]\right]$$

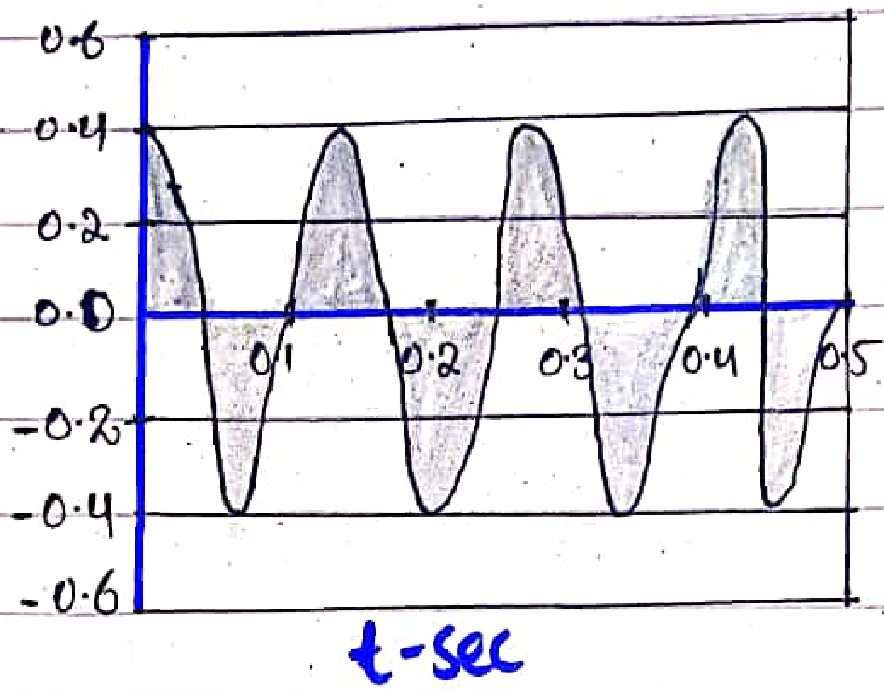
$$= e^{-0.515}\left[0.04166 \cdot \cos(20.60t) + 0.00104 \times \sin(20.60t)\right]$$

$\Rightarrow$

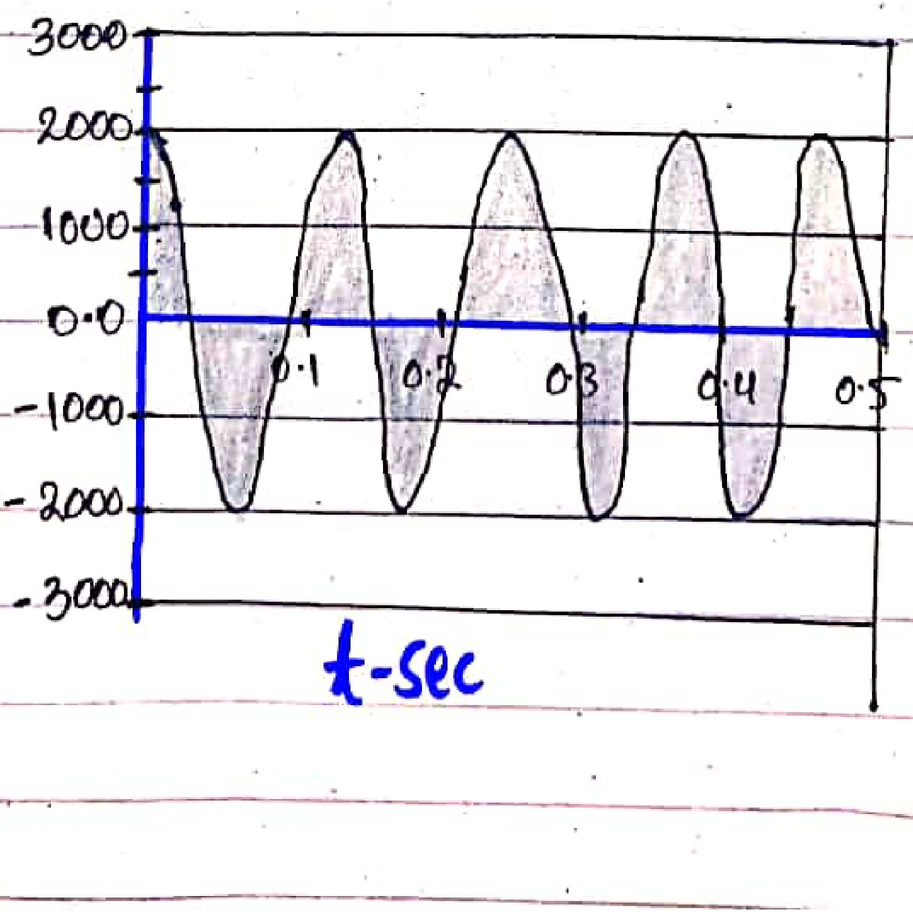
- $f_s(t) = K \cdot u(t) = 90625 \cdot u(t)$

$$f_s(t) = e^{-0.515}\left[3775 \cos(20.60t) + 94.34 \cdot \sin(20.60t)\right]$$

"Damped free vibration";



"Damped free vibration"



"Problem ; 03"

"Given Data";

- Force = 60 Kips
- $U_1 = 6872/1000 = 6.872$ in
- After ; $j = 7$ (cycles)
- Completed = 3.57 sec
- $U_{J+1} = 2.286$ cm $= 0.9$ in
- Ignore the verticle vibration

"Required";

"a" Damping ratios
"b" Natural period of undamped vibration.
"c" Stifness of structures.
"d" Weight of tank.
"e" Damping co-efficient
"f" Number of cycles to reduce the displacement amplitude to 0.5"

Solution;

"a" ζ = Damping ratio = ?

As;

$$\delta = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

By putting values;

$$7 = \frac{1}{2(3.14)\zeta} \ln\left[\frac{6.872}{0.9}\right]$$

$$\zeta(7\times2\times3.14) = 2.032$$

$$\zeta(43.96) = 2.032$$

$$\zeta = 2.032/43.96$$

$$= 0.0462$$

$$\boxed{\zeta = 4.62\,\%}$$

"b" $T_n = ?$

As "Seven" cycles are completed in "3.57" sec

Thus Time required to complete one cycle $= 7/3.57 = 1.96$ sec

$T_D = 1.96$ sec

Now;

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\Rightarrow \frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n}\left(\sqrt{1-\zeta^2}\right)$$

As;

$$T_D = \frac{T_n}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow T_n = T_D\left(\sqrt{1-\zeta^2}\right)$$

$$= 1.96\left(\sqrt{1-(0.0462)^2}\right)$$

$T_n = 1.957$ sec "Natural period of undumped vibration."

"c" Stifness of structure; "K = ?"

As; $K = \frac{F \cdot \cos\theta}{2}$

$K = \frac{60 \cdot \cos(60^\circ)}{2}$ $\left(\begin{matrix} F = 60\,Kips \\ \theta = 60^\circ \end{matrix} \right)$

$= 15\,K/in$

$$K = 18000\,lb/ft$$

"d" "Weight of Tank"; "W = ?"

As;

$\omega_n = \sqrt{K/m} = \sqrt{K/(W/g)} = \sqrt{K \cdot g/W}$

$\Rightarrow \omega_n^2 = K \cdot g/W \Rightarrow \left(W = K \cdot g/\omega_n^2 \right)$

By putting values of $\omega_n = 2\pi/T_n$

$W = K \cdot g / \left(4\pi^2/T_n^2 \right) = K \cdot g \left(\frac{T_n^2}{4\pi^2} \right)$

$W = \frac{18000\,lb}{ft} \cdot \frac{32.2\,ft}{sec^2} \left(\frac{(1.957)^2}{4(3.14)^2} \right)$

$W = 56284.75\,lb$ $= 56.284\,Klb$

"e" "Damping co-efficient"; "C = ?"

It is know that; $\zeta = \frac{C}{2m\omega_n}$

$\Rightarrow C = \zeta(2m\omega_n) = \zeta\left(2m\left(\frac{2\pi}{T_n}\right)\right)$

By putting values;

$C = 0.0462\left(2\left(\frac{56284}{32.2}\right)\left(\frac{2(3.14)}{1.957}\right)\right)$

1.957

$C = 518.286$ lb.sec/ft

"f" No of cycles to reduce displacement altitude from "6.872 in to 0.5in"

$j = ?$

$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$

$= \frac{1}{2(3.14)(0.0462)} \ln\left[\frac{6.872}{0.9}\right]$

$= 7.006$ OR

$j = 7$ cycles