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SECTION: "B"

DEPARTMENT: BE Civil

Subject: Differential equations.

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Q.No1 ⇒ Solve the following objective type questions.

i) The order of matrix A is $m \times p$ and the order of B is $p \times n$. Then the order of matrix AB is?

Ans: ⇒ AB order $= m \times n$

ii) The number of non-zero rows in an Echelon form?

Ans ⇒ As we know that, no. of ~~non-zer~~ ~~rows~~ non-zero rows in an Echelon form is rank of matrix.

iii) If matrix $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is singular matrix then $a = ?$

Ans ⇒ $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$

the determinant of B will be;

$B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix} \Rightarrow |B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$

$= 1 \times a - 4 \times 2 \Rightarrow a - 8 = 0$

$a = 8.$

iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Ans => the determinat of A will be;

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix} = (2i \times (-i)) - (i \times i)$$

$$|A| = -2i^2 - i^2$$

$$|A| = -3i^2$$

$$|A| = -3(-1) \quad \therefore i^2 = -1$$

$$\boxed{\Rightarrow |A| = 3.}$$

v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Ans => The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a scalar matrix.

vi) Solution of $\frac{dy}{dx} + 2xy = y$?

Ans => $\frac{dy}{dx} + 2xy = y$

subtracting $(2xy)$ on b/s.

$$\frac{dy}{dx} + \cancel{2xy} - \cancel{2xy} = y - 2xy$$

$$\frac{dy}{dx} = y - 2xy$$

taking (y) as common here.

$$\frac{dy}{dx} = y(1-2x).$$

multiplying (dx) on b/s.

$$\cancel{dx} \times \frac{dy}{\cancel{dx}} = y(1-2x)\,dx$$

$$dy = y(1-2x)\,dx.$$

Divide (y) on b/s.

$$\frac{1}{y}(dy) = (1-2x)\,dx$$

Taking integration

$$\int \frac{1}{y}.\,dy = \int 1-2x.\,dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + C$$

$$\ln y = x - x^2 + C.$$

$$e^{\ln y} = e^{x-x^2} + C$$

OR.

$$\boxed{e^{\ln y} = e^{x-x^2+c}}$$

vii) The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \quad \text{is}$$

Ans ⇒ The order of this given differential equation is 1, while the degree is 6.

viii) The order and degree of

$$\frac{d^2y}{dx^2} - 4xy = \sin\frac{d^2y}{dx^2} \quad \text{is ?}$$

Ans ⇒ The order of this given differential equation is 2 while the degree does not exist.

ix) The differential equation

$$2\frac{dy}{dx} + x^2y = 2x + 3, \; y(0) = 5 \quad \text{is ?}$$

Ans ⇒ $2\frac{dy}{dx} + x^2y = 2x + 3$

$$\int 2\frac{dy}{\cancel{dx}} = \int (2x + 3 - x^2y)\,dx.$$

$$2y = \frac{2x^2}{2} + 3x - y\frac{x^3}{3} + C$$

$$2y = \frac{2x^2}{2} + 3x - y\frac{x^3}{3} + C.$$

Divide 2 on b/s.

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + C.$$

put $x = 0$ and $y = 5$ ∴ (given)

$$5 = 0 + 0 - 0 + C.$$

$C = 5$. $\Rightarrow$ { So, it is homogenous equation. }

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x.) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Sol:-

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \underset{\sim}{R} \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \quad R_1 - R_2$$

Taking $(a-b)$ common from R_1

$$\underset{\sim}{R} = a-b \begin{vmatrix} 0 & 1 & a+b \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Now $R_2 - R_3$

$$= \underset{\sim}{R} \quad (a-b) \begin{vmatrix} 0 & 1 & a+b \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

taking $(b-c)$ common from R_2.

$$= (a-b)(b-c) \begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Expanding along C_1.

$$= (a-b)(b-c) \left[0 - 0 + 1 \begin{vmatrix} 1 & a+b \\ 1 & b+c \end{vmatrix} \right]$$

$$= (a-b)(b-c) \left[1 (\cancel{b} + c - a - \cancel{b}) \right]$$

$$= (a-b)(b-c)(c-a).$$

Q.2
i) Express the determinant.

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

As the product of factors which are linear in a, b, c ?

Solution:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1.

$$= a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a\{b^2\times c^3 - b^3\times c^2\} - b(a^2\times c^3 - a^3\times c^2) + c(a^2\times b^3 - a^3\times b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2b^3c - a^3b^2c$$

taking common as (a, b, c)

$$= abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$= abc[bc(c-b) - ac(c-a) + ab(b-a)]$$

This is the required answer.

Question : 02 (ii) :

Find the Eigen value:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

=> Solution:-

Let;

$$A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$= \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

Expand By R_1

$$= (2-\lambda)\begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix} - (-1)\begin{bmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

$$+ (-1)\begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix} + 0 = 0 \longrightarrow \text{A}$$

$\Rightarrow$ Now,

$$\begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix}$$

Expand By R_1

$$= 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + (-1)\begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\{(3-\lambda)(2-\lambda) - (-1 \times (-1))\} (+1)\{(-1)(2-\lambda) - (-1 \times -1)\}$$

$$(-1)\{(-1 \times -1) - (-1 \times (3 \times \lambda))\}$$

$$= (3-\lambda)(6 - 5\lambda + \lambda^2 - 1) + (-2 + \lambda - 1) - (1 + 3 - \lambda)$$

$$= (3-\lambda)(5 - 5\lambda + \lambda^2) + (-3 + \lambda) - (4 - \lambda)$$

$= 15 - 15\lambda + 3\lambda^2 - 5\lambda + 5\lambda^2 - \lambda^3 - 3 + \lambda - 4 + \lambda$

$= -\lambda^3 + 8\lambda^2 - 18\lambda + 8$ ——— (i)

$\Rightarrow$ Now,

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

Expand by R_1

$$= -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ 0 & 2-\lambda \end{vmatrix} + (-1)\begin{vmatrix} -1 & 3-\lambda \\ 0 & -1 \end{vmatrix}$$

$= (-1)\{(3-\lambda)(2-\lambda) - (-1\times -1)\} + \{(-1)(2-\lambda) - 0\}$

$- \{(-1\times -1) - 0\}$

$= (-1)(6 - 5\lambda + \lambda^2 - 1) + (-2 + \lambda) - 1$

$= (-1)(6 - 5\lambda + \lambda^2 - 1) - 2 + \lambda - 1$

$= (-1)(5 - 5\lambda + \lambda^2) - 3 + \lambda$

$= -5 + 5\lambda - \lambda^2 + 1 - 3 + \lambda$

$= -\lambda^2 + 6\lambda - 8$ ——— (ii)

Now,

$$\begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

Expand By C_1:

$$= -1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$= -1 \{ (-1 \times (2-\lambda)) - (-1 \times -1) \} + 1 \{ (3-\lambda)(2-\lambda) - (-1 \times -1) \}$$

$$= (-1) (-2 + \lambda - 1) + (6 - 5\lambda + \lambda^2 - 1)$$

$$= -1 (-2 + \lambda - 1) + (5 - 5\lambda + \lambda^2)$$

$$= (-1) (-3 + \lambda) + 5 - 5\lambda + \lambda^2$$

$$= 3 - \lambda + 5 - 5\lambda + \lambda^2$$

$$= \lambda^2 - 6\lambda + 8 \quad \text{———} \quad (iii)$$

Now putting these three equations i, ii and iii in eq (A)

$$\Rightarrow (2-\lambda)(-\lambda^3 + 8\lambda^2 - 18\lambda + 8) + (-\lambda^2 + 6\lambda - 8)$$
$$- (\lambda^2 - 6\lambda + 8) = 0$$

$$\Rightarrow -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda - \lambda^2 + 6\lambda - 8 - \lambda^2 + 6\lambda - 8 = 0$$

$$\Rightarrow \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 18\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda - 8\lambda + 6\lambda + 6\lambda + 16 - 8 - 8 = 0$$

$$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$$

$\Rightarrow$ By Synthetic Division.

	1	-10	32	-32
2		2	-16	+32
	1	-8	+16	0

$$(\lambda-2)(\lambda^3 - 8\lambda^2 + 16\lambda) = 0$$

$$(\lambda-2)\lambda(\lambda^2 - 8\lambda + 16) = 0$$

So,

$\lambda = 0$, $\lambda - 2 = 0$, $\lambda^2 - 8\lambda + 16 = 0$

$\lambda = 2$

$\Rightarrow \lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\Rightarrow \lambda(\lambda-4) - 4(\lambda-4) = 0$

~~$\Rightarrow \lambda - 4 = 0$~~

$\Rightarrow (\lambda-4)(\lambda-4) = 0$

$\Rightarrow \sqrt{(\lambda-4)^2} = \sqrt{0}$

$\boxed{\lambda = 4}$

Hence,

$\lambda = 0, \lambda = 2\ \lambda = 4$

Question : 03 :-

The rate of change in the form of differential equation is given by $(x^2+3y^2)dx - 2xy\,dy = 0$. Find the general solution at $x=2$ & $y=6$

Solution:-

Given differential equation =>

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$(x^2+3y^2)dx = 2xy\,dy = 0.$$

Now,

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{x}{2y} + \frac{3y}{2x}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{y} + \frac{3y}{x}\right) \text{ — (A)} \quad \left\{ \text{taking common as "1/2"} \right.$$

=> comparing eq Ⓐ with $\frac{dy}{dx} = g(y/x)$

eq. Ⓐ is homogenous equation of degree 1.

Put $\boxed{y/x = v}$ or $\boxed{y = vx}$

differentiate w.r.t x.

$$\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$$

put above eq in eq Ⓐ

$$v + x.\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

$$2v + 2x.\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x.\frac{dv}{dx} = \frac{1}{v} + 3v - 2v.$$

$$2x.\frac{d\cancel{x}}{dx} = \frac{1}{v} + v.$$

$$2x\frac{dv}{dx} = \frac{1 + v^2}{v}$$

$$\frac{2v}{1+v^2}dv = \frac{1}{x}dx$$

Taking integration:

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$\cancel{\ln}(1+v^2) = \cancel{\ln}(xc)$$

$$1+v^2 = xc \quad \text{———} \quad B$$

putting $v = y/x$ in eq B

$$1 + \left(\frac{y}{x}\right)^2 = xc.$$

$$1 + y^2/x^2 = xc.$$

$$\frac{x^2+y^2}{x^2} = xc.$$

$$x^2 + y^2 = x^3 c. \quad \text{———} \quad C$$

Given that :- initial value:

$x = 2,\ y = 6$

$$(2)^2 + (6)^2 = (2)^3 c.$$

$$4 + 36 = 8c.$$

$$\boxed{c = 5}.$$

putting $c = 5$ in eq (c)

$$x^2 + y^2 = x^3 \times 5.$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x - 1).$$

Taking under root on b/s.

$$\sqrt{(y)^2} = \sqrt{x^2(5x-1)}$$

$$y = \pm x\sqrt{5x-1}.$$

Answer:-

$$\boxed{\begin{array}{l} y = +x\sqrt{5x-1} \\ y = -x\sqrt{5x-1} \end{array}}$$