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Subject: Advanced Fluid

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Submitted to: Engr Abdul Waheed Sir

Question no 1

Drag:-

A body which is wholly immersed in a homogeneous fluid may be subjected to two kind of force arising from relative motion between body and fluid these forces are termed as drag and lift. If the force parallel to the motion then it is termed as drag force

There are two Components

1) Pressure Drag (FP)

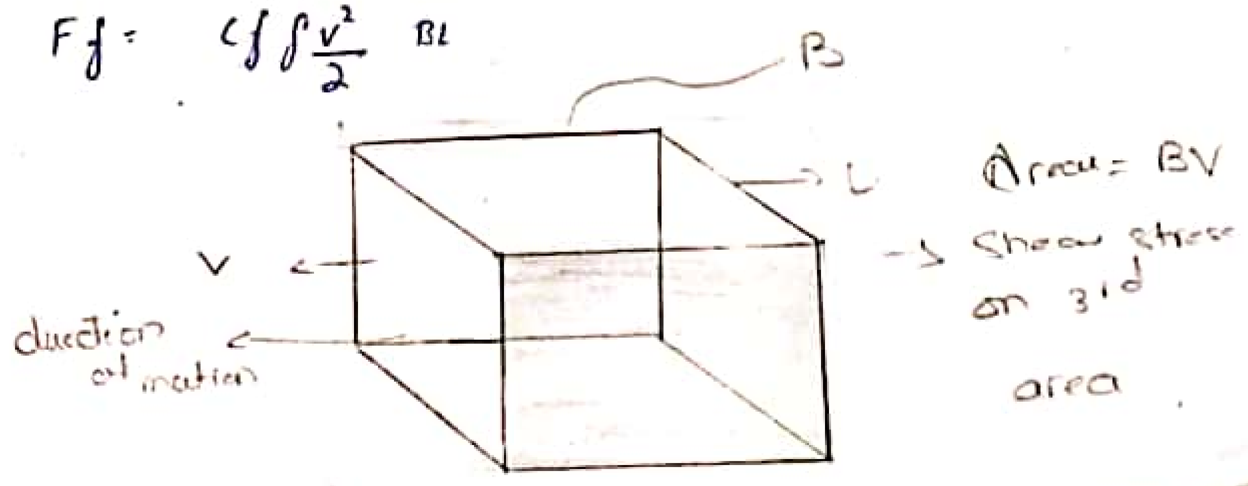
It is equal to integration of Components in direction of motion of all pressure forces exerted on surface of body.

$$FP = C_P \int \frac{v^2}{2} A$$

2) Friction Drag: (Ff)

It is equal to integration of Components of shear stress along surface of body in direction of motion.

$$Ff = C_f \int \rho \frac{v^2}{2} \; BL$$

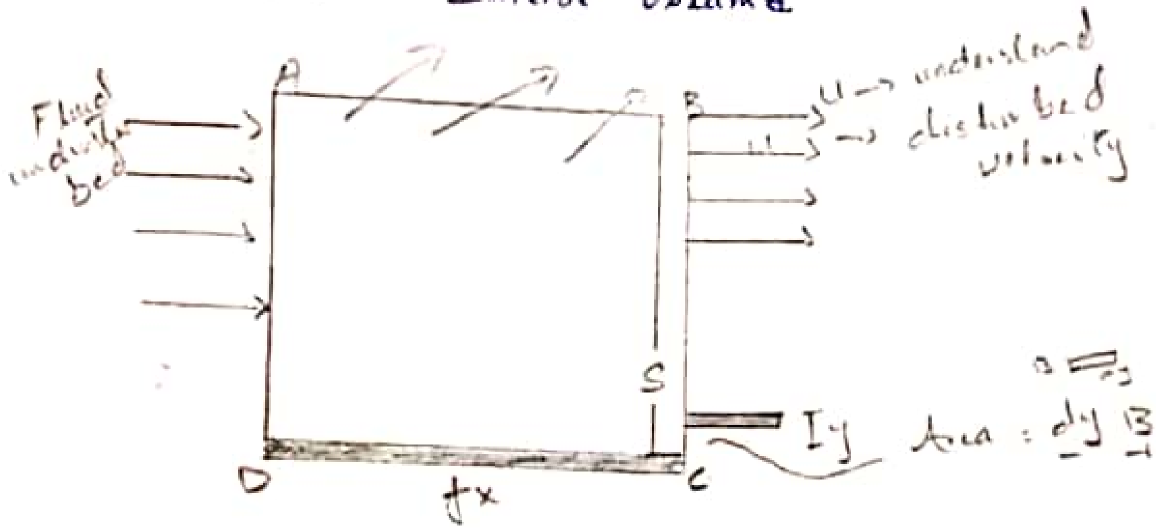


(2)

→ Friction Drag of Boundry layer

Fig shows growth of boundry layer along one site of smooth plate inside the fluid

Now Consider a Control volume



Where δ is thickness of boundry layer and U is Undisturbed velocity

Thus

$-F_x$ = drag = (rate in momentum in x direction leaving through BC + rate of momentum through AB) - rate of momentum entering through DA)

$\Delta P = P_{out} - P_{in}$

Thus according to momentum

$$\Sigma F = \frac{d}{dt}(P) = \frac{d_{mor}}{dt}$$

where
$$\frac{dm}{dt} = \rho Q \quad \text{thus}$$

$$F = \rho Q v$$
or
$$F = \rho A\, v \cdot v$$
$$F = \rho A v^2$$

$$DA \longrightarrow \rho v \,(vBs)$$
$$BC \longrightarrow \rho B \int_0^s v^2\, dy$$
$$AB \longrightarrow \rho v \left(vBs - B\int_0^s v\, dy\right)$$

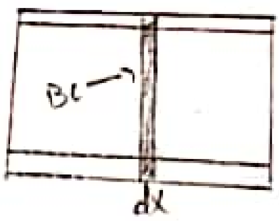
Putting value

$$F_x = \rho B \int_0^s u(U-u)\, dy$$

$$F_x = \rho B U^2 S_\alpha$$

Now to find local wall shear stress

$$\tau_0 = \frac{dF_x}{B \cdot dx - \text{area}}$$

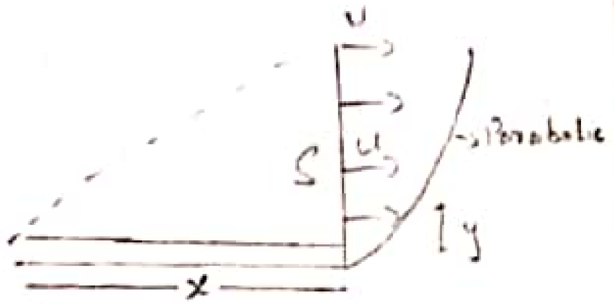


$$F_x = \rho B U^2 S_\alpha$$

$$\tau_0 = \rho U^2 \alpha \frac{ds}{dx}$$ in general equation of shear stress

→ Laminar boundry layer:

$$\frac{u}{V} = f\left(\frac{y}{\delta}\right)$$



Assume

$$\eta = \frac{y}{\delta} \quad \text{or} \quad y = \eta\delta$$

thus $\frac{u}{V} = f(x)$ or $u = uf(\eta)$

In case of Laminar flow

$$\tau_o = \mu\left(\frac{du}{dy}\right)$$

$$= \frac{u}{\delta}\left(\frac{du}{dx}\right) = \frac{\mu U}{\delta}\left[\frac{df}{dx}(x)\right]$$

Solving the equation

$$\tau_o = \frac{\mu U B}{\delta} \longrightarrow (1)$$

As general equation is $\tau_o = \rho u^2 \alpha \frac{d\delta}{dx}$

Equating both equation

$$\frac{\mu U B}{\delta} = \rho v^2 \alpha \frac{d\delta}{dx}$$

or

$$\delta d\delta = \frac{\mu B}{\rho u \alpha} dx$$

Integration the equation

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho u \alpha} x + C$$

Now at $x = 0$ $\delta = 0$ then $C = 0$

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho u \alpha}$$

or

$$\delta = \sqrt{\frac{2\mu B}{\rho u \alpha}}\, x \quad \text{or} \quad \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho u}}$$

xing and $\div$ing by "x"

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho u}} \cdot \frac{x}{\sqrt{x} \cdot \sqrt{x}}$$

where $\alpha = 0.135$
$B = 1.63$
$R_x = \frac{\rho u x}{\mu}$

$$\delta = \frac{4.91}{\sqrt{R_x}}\, x \quad \text{or} \quad \frac{\delta}{x} = \frac{4.91}{\sqrt{R_x}}$$

Now $\tau_0 = \frac{\mu U B}{\delta}$

Thus putting value

$$\boxed{\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}}$$

where R_x is local Reynold number

Now $F_D = B \int_0^L \underbrace{\tau_0\, dx}_{\text{stress}}$

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Putting values

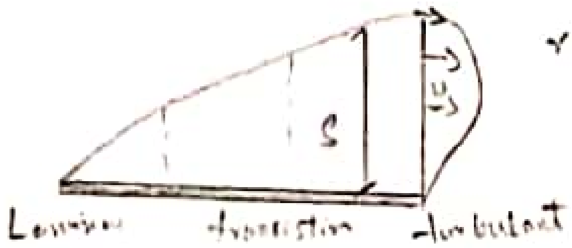
$$F_f = 0.664 B \sqrt{\mu U L U^2}$$

As general equation is

$$F_f = c_f \frac{\rho v^2}{2} B L \longrightarrow \text{equating both equation}$$

$$c_f = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \frac{1.328}{\sqrt{R}}$$

Turbulant boundry Layer



resistance is less
so curve become
straight

fig show that velocity distribution in turbulant boundry layer shows a much steeper gradient near wall and flatter through out remaining layer

The shear stress greater than turbulant than in laminar layer.

As we have

$$\tau_0 = f \frac{\rho v^2}{8}$$

Now we have obtained an approximate relation between v and U by using pipe factor equation of

$$\frac{v}{U_{max}} = \frac{1}{1+1.33\sqrt{f}}$$

Using friction factor of 0.028 from chart which is middle critical value

So $\boxed{U = 1.235v}$

Now we have

$$\tau_o = f\rho\frac{v^2}{8}$$

As we know

$$f = \frac{0.316}{R^{0.25}}$$

thus
$$\tau_o = \frac{0.316}{\left(\frac{Dv}{\nu}\right)^{1/4}} \frac{\rho v^2}{8}$$

where $v = \frac{U}{1.235}$ Thus

$$\tau_o = \frac{0.316}{\left(\frac{D}{\nu}\left(\frac{U}{1.235}\right)\right)^{1/4}} \frac{\rho}{8}\left(\frac{U}{1.235}\right)^2$$

$$D = 2S$$

thus
$$\tau_o = \frac{0.023\rho U^2}{\left(\frac{SU}{\nu}\right)^{1/4}}$$

As we have

$$\tau_o = \rho U^2 \alpha \frac{dS}{dx}$$

Equating both and integration for boundary condition of

$x = 0$, $\delta = 0$

thus

$$\delta = \left(\frac{0.0267}{\alpha}\right)^{4/5} \left(\frac{v}{Ux}\right)^{1/5} x$$

For $\alpha = 0.0972$

$$\boxed{\frac{\delta}{x} = \frac{0.377}{(R_x)^{1/5}}}$$

Putting values in equation

$$\tau_0 = 0.0587\, \rho \frac{U^2}{2} \left(\frac{v}{Ux}\right)^{1/5}$$

Now

$$F_f = B \int_0^L \tau_0 \, dx$$

$$F_f = 0.0735\, \rho \frac{U^2}{2} \left(\frac{v}{UL}\right) BL$$

As

$$F_f = C_f \, \rho \frac{U^2}{2} BL$$

equating both

$$\boxed{C_f = \frac{0.0735}{R^{1/5}}}$$

For $R > 10^7$

$$\boxed{C_f = \frac{0.455}{(\log R)^{2.58}}}$$ Ans

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Question 1

Part b

As Specific energy

$$E = y + \frac{v^2}{2g}$$

The flow Q per unit width b can be expressed as

$$q = \frac{Q}{b}$$

Now average velocity will be

$$v = \frac{Q}{A} = \frac{q b}{b y} = \frac{q}{y}$$

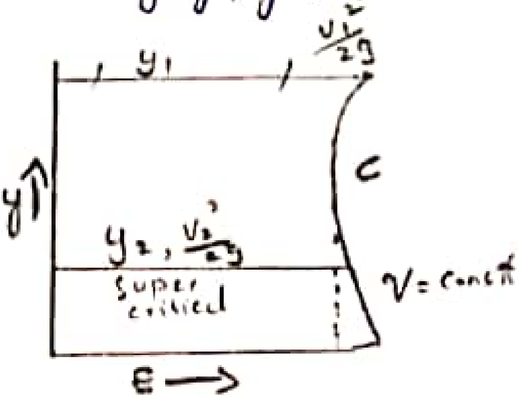
thus

$$E = y + \frac{v^2}{2g} \Rightarrow y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

$$(E - y) = \frac{1}{2g}\left(\frac{q^2}{y^2}\right) \text{ or } (E-y)\,y^2 = \frac{q^2}{2g}$$

Thus plot of E vs y will be parabolic for particular q, there will be two kind of possible values of y, for a given E.

The Equation is cubic with three roots, with third root being negative point c represents dividing point b/w two regime of flow



Thus for given q, & value of E is minimum & Flow at That point is critical depth y_c & velocity at that point is critical velocity V_c

Thus

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

For minimum Specific energy $\frac{dE}{dy} = 0$

Thus

$$\frac{dE}{dy} = 1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right) = 0$$

$$\Rightarrow \frac{q^2}{gy^3} = 1 \Rightarrow q^2 = gy^3$$

$$\frac{q^2}{g} = y^3 \Rightarrow \left(\frac{q^2}{g}\right)^{1/3} = y_{cr}$$

Now

$$q^2 = gy^3$$

&

$$q = vy \Rightarrow v^2 y^2 = gy^3$$

or

$$v^2 = gy_{cr}$$

or

$$v_{cr} = \sqrt{g y_{cr}}$$

Question no 2

Given:

Water flows at rate . $Q = 35\ m^3/s$

Bed Slope, $S_0 = 0.0005$

$n = 0.0219$

Width of bed in Student ID 7772 mm

Required:

Depth of rectangular channel = ?

Critical Depth . $y_c = ?$

Critical velocity, $v_c = ?$

Flow is critical or subcritical = ?

Sol:

Manning Equation

$$Q = \left(\frac{1}{n} R_n^{2/3} S_0^{1/2}\right) A \quad \text{— (1)}$$

Area = 7772 x d

Perameter = d + 7772 + d

$$\text{Hydraulic Radius } R_n = \frac{\text{Area}}{\text{Perameter}}$$

$$= \frac{7772}{2d + 7772}$$

Put values in eq 1

$$Q = \left(\frac{1}{n} R_n^{2/3} S_0^{1/2}\right) A$$

$$3.5 \left(\frac{1}{0.0219} \cdot \left(\frac{7772d}{2d+7772}\right)^{2/3} \times (0.0008)^{1/2} \times 7772d\right.$$

$$\frac{3.5 \times 0.0219}{(0.0008)^{1/2}} = \left(\frac{7772d}{2d+7772}\right)^{2/3} \times 7772d$$

$$2.709 = \sqrt[2/3]{\left(\frac{7772d}{2d+7772}\right)^{2/3}} \times 7772d$$

$$2.709 = \frac{7772d}{2d+7772} \times 7772d$$

$$5.418d + 21.046 = 60.35d$$

$$21.046 = 60.35d - 5.418d$$

$$21.046 = 54.932d$$

$$\frac{21.046}{54.932} = d$$

$$\boxed{d = 0.383m}$$

So the depth of channel is 0.383 m

Now

As q = discharge per unit width

$$q = \frac{Q}{b}$$

$$= \frac{3.5}{7772}$$

$$q = 0.450$$

→ Critical Depth y_{cr}

using equation

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \left(\frac{(0.450)^2}{9.81}\right)^{1/3}$$

$$y_{cr} = 0.274m$$

→ Critical Velocity, V_{cr}

Using equation

$$V_{cr} = g y_{cr}$$

$$V_{cr} = \sqrt{(9.81)(0.274)}$$

$$V_{cr} = 1.63\ m/s$$

$$V = \frac{Q}{A} = \frac{35}{7772 \times 0.383}$$

$$V = 1.176\ m$$

→ $y = 0.383m \quad y_{cr} = 0.274m$

$V_{cr} = 1.63m/s$

As $y > y_{cr}$

and

$V < V_{cr}$

So flow is Subcritical flow.

Question no 3

Given Data

Width of Smooth plate, $B = 200mm$
$= 0.2m$

Length of Smooth plate, $L = 800mm$
$= 0.8m$

Oil with Specific gravity, $S = 0.89$

Undisturbed velocity, $U = 5 m/s$

Kinematic viscosity, $\nu = 0.93 \times 10^{-4} m^2/s$

Required Data

Friction drag on one side of a smooth plate, $F_f = ?$

Sol

Check the flow

As $\nu = 0.93 \times 10^{-4} m^2/s$

$$R = \frac{LU}{\nu} = \frac{(0.8)(5)}{0.93 \times 10^{-4}}$$

$$R = 43010.75 < 500,000$$

thus flow is laminar

Now

$$C_f = \frac{1.328}{\sqrt{R}} \Rightarrow \frac{1.328}{\sqrt{43010.75}}$$

$$C_f = 6.403 \times 10^{-3}$$

$$C_f = 0.0064$$

$$\Rightarrow \quad F_f = C_{ff} \frac{v^2}{2} BL$$

$$= (0.0064)(S_{oil} \times \gamma_{water}) \times \frac{(5)^2}{2} \times (0.2)(0.8)$$

$$= ((0.0064)(0.89 \times 1000) \times \frac{5^2}{2} \times (0.2)(0.8)$$

$$\boxed{11.392\ N}$$

——————X——————X——————